8.10.5 Numerical Model of a Regenerator with no Entrained Heat Capacity

The development of a flexible, numerical model of a regenerator in which the entrained heat capacity of the fluid is neglected is illustrated in this section in the context of the single bed regenerator application shown in Figure 8- 76.

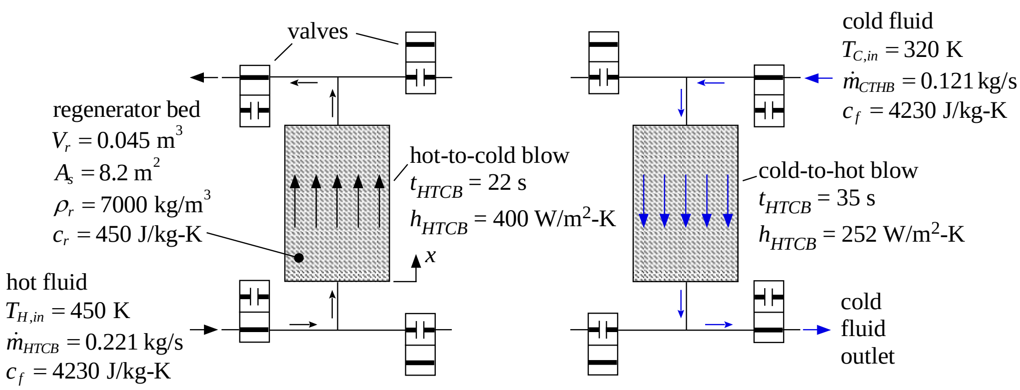

Figure 8- 76: Single bed regenerator system.

During the hot-to-cold blow process, fluid with specific heat capacity c_f = 4230 J/kg-K enters the regenerator at its hot end (at x = 0) with temperature $T_{H,in}$ = 450 K and mass flow rate $\dot{m}_{HTCB}$ = 0.221 kg/s. The hot-to-cold blow process lasts t_{HTCB} = 22 s and during this time the heat transfer coefficient between the fluid and the regenerator matrix is h_{HTCB} = 400 W/m^2-K. During the cold-to-hot blow process, the same fluid, with heat capacity c_f = 4230 J/kg-K, enters the regenerator at its cold end (at $x = L$) with temperature $T_{C,in}$ = 320 K and mass flow rate $\dot{m}_{CTHB}$ = 0.121 kg/s. The cold-to-hot blow process lasts t_{CTHB} = 35 s and during this time the heat transfer coefficient between the fluid and the regenerator matrix is h_{CTHB} = 252 W/m^2-K. The regenerator bed is composed of a material with heat capacity c_r = 450 J/kg-K and density ρ_r = 7000 kg/m^3. The total volume of the regenerator material is V_r = 0.045 m^3 and the surface area exposed to the fluid is A_s = 8.2 m^2.

The application shown in Figure 8- 76 is not symmetric (the heat transfer coefficient during the hot-to-cold blow process is different from its value during the cold-to-hot blow process) or balanced (the total heat capacity of the fluid that passes through the regenerator during the hot-to-cold blow process is different from the total heat capacity during the cold-to-hot blow process). Therefore, the solution presented in Section 8.10.3 and programmed in EES cannot be used to analyze the problem.

In this section, a numerical model of the system will be developed using a finite difference technique. It is possible to model this regenerator by developing the state equations that provide the time rate of change of the temperature of the regenerator material within each node and then integrating these equations forward in time from an assumed initial condition. If the integration process is continued for a sufficiently long period of time then the behavior of the regenerator will eventually achieve the periodic steady-state condition of interest. However, this process can take many cycles of integration and therefore it may be excessively computationally intensive. Since we are only interested in the periodic steady-state solution, a better approach discretizes the length of the regenerator and the time associated with one cycle. The associated regenerator segments occupy the entire time and space associated with one cycle. Energy balances on these segments lead to a system of equations that provide the fluid and regenerator temperatures throughout one cycle and can be solved using a single matrix inversion. The advantage of this approach is that one of the boundary conditions included in the system of equations enforces the periodic steady-state condition so only one matrix inversion is required.

