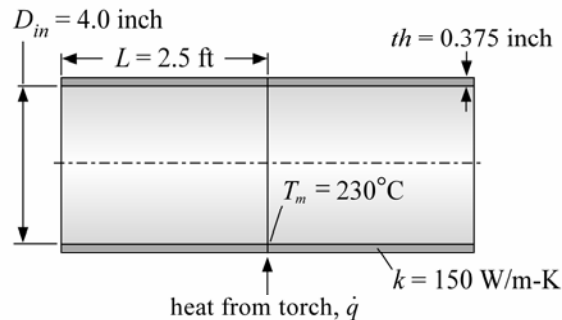


EXAMPLE 1.6-1: Soldering Tubes

Two large pipes must be soldered together using a propane torch, as shown in Figure 1.



$$T_{\infty} = 20^{\circ}\text{C}, \bar{h} = 20 \text{ W/m}^2\text{-K}$$

Figure 1: Two bare pipes being soldered together

Each of the two pipes is $L = 2.5$ ft long with inner diameter $D_{in} = 4.0$ inches and a thickness $th = 0.375$ inch. The pipe material has conductivity $k = 150$ W/m-K. The surrounding air is at $T_{\infty} = 20^{\circ}\text{C}$ and the heat transfer coefficient between the external surface of the pipe and the air is $\bar{h} = 20$ W/m²-K. Assume that convection from the internal surface of the pipe can be neglected.

- a.) The temperature of the interface between the two pipes must be elevated to $T_m = 230^{\circ}\text{C}$ in order to melt the solder; estimate the heat transfer rate, \dot{q} , that must be applied by the propane torch in order to accomplish this.

This problem is solved using EES. The initial section of the code provides the stated inputs (converted to SI units).

```
"EXAMPLE 1.6-1: Soldering Tubes"
$UnitSystem SI MASS DEG PA C J
$Tabstops 0.2 0.4 0.6 0.8 3.5
```

```
"Inputs"
```

```
D_in=4.0 [inch]*convert(inch,m)           "Inner diameter"
th=0.375 [inch]*convert(inch,m)          "Pipe thickness"
k = 150 [W/m-K]                            "Conductivity"
h_bar = 20 [W/m^2-K]                       "Heat transfer coefficient"
T_infinity=converttemp(C,K,20 [C])        "Air temperature"
L=2.5 [ft]*convert(ft,m)                  "Pipe length"
T_m=converttemp(C,K,230 [C])              "Melt temperature"
```

The two pipes can be treated as constant cross sectional area fins; the solutions obtained in Section 1.6 are valid provided that the Biot number characterizing the temperature gradient within the pipe in the radial direction is sufficiently small:

$$Bi = \frac{\bar{h} th}{k}$$

$$Bi = h_{\text{bar}} \cdot th / k$$

"Biot number"

The Biot number is 0.0013, which is much less than unity. The cross-sectional area for conduction (A_c) is:

$$A_c = \frac{\pi}{4} \left[(D_{in} + 2th)^2 - D_{in}^2 \right]$$

and the perimeter exposed to air (per) is:

$$per = \pi (D_{in} + 2th)$$

Notice that the internal surface of the pipe (which is assumed to be adiabatic) is not included in the perimeter. The ratio of the area of the exposed ends of the pipe to the external surface area (AR_{tip}) is calculated according to:

$$AR_{tip} = \frac{A_c}{per L}$$

$$\begin{aligned} A_c &= \pi \cdot ((D_{in} + 2th)^2 - D_{in}^2) / 4 \\ per &= \pi \cdot (D_{in} + 2th) \\ AR_{tip} &= A_c / (per \cdot L) \end{aligned}$$

"area for conduction"
"perimeter"
"area ratio"

The area ratio is 0.012 and therefore according to Figure 1-38 the adiabatic tip fin solution can be used with no loss of accuracy. The fin constant (mL) is:

$$mL = \sqrt{\frac{per \bar{h}}{k A_c}} L$$

and the fin efficiency (η_{fin}) is:

$$\eta_{fin} = \frac{\tanh(mL)}{mL}$$

Therefore, the resistance of each fin (R_{fin}) is:

$$R_{fin} = \frac{1}{\eta_{fin} \bar{h} p L}$$

$$\begin{aligned} mL &= \sqrt{(h_{\text{bar}} \cdot per / (k \cdot A_c))} \cdot L \\ \eta_{fin} &= \tanh(mL) / mL \\ R_{fin} &= 1 / (\eta_{fin} \cdot h_{\text{bar}} \cdot per \cdot L) \end{aligned}$$

"fin parameter"
"fin efficiency"
"fin resistance"

The problem can be represented by the resistance network shown in Figure 2; the two pipes can be represented by two resistors connecting the interface to the air and the heat input from the propane torch enters at the interface. In order for the solder to melt, the interface temperature must reach T_m .

