A.5: Introduction to Economics

A.5.1 Introduction

Thermal systems are generally capital intensive. That is, the equipment needed for a specific application, such as a heat exchanger or a furnace, is relatively expensive. This equipment must be purchased when the system is installed and its costs are referred to as the first costs. The owner of the equipment may not be willing or able to pay for the equipment at the time it is purchased so that some or all of the first costs are borrowed. There is often a range of available, competing equipment that can be purchased, each with different costs and different operating efficiencies. Usually, cost and efficiency are related; equipment that operates more efficiently results in reduced operating costs but usually also has higher first costs. The system designer is thus faced with the problem of deciding what equipment should be purchased to minimize the total of the first and operating costs incurred over the time period that the equipment will be operating. The decision process is complicated by the fact that the value of money is not constant but rather a function of time. An additional complication is that the future costs cannot be accurately predicted because they depend upon economic parameters such as the cost of fuel, taxes, interest rates that are not fixed but change with time in an unpredictable manner. Despite these uncertainties, the equipment must be purchased now and therefore the system designer must make his or her choices using a combined thermal and economic analysis.

A.5.2 Present Worth

The expression "time is money" is really true. A fixed sum of money in hand today has greater value than that same sum of money will have some time in the future. Why is this so? Suppose you currently have $1000. You could invest that sum of money and obtain interest on your investment. The interest rate you could obtain from your investment is d, the *market discount rate*. If, for example, the market discount rate is 10% then next year you investment would be worth 1.1 ($1000) or $1100. The following year you would have 1.1 ($1100) or $1210. Each additional year, your investment will continue to grow according to the market discount rate. The $1000 you own today represents a larger sum of money in the future.

Since time and money are interrelated, it is convenient to normalize all costs with respect to its worth today or its *present worth*. The present worth (PW_N) of a sum of money (F) N years in the future can be calculated in terms of the market discount rate using the following simple relation.

$$PW_N = \frac{F}{\left(1+d\right)^N} \tag{A.5-1}$$

A financial commitment must often be paid off in a series of payments, rather than a single lump sum. For example, monthly or annual mortgage payments may be required to repay the loan used to purchase equipment. Alternatively, fuel must be purchased at regular intervals to operate the equipment. The amount of fuel required may be constant, but the cost of the fuel will increase in time due to inflation. The present worth of a series of payments could be calculated by summing the present worth of the individual payments. For example, consider the present worth of a series of annual fuel costs representing a constant amount of fuel use each year. The price of fuel inflates at an annual rate *f*, which in general, is not equal to *d*, the market discount rate. The first year fuel cost is *F*. The present worth of this fuel cost, assuming that it is paid at the end of the year is, from Eq. (A.5-1), $F/(1+d)$. The second year, the fuel price inflates at rate *f* so that the actual amount of money which must be paid at the end of the second year is $F(1+f)$. However, Eq. (A.5-1) indicates that the present worth of this second year payment is $F(1+f) / (1+d)^2$. A series of *N* such payments would have a total present worth of *PW* where

$$PW = PW_1 + PW_2 + \ldots + PW_N = F \sum_{j=1}^{N} \frac{(1+f)^{j-1}}{(1+d)^{j}} \qquad \text{(A.5-2)}$$

The value of the summation in Eq. (A.5-2) is called the present worth factor, *PWF*(*N*, *f*, *d*):

$$PW = F\ PWF\left(N, f, d\right) \qquad \text{(A.5-3)}$$

The present worth factor summation has the analytical solution provided Eq. (A.5-4).

$$PWF(N,f,d) = \sum_{j=1}^{N} \frac{(1+f)^{j-1}}{(1+d)^{j}} = \begin{cases} \dfrac{1}{(d-f)}\left[1-\left(\dfrac{(1+f)}{(1+d)}\right)^{N}\right] & if\ \ f \neq d \\ \dfrac{N}{(f+1)} & if\ \ f = d \end{cases} \qquad \text{(A.5-4)}$$

The PWF function is built into EES and can be accessed by selecting Function Information from the Options Menu and selecting the External routines radio button (select PWF from the list of external routines, as shown in Figure A.5-1).

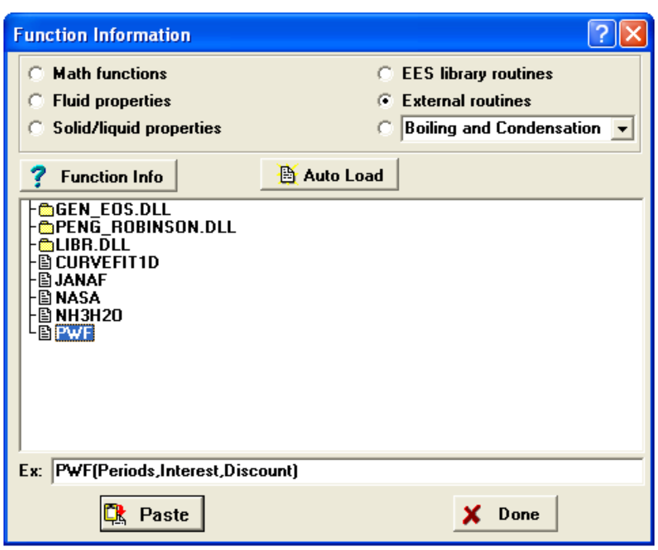

Figure A.5-1: Function Information for the PWF routine.

