

### EXAMPLE 1.7-1: Bent-beam Actuator

One design of a micro-scale, lithographically fabricated (i.e., MEMS) device that can produce in-plane motions is called a bent-beam actuator (Que (2000)). A V-shaped structure (the bent-beam in Figure 1) is suspended between two anchors. The anchors are thermally staked to the underlying substrate and therefore keep the ends of the bent-beam at room temperature ( $T_a = 20^\circ$ ). An elevated voltage is applied to one pillar and the other is grounded. The voltage difference causes current ( $I$ ) to flow through the bent-beam structure. The temperature of the bent-beam rises as a result of ohmic heating and the thermally induced expansion causes the apex of the bent-beam to move outwards. The result is a voltage-controlled actuator capable of producing in-plane motion.

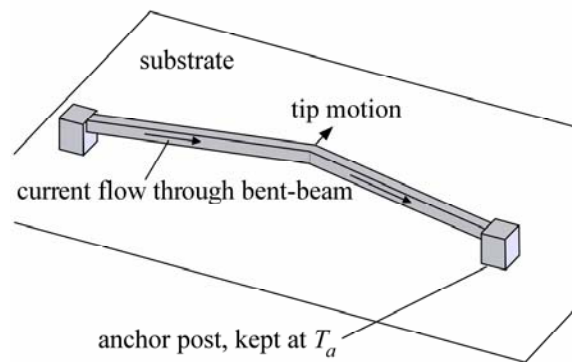


Figure 1: Bent-beam actuator.

The anchors of the bent-beam actuator are placed  $L_a = 1$  mm apart and the beam structure has a cross-section of  $W = 10 \mu\text{m}$  by  $th = 5 \mu\text{m}$ . The slope of the beams (with respect to a line connecting the two pillars) is  $\theta = 0.5$  rad, as shown in Figure 2. The bent-beam material has conductivity  $k = 80$  W/m-K, electrical resistivity  $\rho_e = 1 \times 10^{-5}$  ohm-m and coefficient of thermal expansion  $CTE = 3.5 \times 10^{-5}$  K $^{-1}$ . You may neglect radiation from the beam and assume all of the heat that is generated is convected to the surrounding air at temperature  $T_\infty = 20^\circ\text{C}$  with average heat transfer coefficient  $\bar{h} = 100$  W/m $^2$ -K or transferred conductively to the pillars (which remain at  $T_a = 20^\circ\text{C}$ ). The actuator is activated with  $I = 10$  mA of current.

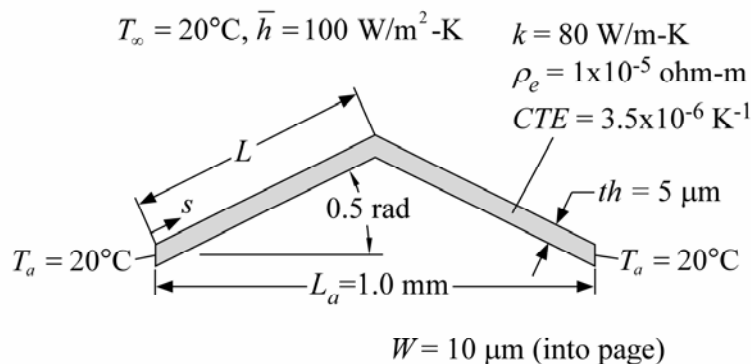


Figure 2: Dimensions and conditions associated with bent-beam actuator.

a.) Is it appropriate to treat the bent-beam as an extended surface?

The input parameters for the problem are entered into EES:

"EXAMPLE 1.7-1: Bent-beam Actuator"

\$UnitSystem SI MASS RAD PA C J

\$Tabstops 0.2 0.4 0.6 0.8 3.5

"Inputs"

L\_a=1 [mm]\*convert(mm,m)

"distance between anchors"

w=10 [micron]\*convert(micron,m)

"width of beam"

th=5 [micron]\*convert(micron,m)

"thickness of beam"

I=0.010 [Amp]

"current"

theta=0.5 [rad]

"slope of beam"

T\_a=converttemp(C,K,20 [C])

"temperature of pillars"

T\_infinity=converttemp(C,K,20 [C])

"temperature of air"

h\_bar=100 [W/m^2-K]

"heat transfer coefficient"

k=80 [W/m-K]

"conductivity"

rho\_e=1e-5 [ohm-m]

"electrical resistivity"

CTE=3.5e-6 [1/K]

"coefficient of thermal expansion"

The extended surface approximation requires that the 3-D temperature distribution within the bent-beam be approximated as 1-D; that is, temperature gradients within the beam that are perpendicular to the surface will be ignored so that the temperature may be approximated as a function only of  $s$ , the coordinate that follows the beam (see Figure 2). The resistance that must be neglected in order to use the extended surface approximation is conduction in the lateral direction ( $R_{cond,lat}$ ). The extended surface approximation is justified provided that the lateral conduction resistance is small relative to the resistance that is being considered, convection from the outer surface ( $R_{conv}$ ). The Biot number is therefore:

$$Bi = \frac{R_{cond,lat}}{R_{conv}}$$

The heat transfer will take the shortest path to the surface and therefore it is appropriate to use the smallest lateral dimension ( $th/2$ ) to compute the lateral conduction resistance.

$$Bi = \left( \frac{th}{2kWL} \right) \left( \frac{\bar{h}WL}{1} \right) = \frac{th\bar{h}}{2k}$$

where  $L$  is the length of the beam from pillar to apex (see Figure 2).