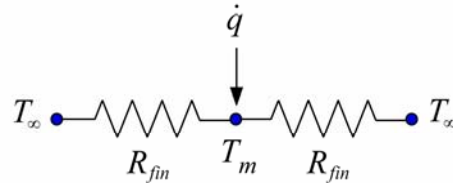


Figure 2: Resistance network associated with soldering two pipes.

The heat transfer required from the torch is therefore:

$$\dot{q} = 2 \frac{(T_m - T_\infty)}{R_{fin}}$$

$q_dot=2*(T_m-T_infinity)/R_fin$

"required torch heat transfer rate"

The factor 2 appears because there are two pipes, each of which acts as a fin. The EES solution indicates that the propane torch must provide at least 812 W to accomplish this job.

b.) Unfortunately, the propane torch cannot provide 812 W and therefore it is not possible to melt the solder. Therefore, you decide to place insulating sleeves over the pipes adjacent to the soldering torch, as shown in Figure 3.

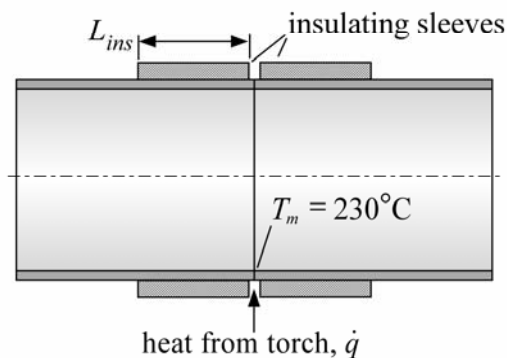


Figure 3: Tubes with insulating sleeves placed over them to reduce the heat transfer required.

If the insulation is perfect (i.e., convection is eliminated from the section of the pipe covered by the insulating sleeves), then how long must the sleeves be (L_{ins}) in order to reduce the heat required to $\dot{q} = 500 \text{ W}$?

The tubes with insulating sleeves can be represented by a resistance network similar to the one shown in Figure 2, but with additional resistances inserted between the interface and the base of the fins that correspond to the insulated sections of pipes, as shown in Figure 4.

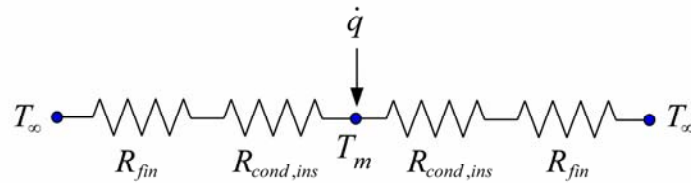


Figure 4: Resistance network with additional resistors associated with insulated tubes.

The resistance of the insulated sections of the pipe ($R_{cond,tube}$):

$$R_{cond,ins} = \frac{L_{ins}}{k A_c}$$

$R_{cond,ins} = L_{ins} / (k * A_c)$

"resistance of insulated portion of pipe"

The length of the un-insulated section of pipe is reduced and therefore the fin efficiency and fin resistance must be re-calculated. The fin efficiency (η_{fin}) becomes:

$$\eta_{fin} = \frac{\tanh[m(L - L_{ins})]}{m(L - L_{ins})}$$

and the resistance of each fin (R_{fin}) is:

$$R_{fin} = \frac{1}{\eta_{fin} \bar{h} per(L - L_{ins})}$$

$mL = \sqrt{h_{bar} * per / (k * A_c)} * (L - L_{ins})$

"fin parameter"

$\eta_{fin} = \tanh(mL) / mL$

"fin efficiency"

$R_{fin} = 1 / (\eta_{fin} * h_{bar} * per * (L - L_{ins}))$

"fin resistance"

Using the resistance network shown in Figure 4, the required heat transfer rate can be expressed as:

$$\dot{q} = 2 \frac{(T_m - T_{\infty})}{R_{cond,ins} + R_{fin}}$$

EES can solve for the length of insulation that is required by setting the required heat transfer rate to the available heat transfer rate,

$q_{dot} = 2 * (T_m - T_{infinity}) / (R_{fin} + R_{cond,ins})$

"required torch heat transfer"

$q_{\text{dot}}=500$ [W]

$L_{\text{ins_ft}}=L_{\text{ins}}*\text{convert}(\text{m},\text{ft})$

"available torch heat transfer"

"length of insulation in ft"

The solution indicates that the insulating sleeves must be at least 0.16 m (0.52 ft) in order to reduce the required heat transfer rate to the point where 500 W will suffice.