The solution is implemented in MATLAB and the inputs are entered at the top of the script:

```
clear all;
T_H_in=450;                  % hot fluid inlet temperature (K)
T_C_in=310;                  % cold fluid inlet temperature (K)
c_f=4230;                    % specific heat capacity of the fluid (J/kg-K)
rho_r=7000;                  % density of regenerator material (kg/m^3)
c_r=450;                     % specific heat capacity of regenerator material (J/kg-K)
t_HTCB=22;                   % time for hot-to-cold blow process (s)
t_CTHB=35;                   % time for cold-to-hot blow process (s)
h_HTCB=400;                  % heat xfer coefficient, hot-to-cold blow (W/m^2-K)
h_CTHB=252;                  % heat xfer coefficient, cold-to-hot blow (W/m^2-K)
V_r=0.045;                   % volume of regenerator material (m^3)
A_s=8.2;                     % total surface area for heat transfer (m^2)
m_dot_HTCB=0.221;            % hot-to-cold blow mass flow rate (kg/s)
m_dot_CTHB=0.121;            % cold-to-hot blow mass flow rate (kg/s)
```

Figure 8-77 illustrates the discretization of the regenerator and the positions of the fluid and regenerator temperature nodes.

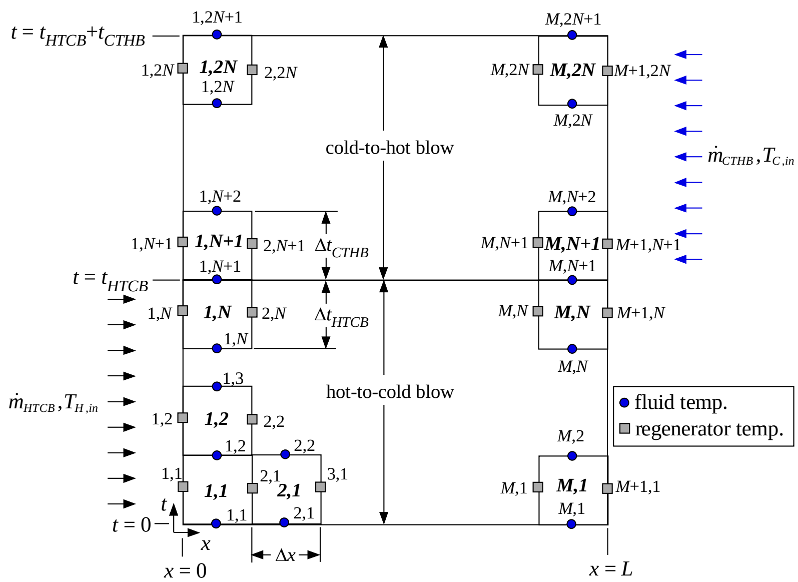

Figure 8-77: Regenerator discretized in space and time.

The spatial and temporal location of each of the regenerator temperature nodes are given by:

$$\tilde{x}_{r\,i,j} = \frac{x_{r\,i,j}}{L} = \frac{1}{M}\left(i - \frac{1}{2}\right) \quad \text{for } i = 1..M \quad \text{for } j = 1..\left(2N+1\right) \tag{8-398}$$

$$t_{r\,i,j} = \begin{cases} \frac{t_{HTCB}}{N}\left(j-1\right) & \text{for } i=1..M \quad \text{for } j=1..\left(N+1\right) \\ t_{HTCB}+\frac{t_{CTHB}}{N}\left(j-N-1\right) & \text{for } i=1..M \quad \text{for } j=\left(N+2\right)..\left(2N+1\right) \end{cases} \tag{8-399}$$

where M is the number of sections in space and N is the number of sections in time used to discretize each of the blow processes.

```
M=10;                        % number of nodes in space
N=20;                        % number of nodes in time for each blow process

%position regenerator temperature nodes
for i=1:M
    for j=1:(N+1)
        x_hat_r(i,j)=(i-1/2)/M;
        t_r(i,j)=t_HTCB*(j-1)/N;
    end
    for j=(N+2):(2*N+1)
        x_hat_r(i,j)=(i-1/2)/M;
        t_r(i,j)=t_HTCB+t_CTHB*(j-N-1)/N;
    end
end
```