Bi=th\*h\_bar/(2\*k)

"Biot number"

The Biot number is small ( $3 \times 10^{-6}$ ) and therefore the extended surface approximation is justified.

b.) Develop an analytical solution that can predict the temperature of one leg of the bent-beam as a function of position along the beam,  $s$ .

The general solution for an extended surface with a constant cross-sectional area and spatially uniform generation was derived in Section 1.7.2:

$$T = C_1 \exp(ms) + C_2 \exp(-ms) + T_\infty + \frac{\dot{g}''' A_c}{h \text{ per}} \quad (1)$$

For the bent-beam actuator, the perimeter ( $\text{per}$ ), cross-sectional area ( $A_c$ ), and fin parameter ( $m$ ) are

$$\text{per} = 2(W + th)$$

$$A_c = W th$$

$$m = \sqrt{\frac{h \text{ per}}{k A_c}}$$

$$\text{per} = 2(W + th)$$

$$A_c = W th$$

$$m = \sqrt{h_{\text{bar}} \text{per} / (k A_c)}$$

"perimeter"

"area"

"fin parameter"

The volumetric generation,  $\dot{g}'''$ , is related to ohmic heating. The electrical resistance of the bent-beam structure ( $R_e$ ) is:

$$R_e = \frac{\rho_e 2L}{A_c}$$

where

$$L = \frac{L_a}{2 \cos(\theta)}$$

The volumetric rate of electrical dissipation is the ratio of ohmic dissipation to the volume of the structure:

$$\dot{g}''' = \frac{I^2 R_e}{2L A_c}$$

$$L = L_a / (2 \cos(\theta))$$

$$R_e = \rho_e L^2 / A_c$$

$$\dot{g}''' = I^2 R_e / (2 L A_c)$$

"length of half-beam"

"resistance of beam structure"

"volumetric generation"

The constants  $C_1$  and  $C_2$  in Eq. (1) are determined using the boundary conditions. The temperature of the beam where it meets the pillar is specified:

$$T_{s=0} = T_a \quad (2)$$

Substituting Eq. (1) into Eq. (2) leads to:

$$C_1 + C_2 + T_\infty + \frac{\dot{g}''' A_c}{h \text{ per}} = T_a \quad (3)$$

A half-symmetry model of the bent-beam actuator will be developed; because both legs of the bent-beam see identical conditions there is nothing to drive heat from one leg to the other and therefore there will be no conduction through the end of the half-beam (at  $s = L$ ):

$$\dot{q}_{s=L} = -k \left. \frac{dT}{ds} \right|_{s=L} = 0$$

or

$$\left. \frac{dT}{ds} \right|_{s=L} = 0 \quad (4)$$

Substituting Eq. (1) into Eq. (4) leads to:

$$C_1 m \exp(mL) - C_2 m \exp(-mL) = 0 \quad (5)$$

Equations (3) and (5) can be entered in EES and used to determine  $C_1$  and  $C_2$ .

$T_\infty + C_1 + C_2 + \dot{g}''' A_c / (h \text{ per}) = T_a$  "from boundary condition at  $s=0$ "  
 $C_1 m \exp(mL) - C_2 m \exp(-mL) = 0$  "from boundary condition at  $s=L$ "

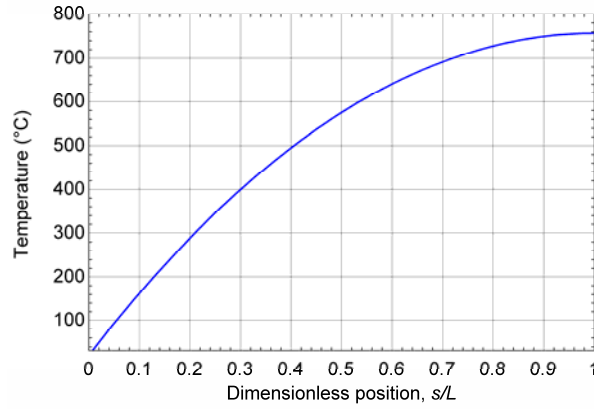
A variable  $s_{\text{bar}}$  is defined as  $s/L$  so that  $s_{\text{bar}} = 0$  corresponds to the pillar and  $s_{\text{bar}} = 1$  to the apex; the variable  $s_{\text{bar}}$  is defined only for convenience so that it is easy to generate a parametric tables in which  $s$  is varied from 0 to  $L$  even if parameters such as  $\theta$  and  $L_a$  change.

