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IN MEMORIAM: SOLOMON FEFERMAN (1928–2016)

§**1. Sketch of a remarkable life.** Solomon Feferman, Patrick Suppes Family Professor of Humanities and Science and Professor of Mathematics and Philosophy, *Emeritus*, at Stanford University and a former President of the Association for Symbolic Logic, died at his home in Stanford on July 26, 2016. Only three months earlier, he had traveled to New York to participate in a symposium at Columbia University organized by the *Journal of Philosophy* honoring Charles Parsons. His paper (published as [2016]) was as lucid as ever, but he had difficulty maintaining his balance, and in some of his movements on the Columbia campus he used a wheelchair. On his return home, he was diagnosed as having suffered a mild stroke. He went into treatment with a good prognosis, but recovery did not come.

Feferman was born in the Bronx borough of New York City on December 13, 1928, of immigrant working-class parents. In 1938, when he was 9, the family moved to Los Angeles. He was an undergraduate at the California Institute of Technology. Originally, he planned to study physics, but he concluded that he did not have talent for that subject and switched to mathematics. After his graduation in 1948, he entered the graduate program in mathematics at the University of California, Berkeley. Stimulated by a seminar of Alfred Tarski, his interest turned to mathematical logic, and he began a dissertation under Tarski. The latter suggested two thesis projects to him, namely, (i) to prove a representation theorem for locally finite cylindric algebras and (ii) to provide a decision procedure for the theory of ordinals under addition. Feferman established the representation theorem with a proof Tarski judged not to be sufficiently algebraic; he also reduced the decision problem to that for the weak second-order theory of ordinals under the less-than relation, but did not solve the latter problem. Feferman thought that these results together "would be satisfactory for a thesis", but Tarski refused to accept it.

In 1953 he was drafted into the US Army and served for two years. During those two years he kept up his logical studies by reading Kleene's *Introduction to Metamathematics*. The interruption of his official graduate studies turned out to lead to a new topic for his thesis, away from the Tarskian suggestions. "Out of the blue", he received an invitation from Alonzo Church to review for the *Journal of Symbolic Logic* an article [1951] by Hao Wang on the arithmetization of the completeness theorem for classical first-order logic. Feferman noticed in his review [1955] that Wang's proof could be considerably simplified. More importantly, he observed that the statement of the theorem contained an ambiguity, since the arithmetic definition of the

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consistency statement for an infinite recursive set T of sentences can be given in nonequivalent ways; it is not canonical in any sense. Thus, the question arose what conditions have to be imposed on its arithmetic definition in order to obtain a precise version not only of Wang's theorem, but also of Gödel's second incompleteness theorem for arbitrary recursive theories T. Feferman remarked in his autobiography [2017, p. 10], "my work on that review led me directly down the path to my dissertation".

That path and Leon Henkin's role in it are described in detail in [2014]. While the thesis work was still not yet complete, Feferman was appointed as instructor in mathematics and philosophy at Stanford University for the academic year 1956–1957—an appointment that was subsequently extended for a second year. Except for sabbatical leaves and a year as visitor at MIT, Stanford remained Feferman's academic home: from 1958 through 1962 as Assistant Professor and from 1962 through 1968 as Associate Professor. He was promoted to full Professor in 1968 and retained that position until his retirement in 2004, receiving the Patrick Suppes Family Chair in 1993. Feferman served as Chair of the Mathematics Department from 1985 through 1992. The period of his chairmanship was "interrupted" with two sabbatical years: 1986–1987 as a Guggenheim Fellow at the Stanford Center for Advanced Study in the Behavioral Sciences and 1989–1990 as a Fellow at the Stanford Humanities Center. He was President of the Association for Symbolic Logic from 1980 through 1982. In 1990 he was elected Fellow of the American Academy of Arts and Sciences and in 2003 he was awarded the Rolf Schock Prize in Logic and Philosophy.