The spatial and temporal location of each of the fluid temperature nodes are given by:

$$\tilde{x}_{f\,i,j} = \frac{x_{f\,i,j}}{L} = \frac{1}{M}\left(i-1\right) \quad \text{for } i=1..\left(M+1\right) \quad \text{for } j=1..2N \tag{8-400}$$

$$t_{f\,i,j} = \begin{cases} \frac{t_{HTCB}}{N}\left(j-\frac{1}{2}\right) & \text{for } i=1..\left(M+1\right) \quad \text{for } j=1..N \\ t_{HTCB}+\frac{t_{CTHB}}{N}\left(j-N-\frac{1}{2}\right) & \text{for } i=1..\left(M+1\right) \quad \text{for } j=\left(N+1\right)..2N \end{cases} \tag{8-401}$$

```
% position fluid temperature nodes
for i=1:(M+1)
    for j=1:N
        x_hat_f(i,j)=(i-1)/M;
        t_f(i,j)=t_HTCB*(j-1/2)/N;
    end
    for j=(N+1):(2*N)
        x_hat_f(i,j)=(i-1)/M;
        t_f(i,j)=t_HTCB+t_CTHB*(j-N-1/2)/N;
    end
end
```

The governing equations are derived by balancing the total energy transferred to and from the fluid and regenerator during each section. The regenerator energy balance for node *i*, *j* during the hot-to-cold blow process, shown in Figure 8-78(a), is:

$$\underbrace{\frac{h_{HTCB}\,A_s}{M}\left[\frac{\left(T_{f\,i,j}+T_{f\,i+1,j}\right)}{2}-\frac{\left(T_{r\,i,j}+T_{r\,i,j+1}\right)}{2}\right]\frac{t_{HTCB}}{N}}_{\text{amount of energy associated with convection from fluid}}=\underbrace{\frac{\rho_r\,c_r\,V_r}{M}\left(T_{r\,i,j+1}-T_{r\,i,j}\right)}_{\text{energy stored in regenerator}} \tag{8-402}$$

$$\text{for } i=1..M \quad \text{for } j=1..N$$

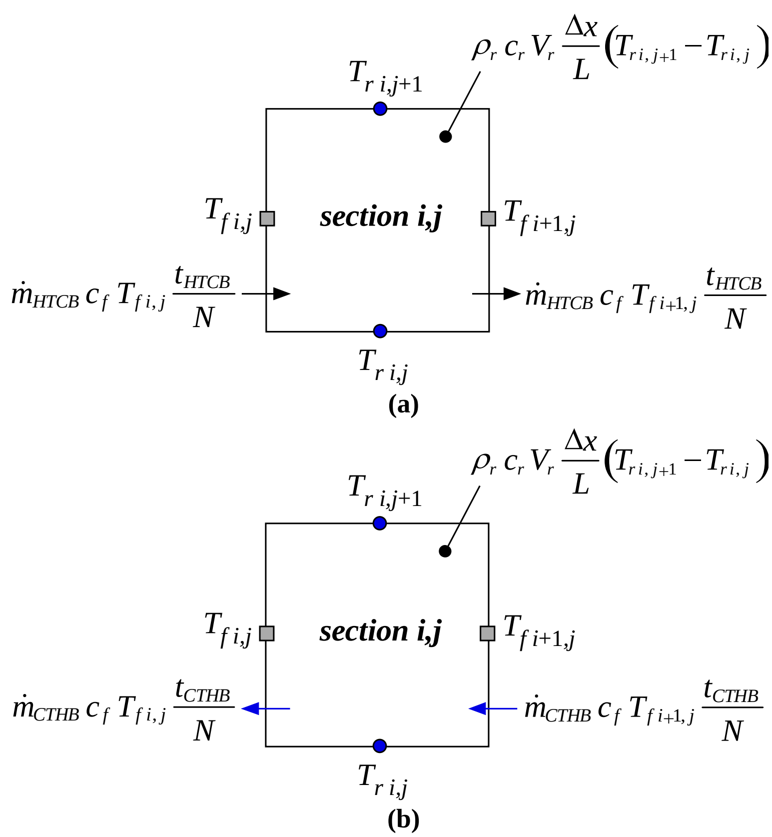

Figure 8-78: Energy balance for section *i, j* (a) during the hot-to-cold blow process and (b) during the cold-to-hot blow process.