EXAMPLE A.5-1: Calculation of Annual Mortgage Payments

Determine the annual payment for a *Loan* = \$50,000 loan that is to be repaid in *N* = 10 years with equal, annual payments. The mortgage interest rate is *d* = 6%.

a.) Determine the amount of each annual payment.

The inputs are entered in EES:

```
$UnitSystem SI MASS RAD PA  K J
$TABSTOPS   0.2 0.4 0.6 0.8 3.5 in

"Inputs"
Loan=50000 [$]                          "amount of loan"
N=10 [year]                             "period of loan"
d=0.06 [-]                              "interest rate (discount rate)"
```

The present worth of the 10 annual payments that you make must be equal to the amount of the loan:

$$Loan = AP\ PWF\left(N, 0, d\right) \quad \text{(A.5-5)}$$

where *AP* is the amount of each payment; note that each payment is equal and so the inflation factor (*f*) is set to 0.

```
Loan=AP*PWF(N,0,d)          "the present worth of your payments must equal the amount of the loan"
```

This leads to *AP* = \$6793/year.

A.5.3 Life-Cycle Costing Method

The present worth of the sum of all costs associated with owning and operating a piece of equipment or a system over its estimated life is called the life-cycle cost. It is the life-cycle cost (*LCC*), not the first costs or operating costs, which should dictate the selection of equipment for a thermal system. The life-cycle cost considers the time value of money by relating all future costs to present costs. In addition to operating costs, the future costs may include mortgage payments, insurance and maintenance, additional taxes, and there may be a future payment in the form of any salvage value that the equipment has at the end of its estimated life.

For many thermal systems, the amount of energy that must be purchased in order to operate the equipment does not change significantly from year to year. In this case, the life cycle cost can be calculated using the method presented in Duffie and Beckman (2006) where the life-cycle cost is considered to be the sum of two terms. The first term is proportional to the first year operating cost (*F*), and the second term is proportional to the first costs of the system (*E*).

$$LCC = P_1\,F + P_2\,E \quad \text{(A.5-6)}$$

The proportionality constants are referred to as P_1 and P_2 and the method is referred to as the P_1-P_2 method. The constant P_1 is a present worth factor which depends primarily on the number of years that the equipment is expected to operate (N), the inflation rate for expenses related to operation (f, typically the rate at which the cost of the fuel inflates), and the market discount rate (d). For a residential application, the entire cost of the fuel is an expense so:

$$P_1 = PWF(N, f, d) \tag{A.5-7}$$

For a commercial application, fuel expenses may be deductible from income tax, in which case:

$$P_1 = \left(1-\overline{t}\right) PWF(N, f, d) \tag{A.5-8}$$

where $\overline{t}$ is the effective income tax rate. The value of P_1 for commercial applications is typically about half that for residential applications.

The constant of proportionality P_2 depends on many economic parameters, including the down payment on the first costs, the mortgage interest rate, the market discount rate, the term of the economic analysis, the salvage value of the equipment at the end of economic analysis period, and other economic factors related to first costs such as tax credits, property tax, maintenance, and depreciation. Accounting for the present value of all of these economic factors results in the following expression for P_2.

$$\begin{aligned}
P_2 = {} & \underbrace{D}_{\text{down payment}} + \underbrace{(1-D)\frac{PWF(N_{\min},0,d)}{PWF(N_L,0,m)}}_{\text{payments on principal}} \\
& \underbrace{-\overline{t}(1-D)\left[PWF(N_{\min},m,d)\left(m-\frac{1}{PWF(N_L,0,m)}\right)+\frac{PWF(N_{\min},0,d)}{PWF(N_L,0,m)}\right]}_{\text{tax deductions for interest payments}} \\
& +\left[\underbrace{pV(1-\overline{t})}_{\text{property tax}}+\underbrace{M_s\left(1-c\,\overline{t}\right)}_{\text{maintenance}}\right]PWF(N_e,i,d) \\
& \underbrace{-\frac{c\,\overline{t}}{N_D}PWF(N'_{\min},0,d)}_{\text{depreciation}}\underbrace{-\frac{R_v}{\left(1+d\right)^{N_e}}\left(1-c\,\overline{t}\right)}_{\text{Salvage}}
\end{aligned} \tag{A.5-9}$$

where

- c either 1 (for an income producing) or 0 for non-income producing investment
- d market discount rate
- D ratio of the down payment to the total first costs
- i general inflation rate
- m annual mortgage rate

M_s ratio of maintenance, insurance, and other incidental costs in the first year to the first costs
N number of years considered in the economic analysis
N_L term of the mortgage
$N_{\min}$ minimum of N and N_L
N_D depreciation lifetime in years
$N'_{\min}$ minimum of N and N_D
p property tax rate based on assessed value
R_v ratio of the resale value at the end of the economic period to the first costs
$\overline{t}$ effective income tax rate
V ratio of assessed value in the first year to the first costs

Functions for P_1 and P_2, Eqs. (A.5-7) through (A.5-9) are provided in EES (select Function Information from the Options menu and then select EES library routines and scroll down to the P1P2.LIB option. Click on the P1P2 folder icon to view the documentation).