Feferman truly joined logic in its mathematical guise with philosophy; he remarked in [2008], responding to a biographical question:

> I'm a philosopher by temperament but not by training, and a philosopher of logic and mathematics in part . . . by accidents of study and career. Yet it seems to me that if I was destined to be anything it was to be a logician primarily motivated by philosophical concerns.

We are going to discuss aspects of Feferman's work that are directly connected to both mathematical logic and the philosophy of mathematics, but describe also his deep involvement with the history of modern logic.

§**2. Mathematical logic.** Feferman's work touched *all* areas of mathematical logic, though he is best known as a proof and recursion theorist. His early work with Robert Vaught [1959] is still very important in model theory. When Cohen was working on the independence of the axiom of choice and the continuum hypothesis, he consulted very frequently with Feferman. Having been introduced in that way to the new technique of forcing, Feferman was one of the first to use forcing and obtain significant set theoretic results, published in [1965]. However, as we indicated already, we focus on work that is directly connected to foundational programs or broadly motivated by philosophical considerations.

2.1. Progressions of theories. The work in his dissertation on the arithmetization of metamathematics was published in [1960] and turned out to be extremely useful for a fascinating project that was at first a natural continuation and then a dramatic expansion of the work of Turing [1939]. Turing's *ordinal logics* became *progressions of first-order theories*, i.e., sequences of formal theories indexed along paths in the system **O** of constructive ordinals that had been developed by Church and Kleene. Feferman reproved Turing's completeness result for $\prod_1^0$-statements in progressions along (short) paths in **O**. These progressions were based, as the ordinal logics had been, on the iteration of consistency statements. Iterating the local reflection principle along any path in or through **O** did not yield $\prod_2^0$ completeness; however, iterating the global or uniform reflection principle along suitable paths through **O** yielded a remarkable result: completeness for all arithmetic sentences. This work was published as [1962].

2.2. Autonomous progressions and predicativity. Already in 1961, Feferman had begun to take up a suggestion of Kreisel's on how to formulate *autonomy* or "boot-strap" conditions when iterating theories along transfinite ordinals. The rough idea was that an ordinal could be used in the indexing only when it had been shown to be an ordinal at an earlier stage. That led to Feferman's work on *autonomous ramified progressions*, where systems of ramified second-order number theory were considered. This allowed him in [1964], simultaneously with Schütte, to characterize *predicative analysis* by a progression of length Γ_0, the Feferman-Schütte ordinal. The connection to the logical-philosophical ideas of Poincaré and Russell, but also Weyl, was quite explicit. The complementary mathematical task was to explore which parts of analysis could actually be carried through in a predicative way. That task was taken on already at this early stage and was continued throughout Feferman's career; perhaps most distinctively in [1988]. Feferman noted that the mathematics applied in science could be developed in predicative theories, in fact, conservative extensions of PA. That led him to reject indispensability arguments for set theory. See [1993].

2.3. Subsystems of analysis and inductive definitions. Doing mathematics in ramified theories is difficult, as witnessed by the introduction of the axiom of reducibility in Whitehead and Russell's *Principia Mathematica*. So Feferman went on to characterize predicative mathematics by autonomous unramified progressions and, in addition, he connected segments of the progressions to subsystems of analysis, i.e., subsystems of second-order arithmetic with restricted comprehension or choice principles. That fit well with an emerging interest in the proof theoretic investigation of impredicative theories, in the tradition of Hilbert's program. That, of course, was no longer carried out from a finitist standpoint. The predicative systems, in their ramified form, are easily reducible to their intuitionist versions. But for the stronger theories with impredicative principles, there were no obvious routes to be taken.[1]

[1]Spector's functional interpretation of full classical analysis generated deep interest in obtaining constructive consistency proofs for stronger systems. The Stanford Seminar in 1963, led by Kreisel, was instrumental in analyzing Spector's proof and recognizing it as *not* providing a "constructive" interpretation of analysis.