The left side of Eq. (8-402) is the total amount of energy transferred by convection from the fluid to the regenerator in the section and the right side of Eq. (8-402) is the amount of energy stored in the regenerator. The regenerator energy balance for node *i*, *j* during the cold-to-hot blow process, shown in Figure 8-78(b), is:

$$\frac{h_{CTHB}\,A_s}{M}\left[\frac{\left(T_{r\,i,j}+T_{r\,i,j+1}\right)}{2}-\frac{\left(T_{f\,i,j}+T_{f\,i+1,j}\right)}{2}\right]\frac{t_{CTHB}}{N}=\frac{\rho_r\,c_r\,V_r}{M}\left(T_{r\,i,j}-T_{r\,i,j+1}\right) \tag{8-403}$$

$$\text{for } i=1..M \quad \text{for } j=\left(N+1\right)..2N$$

where the left side of Eq. (8-403) is the total amount of energy transferred by convection from the regenerator to the fluid in the section and the right side of Eq. (8-402) is the amount of energy released by the regenerator.

The fluid energy balance for node *i*, *j* during the hot-to-cold blow process is:

$$\underbrace{\dot{m}_{HTCB}\, c_f \left(T_{f\,i,j} - T_{f\,i+1,j}\right)\frac{t_{HTCB}}{N}}_{\text{amount of enthalpy change of fluid}} = \underbrace{\frac{h_{HTCB}\, A_s}{M}\left[\frac{\left(T_{f\,i,j}+T_{f\,i+1,j}\right)}{2} - \frac{\left(T_{r\,i,j}+T_{r\,i,j+1}\right)}{2}\right]\frac{t_{HTCB}}{N}}_{\text{amount of energy transferred to matrix}} \tag{8-404}$$
$$\text{for } i = 1..M \quad \text{for } j = 1..N$$

The left side of Eq. (8-404) is the enthalpy change of the fluid flow and the right side of Eq. (8-404) is the total amount of energy transferred by convection from the fluid to the regenerator. The fluid energy balance for node *i*, *j* during the cold-to-hot blow process is:

$$\dot{m}_{CTHB}\, c_f \left(T_{f\,i+1,j} - T_{f\,i,j}\right)\frac{t_{CTHB}}{N} = \frac{h_{CTHB}\, A_s}{M}\left[\frac{\left(T_{f\,i,j}+T_{f\,i+1,j}\right)}{2} - \frac{\left(T_{r\,i,j}+T_{r\,i,j+1}\right)}{2}\right]\frac{t_{CTHB}}{N} \tag{8-405}$$
$$\text{for } i = 1..M \quad \text{for } j = \left(N+1\right)..2N$$

The boundary conditions on the regenerator temperature nodes that enforce a periodic, steady-state solution are:

$$T_{r\,i,1} = T_{r\,i,2N+1} \quad \text{for } i = 1..M \tag{8-406}$$

The boundary condition on the fluid temperature nodes during the hot-to-cold blow process specify that the fluid entering at $x = 0$ is the hot inlet temperature:

$$T_{f\,1,j} = T_{H,in} \quad \text{for } j = 1..N \tag{8-407}$$

The boundary condition on the fluid temperature nodes during the cold-to-hot blow process specify that the fluid entering at $x = L$ is the cold inlet temperature:

$$T_{f\,M+1,j} = T_{C,in} \quad \text{for } j = \left(N+1\right)..2N \tag{8-408}$$

Equations (8-402) through (8-408) are $(M+1)2N+M(2N+1)$ equations in an equal number of unknown fluid and regenerator temperatures. In order to solve this sytem of equations using MATLAB, it is necessary to place them in matrix format:

$$\underline{\underline{A}}\,\underline{X} = \underline{b} \tag{8-409}$$

The matrix $\underline{\underline{A}}$ and vector $\underline{b}$ are initialized:

```
A=spalloc(M*(2*N+1)+2*N*(M+1),M*(2*N+1)+2*N*(M+1),(M*(2*N+1)+2*N*(M+1))*4);
b=zeros(M*(2*N+1)+2*N*(M+1),1);
```