$s_{\text{bar}} = s/L$  "non-dimensional position"

The temperature is evaluated using Eq. (1).

$T = T_\infty + C_1 \exp(m*s) + C_2 \exp(-m*s) + \dot{g}''' A_c / (h \text{ per})$  "temperature"  
 $T_C = \text{converttemp}(K, C, T)$  "in C"

A parametric table is generated that includes the variables  $s_{\text{bar}}$  and  $T_C$ . The temperature distribution through a half-leg of the beam is shown in Figure 3.



**Figure 3: Temperature as a function of dimensionless position along half-length of beam.**

- c.) The thermally induced elongation of a differential segment of the beam (of length  $ds$ ) is given by:

$$dL = CTE(T - T_a) ds$$

Estimate the displacement of the apex of the beam. Plot the displacement as a function of voltage.

The total displacement of the beam ( $\Delta L$ ) is obtained by integrating the differential elongation along the beam:

$$\Delta L = \int_0^L CTE(T - T_a) ds \quad (6)$$

Substituting the solution for the temperature distribution, Eq. (1) into Eq. (6) leads to:

$$\Delta L = \int_0^L CTE \left( C_1 \exp(ms) + C_2 \exp(-ms) + T_\infty + \frac{\dot{g}''' A_c}{h \text{ per}} - T_a \right) ds$$

Evaluating the integral:

$$\Delta L = CTE \left[ \left( T_\infty - T_a + \frac{\dot{g}''' A_c}{h \text{ per}} \right) x + \frac{C_1}{m} \exp(ms) - \frac{C_2}{m} \exp(-ms) \right]_0^L$$

Substituting the integration limits:

$$\Delta L = CTE \left\{ \left( T_\infty - T_a + \frac{\dot{g}''' A_c}{h \text{ per}} \right) L + \frac{C_1}{m} [\exp(mL) - 1] - \frac{C_2}{m} [\exp(-mL) - 1] \right\}$$

$\Delta L = CTE \cdot (T_{\infty} - T_a) \cdot \frac{A_c}{h \cdot \bar{r}} \cdot L + C_1 \cdot \frac{\exp(m \cdot L) - 1}{m} - C_2 \cdot \frac{\exp(-m \cdot L) - 1}{m}$   
 "displacement of beam"

Assuming that the joint associated with the apex does not provide a torque on either leg of the beam, the displacement of the apex can be estimated using trigonometry (Figure 4).

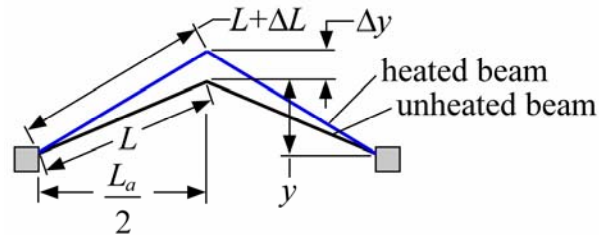


Figure 4: Trigonometry associated with apex motion

The original position of the apex ( $y$ ) is given by:

$$y = \sqrt{L^2 - \left(\frac{L_a}{2}\right)^2}$$

therefore, the motion of the apex ( $\Delta y$ ) is:

$$\Delta y = \sqrt{(L + \Delta L)^2 - \left(\frac{L_a}{2}\right)^2} - \sqrt{L^2 - \left(\frac{L_a}{2}\right)^2}$$

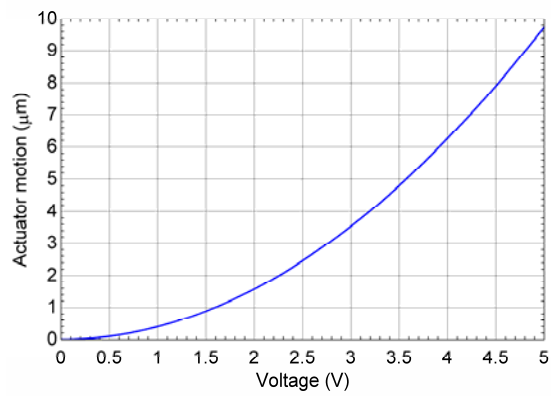
$\Delta y = \sqrt{(L + \Delta L)^2 - (L_a/2)^2} - \sqrt{L^2 - (L_a/2)^2}$  "displacement of apex"  
 $\Delta y_{\text{micron}} = \Delta y \cdot \text{convert}(m, \text{micron})$  "in  $\mu\text{m}$ "

The voltage across the beam ( $V$ ) is:

$$V = I R_e$$

$V = I \cdot R_e$  "voltage"

Figure 5 illustrates the actuator displacement as a function of voltage. This plot was generated using a parametric table including the variables  $\Delta y_{\text{micron}}$  and  $V$ ; the variable  $I$  was commented out in order to make the table.



**Figure 5: Actuator displacement as a function of the applied voltage.**