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The work of Friedman [1970] and Feferman [1970] seemed to open a way by relating impredicative subsystems of analysis to theories of generalized inductive definitions. The hope that the latter theories might be reducible to constructive theories was realized in [1981]. Feferman presented in [1988a] his foundational perspective on such reductions, which were seen as part of a generalized Hilbert Program. For a substantial part of these proof theoretic investigations, one needed strong systems of ordinal notation: Feferman contributed immensely to that work.

2.4. Explicit mathematics. Formalizing parts of classical mathematics in predicative subsystems of analysis was a direct answer to the question, how much of mathematical practice can be obtained on a predicative basis? For the systems of explicit mathematics the order of influence between "theory" and "practice" was exactly reversed. Feferman had been deeply impressed by the novel, informal development of constructive analysis in Bishop [1967] and began to search for an axiomatic foundation that would capture the principles underlying Bishop's mathematics. An initial formulation was given in [1975]. At the time, the programmatic goal was to explain, "how it [Bishop's mathematics] managed to look so much like classical analysis in practice while admitting a constructive interpretation" [2016a, p. 272]. The basic answer appealed to the insight that mathematical "objects" (from numbers through functions to classes) are given by explicit presentations. This work has been expanded to a full-fledged foundational perspective, the "operational" one that is concisely and comprehensively exposed in [2016a].

§**3**. **Philosophical reflections.** Although Feferman was first of all a mathematical logician, he said of himself as we have noted above that he was "a philosopher by temperament"—a philosopher of logic and mathematics. One can find philosophical motivation in much of his mathematical work even early in his career, but he did not express directly philosophical views in these writings. The papers in his collection *In the Light of Logic* [1998], first published between 1979 and the early 90s, certainly have a philosophical motivation, but the work on foundations presented and discussed there is always essentially mathematical. However, they display features that belong to his philosophical stance: distance from set theory, sympathy for predicative methods and modest extensions of them, and preference for axiomatically more economical methods, as expressed in the slogan "A little bit goes a long way". They show both wide knowledge and catholic appreciation of the different kinds of foundational work and the motives for them.

Feferman himself described his paper "Mathematics as objective subjectivity", written for a symposium at Columbia University in December 1977, as his "first foray into philosophy" ([2016], p. 234). The article was never published, but in some of his later articles one can find its main ideas, for example, his anti-platonism. It is not obvious on the surface what he understands by "platonism", but he clearly thought it is presupposed by set theory

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and even classical second-order arithmetic. The "conceptual structuralism" he articulated later is also foreshadowed in "Mathematics as objective subjectivity". What is distinctive is spelled out in the first of his ten theses for conceptual structuralism:

> The basic objects of mathematical thought exist only as mental conceptions, though the source of these conceptions lie in everyday experience in manifold ways, in the process of counting, ordering, matching, combining, separating, and locating in space and time. ([2009], p. 170)

He does not say that mathematical *objects* "exist only as mental conceptions" but says that the basic conceptions "are not of objects in isolation but of structures" (ibid.). Hence he called his view conceptual structuralism. He describes the basic conceptions as of "relatively simple ideal-world pictures" ([2009], p. 171), which would suggest that they are imagined. Nevertheless, mathematics is objective, because of its stability and coherence under repeated communication, where many individuals, often working independently of one another and in some cases over a long period of time, are involved. He compares the objectivity of mathematics to that of concepts that originate in social life, such as marriage and property (ibid.).[2]

A point of agreement with many researchers in the foundations of mathematics is that conceptions (in particular, of basic structures such as the number systems and the universe of sets) differ in clarity, and that the ones higher in hierarchies like that of sets are less clear. Although Feferman's reserve about set theory was motivated by his mathematical experience and preferences, he clearly thought that for higher set theory (or even third-order number theory) objects could not take up the slack left by the lesser clarity of concepts. That was one important aspect of his anti-platonism. Because of the greater clarity of the concept of natural number, he was confident that statements in the language of first-order number theory have determinate truth-values but doubted this already for second-order arithmetic. He undertook to give actual arguments for the claim that the continuum hypothesis does not have a determinate truth-value; see [forthcoming]. His skepticism about set theory did not mean, however, that he questioned the consistency of set theory, at least not at the level of ZFC.