The unknown temperatures are placed in the vector of unknowns, $\underline{X}$ according to:

$$\underline{X} = \begin{bmatrix} X_1 = T_{r\,1,1} \\ X_2 = T_{r\,2,1} \\ \ldots \\ X_M = T_{r\,M,1} \\ X_{M+1} = T_{r\,1,2} \\ \ldots \\ X_{(2N+1)M} = T_{r\,2N+1,M} \\ X_{(2N+1)M+1} = T_{f\,1,1} \\ \ldots \\ X_{(2N+1)M+M+1} = T_{f\,M+1,1} \\ X_{(2N+1)M+(M+1)+1} = T_{f\,1,2} \\ \ldots \\ X_{(2N+1)M+2N(M+1)} = T_{f\,M+1,2N} \end{bmatrix} \tag{8-410}$$

According to Eq. (8-410), the unknown regenerator temperature $T_{r\,i,j}$ corresponds to entry $(j-1)\,M + i$ of $\underline{X}$ and the unknown fluid temperature $T_{f\,i,j}$ corresponds to entry $(2N+1)M + (j-1)\,(M+1) + i$ of $\underline{X}$. The governing equations are placed into the rows of the matrix $\underline{\underline{A}}$ according to:

$$
\underline{\underline{A}} = \begin{bmatrix}
\text{row } 1 \; = \; \text{regenerator energy balance for } 1,1 \\
\text{row } 2 \; = \; \text{regenerator energy balance for } 2,1 \\
\ldots \\
\text{row } M \; = \; \text{regenerator energy balance for } M,1 \\
\text{row } M+1 = \text{regenerator energy balance for HX } 1,2 \\
\ldots \\
\text{row } 2\,M\,N = \text{regenerator energy balance for } M,2N \\
\text{row } 2\,M\,N+1 = \text{fluid energy balance for } 1,1 \\
\ldots \\
\text{row } 4\,M\,N = \text{fluid energy balance for } M,2N \\
\text{row } 4\,M\,N+1 = \text{regenerator boundary condition for } T_{r1,1} \\
\ldots \\
\text{row } 4\,M\,N+M = \text{regenerator boundary condition for } T_{r\,M,1} \\
\text{row } 4\,M\,N+M+1 = \text{fluid boundary condition for } T_{f\,1,1} \\
\ldots \\
\text{row } 4\,M\,N+M+N = \text{fluid boundary condition for } T_{f\,1,N} \\
\text{row } 4\,M\,N+M+N+1 = \text{fluid boundary condition for } T_{f\,M+1,N+1} \\
\ldots \\
\text{row } 4\,M\,N+M+N+2\,N = \text{fluid boundary condition for } T_{f\,M+1,2N}
\end{bmatrix} \tag{8-411}
$$

According to Eq. (8-411), the regenerator energy balance for segment i,j corresponds to row $(j-1)\,M + i$ of $\underline{\underline{A}}$ and the fluid energy balance for segment i,j corresponds to row $2MN + (j-1)\,M + i$ of $\underline{\underline{A}}$. The regenerator boundary condition for node $T_{r\;i,1}$ (related to periodic steady-state) corresponds to row $4\,MN + i$ of $\underline{\underline{A}}$. The fluid boundary condition for node $T_{f\,1,\,j}$ (during the hot-to-cold blow process) corresponds to row $4\,(M\,N) + M + j$ of $\underline{\underline{A}}$ and the fluid boundary condition for node $T_{f\,M+1,\,j}$ (during the cold-to-hot blow process) corresponds to row $4\,M\,N + M + N + j$ of $\underline{\underline{A}}$.

The regenerator energy balances, Eqs. (8-402) and (8-403), are rearranged in order to identify the coefficients that multiply the unknown temperatures:

$$\underbrace{T_{r\,i,j}\left[\rho_r\,c_r\,V_r-\frac{h_{HTCB}\,A_s\,t_{HTCB}}{2\,N}\right]}_{A_{(j-1)M+i,(j-1)M+i}}+\underbrace{T_{r\,i,j+1}\left[-\rho_r\,c_r\,V_r-\frac{h_{HTCB}\,A_s\,t_{HTCB}}{2\,N}\right]}_{A_{(j-1)M+i,(j+1-1)M+i}}$$

$$+T_{f\,i,j}\underbrace{\left[\frac{h_{HTCB}\,A_s\,t_{HTCB}}{2\,N}\right]}_{A_{(j-1)M+i,(2N+1)M+(j-1)(M+1)+i}}+T_{f\,i+1,j}\underbrace{\left[\frac{h_{HTCB}\,A_s\,t_{HTCB}}{2\,N}\right]}_{A_{(j-1)M+i,(2N+1)M+(j-1)(M+1)+i+1}}=0 \tag{8-412}$$

$$\text{for } i=1..M \quad \text{for } j=1..N$$

$$\underbrace{T_{r\,i,j}\left[\rho_r\,c_r\,V_r-\frac{h_{CTHB}\,A_s\,t_{CTHB}}{2\,N}\right]}_{A_{(j-1)M+i,(j-1)M+i}}+\underbrace{T_{r\,i,j+1}\left[-\rho_r\,c_r\,V_r-\frac{h_{CTHB}\,A_s\,t_{CTHB}}{2\,N}\right]}_{A_{(j-1)M+i,(j+1-1)M+i}}$$

$$+T_{f\,i,j}\underbrace{\left[\frac{h_{CTHB}\,A_s\,t_{CTHB}}{2\,N}\right]}_{A_{(j-1)M+i,(2N+1)M+(j-1)(M+1)+i}}+T_{f\,i+1,j}\underbrace{\left[\frac{h_{CTHB}\,A_s\,t_{CTHB}}{2\,N}\right]}_{A_{(j-1)M+i,(2N+1)M+(j-1)(M+1)+i+1}}=0 \tag{8-413}$$

$$\text{for } i=1..M \quad \text{for } j=(N+1)..2N$$

```
% regenerator energy balances
for i=1:M
    % hot-to-cold blow
    for j=1:N
        A((j-1)*M+i,(j-1)*M+i)=rho_r*c_r*V_r-h_HTCB*A_s*t_HTCB/(2*N);
        A((j-1)*M+i,(j+1-1)*M+i)=-rho_r*c_r*V_r-h_HTCB*A_s*t_HTCB/(2*N);
        A((j-1)*M+i,(2*N+1)*M+(j-1)*(M+1)+i)=h_HTCB*A_s*t_HTCB/(2*N);
        A((j-1)*M+i,(2*N+1)*M+(j-1)*(M+1)+i+1)=h_HTCB*A_s*t_HTCB/(2*N);
    end
    % cold-to-hot blow
    for j=(N+1):(2*N)
        A((j-1)*M+i,(j-1)*M+i)=rho_r*c_r*V_r-h_CTHB*A_s*t_CTHB/(2*N);
        A((j-1)*M+i,(j+1-1)*M+i)=-rho_r*c_r*V_r-h_CTHB*A_s*t_CTHB/(2*N);
        A((j-1)*M+i,(2*N+1)*M+(j-1)*(M+1)+i)=h_CTHB*A_s*t_CTHB/(2*N);
        A((j-1)*M+i,(2*N+1)*M+(j-1)*(M+1)+i+1)=h_CTHB*A_s*t_CTHB/(2*N);
    end
end
```

The fluid energy balances, Eqs. (8-404) and (8-413), are also rearranged:

$$T_{f\,i,j}\underbrace{\left[\dot{m}_{HTCB}\,c_f-\frac{h_{HTCB}\,A_s}{2\,M}\right]}_{A_{2MN+(j-1)M+i,(2N+1)M+(j-1)(M+1)+i}}+T_{f\,i+1,j}\underbrace{\left[-\dot{m}_{HTCB}\,c_f-\frac{h_{HTCB}\,A_s}{2\,M}\right]}_{A_{2MN+(j-1)M+i,(2N+1)M+(j-1)(M+1)+i+1}}$$