§**4**. **Historical illuminations.** At a number of occasions, Feferman brought to his mathematical investigations a genuine historical component, for example, reflections on Poincaré and Weyl in the context of predicative analysis. There were, however, two substantial efforts of an almost purely historical character, his role as chief editor of the five volumes of Gödel's *Collected Works* and as co-author, with his wife, Anita Burdman Feferman, of *Alfred Tarski—Life and Logic*. We cannot do better than Elliott Sober who remarked about the book: "The Fefermans provide a richly textured account of the cultural, intellectual, and political worlds in which Tarski

[2]He quotes Searle [1995] here and elsewhere.

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lived and draw highly individualized portraits of the many people who figured in Tarski's life and career. The work that made Tarski one of logic's giants is lucidly explained in a series of compact interludes. A wonderful book on many levels."

During Feferman's presidency of the ASL, the question of preparing an edition of Gödel's collected works was discussed. Feferman proposed that Jean van Heijenoort or the historian of mathematics Gregory Moore lead the project, but they declined. Feferman then stepped up and was chief editor for what turned out to be a twenty-year project.[3] The initial board consisted of Feferman, John W. Dawson, Jr., van Heijenoort, Moore, Stephen Kleene, and Robert Solovay. As volume II was completed, Moore withdrew and van Heijenoort died. They were replaced by Warren Goldfarb and Charles Parsons. For the last volumes of correspondence, Solovay was replaced by Wilfried Sieg, and Dawson became co-editor in chief. Feferman's wide-ranging grasp of logic and experience with grant applications made him the natural leader. He did not seek to dominate or interfere with the work of others, but insisted on the highest level of scholarship. So it was very much a collective effort that led to an exemplary presentation of Gödel's published papers, a highly informative selection of unpublished essays, and the correspondence with, mostly, logical colleagues.

§**5**. **Professional impact and personal inspiration.** Feferman had seventeen Ph.D. students. Among them were Jon Barwise, Jeffery Zucker, Wilfried Sieg, Carolyn Talcott, Ian A. Mason, Thomas Hofweber, Paolo Mancosu, Gianluigi Bellin, and Ulrik Buchholtz. He inspired other young researchers early in their careers, with the result of collaboration over a long period. Wilfried Buchholz and Wolfram Pohlers were led to the proof-theoretic analysis of generalized inductive definitions and were co-authors of Buchholz et al. [1981]. Gerhard Jäger and Thomas Strahm, attracted by his operational perspective, became co-authors of *Foundations of Explicit Mathematics*, which we hope they will see to completion. Jäger and Sieg [2017], also involving collaboration with Feferman, will remain incomplete: His autobiography ends with 1981, and his replies to the many contributed papers remained unwritten.

Stanford was from early in Feferman's career a major center of logic, especially proof theory. Significant colleagues in earlier years were Paul Cohen, Georg Kreisel, Dana Scott, and William Tait, in later years Jon Barwise, Grigory Mints, and Johan van Benthem. Others held short-term appointments or were there as visitors.

Customarily, in notices of this sort in the Bulletin, marriage and family are not mentioned. We must make an exception of Feferman's marriage to Anita Burdman, remarkable for both its closeness and duration. They were married in 1948, and the marriage endured until her death in 2015. Colleagues who got to know Sol invariably got to know Anita. Their older daughter, the artist Rachel Feferman, died in 2010.

[3] See Feferman [2005, pp. 133–134].

Solomon Feferman is survived by his daughter Julie Feferman-Perez and two grandchildren.

We are still mourning the loss of a great logician, a thoughtful scholar, a man of integrity, and a dear friend.

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