$$+T_{r\,i,j}\underbrace{\left[\frac{h_{HTCB}\,A_s}{2\,M}\right]}_{A_{2MN+(j-1)M+i,(j-1)M+i}}+T_{r\,i,j+1}\underbrace{\left[\frac{h_{HTCB}\,A_s}{2\,M}\right]}_{A_{2MN+(j-1)M+i,(j+1-1)M+i}}=0 \qquad (8\text{-}414)$$

$$\text{for } i=1..M \quad \text{for } j=1..N$$

$$T_{f\,i+1,j}\underbrace{\left[\dot{m}_{CTHB}\,c_f-\frac{h_{CTHB}\,A_s}{2M}\right]}_{A_{2MN+(j-1)M+i,(2N+1)M+(j-1)(M+1)+i+1}}+T_{f\,i,j}\underbrace{\left[-\dot{m}_{CTHB}\,c_f-\frac{h_{CTHB}\,A_s}{2M}\right]}_{A_{2MN+(j-1)M+i,(2N+1)M+(j-1)(M+1)+i}}+$$

$$T_{r\,i,j}\underbrace{\left[\frac{h_{CTHB}\,A_s}{2M}\right]}_{A_{2MN+(j-1)M+i,(j-1)M+i}}+T_{r\,i,j+1}\underbrace{\left[\frac{h_{CTHB}\,A_s}{2M}\right]}_{A_{2MN+(j-1)M+i,(j+1-1)M+i}}=0 \qquad (8\text{-}415)$$

$$\text{for } i=1..M \quad \text{for } j=(N+1)..2N$$

```
% fluid energy balances
for i=1:M
     % hot-to-cold blow
     for j=1:N
          A(2*M*N+(j-1)*M+i,(2*N+1)*M+(j-1)*(M+1)+i)=m_dot_HTCB*c_f-h_HTCB*A_s/(2*M);
          A(2*M*N+(j-1)*M+i,(2*N+1)*M+(j-1)*(M+1)+i+1)=-m_dot_HTCB*c_f-h_HTCB*A_s/(2*M);
          A(2*M*N+(j-1)*M+i,(j-1)*M+i)=h_HTCB*A_s/(2*M);
          A(2*M*N+(j-1)*M+i,(j+1-1)*M+i)=h_HTCB*A_s/(2*M);
     end
     % cold-to-hot blow
     for j=(N+1):(2*N)
          A(2*M*N+(j-1)*M+i,(2*N+1)*M+(j-1)*(M+1)+i+1)=m_dot_CTHB*c_f-h_CTHB*A_s/(2*M);
          A(2*M*N+(j-1)*M+i,(2*N+1)*M+(j-1)*(M+1)+i)=-m_dot_CTHB*c_f-h_CTHB*A_s/(2*M);
          A(2*M*N+(j-1)*M+i,(j-1)*M+i)=h_CTHB*A_s/(2*M);
          A(2*M*N+(j-1)*M+i,(j+1-1)*M+i)=h_CTHB*A_s/(2*M);
     end
end
```

The regenerator boundary conditions corresponding to periodic steady-state, Eq. (8-406), is rearranged:

$$T_{r\,i,1}\underbrace{[1]}_{A_{4MN+i,(1-1)M+i}}+T_{r\,i,2N+1}\underbrace{[-1]}_{A_{4MN+i,(2N+1-1)M+i}}=0 \quad \text{for } i=1..M \qquad (8\text{-}416)$$

```
% regenerator boundary conditions (cyclic steady-state)
for i=1:M
     A(4*M*N+i,(1-1)*M+i)=1;
     A(4*M*N+i,(2*N+1-1)*M+i)=-1;
```

```
end
```

The fluid temperature boundary conditions, Eqs. (8-407) and (8-408), are rearranged:

$$T_{f\,1,j}\underbrace{[1]}_{A_{4MN+M+j,(2N+1)M+(j-1)(M+1)+1}} = \underbrace{T_{H,in}}_{b_{4MN+M+j}} \quad \text{for } j=1..N \tag{8-417}$$

$$T_{f\,M+1,j}\underbrace{[1]}_{A_{4MN+M+N+j,(2N+1)M+(j-1)(M+1)+M+1}} = \underbrace{T_{C,in}}_{b_{4MN+M+N+j}} \quad \text{for } j=(N+1)..2N \tag{8-418}$$

```
% fluid boundary conditions (hot-to-cold blow)
for j=1:N
    A(4*M*N+M+j,(2*N+1)*M+(j-1)*(M+1)+1)=1;
    b(4*M*N+M+j)=T_H_in;
end

% fluid boundary conditions (cold-to-hot blow)
for j=(N+1):2*N
    A(4*M*N+M+N+j,(2*N+1)*M+(j-1)*(M+1)+M+1)=1;
    b(4*M*N+M+N+j)=T_C_in;
end
```

The solution is obtained:

```
% get solution
X=A\b;
```

and placed in the matrices $\underline{\underline{T_r}}$ and $\underline{\underline{T_f}}$:

```
% put solution into temperature matrices
for i=1:M
    for j=1:(2*N+1)
        T_r(i,j)=X((j-1)*M+i);
    end
end
for i=1:(M+1)
    for j=1:(2*N)
        T_f(i,j)=X((2*N+1)*M+(j-1)*(M+1)+i);
    end
end
```

The energy transferred from the hot fluid during one cycle is obtained according to the integral given by Eq. (8-369). The integral is approximated numerically according to:

$$Q_H = \int_{t=0}^{t_{HTCB}} \dot{m}_{HTCB}\, c_f \left(T_{H,in} - T_{f,x=L}\right) dt = \sum_{j=1}^{N} \dot{m}_{HTCB}\, c_f \left(T_{H,in} - T_{f\,M+1,j}\right) \frac{t_{HTCB}}{N} \tag{8-419}$$

```
Q_H=0;
for j=1:N
    Q_H=Q_H+m_dot_HTCB*c_f*(T_H_in-T_f(M+1,j))*t_HTCB/N;
end
```

which leads to Q_H = 1.745x10^6 J. The energy transferred to the cold fluid during one cycle is given by Eq. (8-367). The integral is approximated numerically according to:

$$Q_C = \int_{t=t_{HTCB}}^{t_{HTCB}+t_{CTHB}} \dot{m}_{CTHB}\, c_f \left(T_{f,x=0} - T_{C,in}\right) dt = \sum_{j=N+1}^{2N} \dot{m}_{CTHB}\, c_f \left(T_{f\,1,j} - T_{C,in}\right) \frac{t_{CTHB}}{N} \tag{8-420}$$

```
Q_C=0;
for j=(N+1):(2*N)
    Q_C=Q_C+m_dot_CTHB*c_f*(T_f(1,j)-T_C_in)*t_CTHB/N;
end
```

which leads to Q_C = 1.745x10^6 J. The average temperature of the hot fluid leaving the regenerator is:

$$\bar{T}_{H,out} = T_{H,in} - \frac{Q_H}{\dot{m}_{HTCB}\, c_f\, t_{HTCB}} \tag{8-421}$$

```
T_bar_H_out=T_H_in-Q_H/(m_dot_HTCB*c_f*t_HTCB);
```

which leads to $\bar{T}_{H,out}$ = 365.14 K. The average temperature of the cold fluid leaving the regenerator is:

$$\bar{T}_{C,out} = T_{C,in} + \frac{Q_C}{\dot{m}_{CTHB}\, c_f\, t_{CTHB}} \tag{8-422}$$

```
T_bar_C_out=T_C_in+Q_C/(m_dot_CTHB*c_f*t_CTHB);
```

which leads to $\bar{T}_{C,out}$ = 407.43 K. The usual checks should be carried out for this numerical solution; the solution should be compared to the analytical solution discussed in Section 8.10.3 and the solution should be checked to ensure that it is insensitive to the number of segments used in the discretization.

An EES subroutine called RegeneratorHX is provided with the EES heat exchanger library to perform the same calculations as the MATLAB program that is presented in this section. Information about the subprogram can be viewed by selecting Heat Exchangers from the pull-down menu in the Function Info dialog and then Regenerator from the pull-down menu of heat exchanger types.