The relation for P_2 appears complex because it depends on many economic factors related to tax laws. In practice, evaluation of P_2 is easy calculated provided the economic factors are known. The constant P_2 normally ranges between 0.6 and 1.5, depending primarily on the down payment and tax credits. One of the benefits of using the P_1-P_2 method is that it is possible to carry out a carry out a parametric study in which the constant P_2 is varied and relate that to variations in any of the economic parameters listed above.

The life-cycle cost formulation requires the specification of many economic parameters, such as the market discount rate, the fuel inflation rate, and so on. Most of these parameters will be the same for competing designs and therefore when comparing two alternatives it may be simpler and more accurate to evaluate the life-cycle savings (*LCS*) associated with these alternatives (i.e., the difference in the life-cycle costs of the two alternatives). An uncertainty analysis can be used to determine whether the life-cycle savings remains positive when one or more of the economic parameters are given an uncertainty band.

EXAMPLE A.5-2 Choosing a Furnace Based on Life-Cycle Costs

A homeowner must purchase a new natural-gas furnace and has narrowed the choice down to two alternatives: a conventional furnace having an efficiency η_1 = 0.65 (based on the lower heating value of the fuel) and an installed cost of E_1 = \$3,200 or a high-efficiency, pulse-combustion furnace having an efficiency of η_2 = 0.92 and an installed cost of E_2 = \$4,400. A review of past heating bills indicates that the building requires an average heat input of about Q = 70 GJ per year. The current cost of natural gas is ng = \$1.25 / therm. The pulse-combustion furnace has a considerably higher first cost, but its higher efficiency translates into reduced operating costs which may cause the pulse-combustion furnace to be the wiser investment. A rational decision can only be made by comparing the life-cycle costs of the two alternatives.

a.) Which furnace should the homeowner purchase?

The input parameters are entered in EES:

```
$UnitSystem SI MASS RAD PA  K J
$TABSTOPS   0.2 0.4 0.6 0.8 3.5 in

"Inputs"
eta_1=0.65 [-]                                   "efficiency of furnace 1"
E_1=3200 [$]                                     "installed cost of furnace 1"
eta_2=0.92 [-]                                   "efficiency of furnace 2"
E_2=4400 [$]                                     "installed cost of furnace 2"
Q = 70 [GJ]*convert(GJ, J)                       "average heat transfer required per year"
ng = 1.25 [$/therm]*convert($/therm, $/J)        "cost of natural gas"
```

The most difficult part of a life-cycle analysis is the selection of economic parameters. The values chosen for this example are shown in Table 1. Some of the parameters, for example, the fuel inflation rate, require guesswork and judgment. Nevertheless, a choice must be made. It may be wise to repeat the calculations for a range of values of the economic parameters to see if a change in their values that is within their uncertainty band affect the decision process.

Table 1: Parameters Used in the Life-Cycle Cost Analysis

Symbol	Parameter	Nominal value
c	commercial flag	0
d	market discount rate	0.04
D	down payment fraction	0.20
f	fuel inflation rate	0.045
i	general inflation rate	0.035
m	equipment mortgage rate	0.065
N	period of the economic analysis	10 years
N_D	depreciation lifetime	10 years
N_L	period of the mortgage	10 years
M_s	maintenance, insurance as a fraction of first cost	0.01
p	property tax rate	0.035
R_v	salvage value	0
$\bar{t}$	effective tax rate	0.35
V	ratio of assessed value to first cost	1.0

The economic parameters listed in Table 1 are entered in EES:

```
"Economic Parameters"
c = 0 [-]                          "commercial flag"
d = 0.04 [-]                       "market discount rate"
DD=0.20 [-]                        "down payment fraction"
f = 0.045 [-]                      "fuel inflation rate"
i = 0.035 [-]                      "general inflation rate"
m = 0.065 [-]                      "equipment mortgage rate"
N = 10 [year]                      "period of economic analysis"
N_D=10 [year]                      "depreciation lifetime"
```

```
N_L=10 [year]                "period of mortgage"
M_s = 0.01     [-]           "ratio of maintenance and insurance to first cost"
p = 0.035 [-]                "property tax rate"
R_v=0 [-]                    "salvage value"
t_bar = 0.35 [-]             "effective tax rate"
V = 1.0 [-]                  "ratio of assessed value to first cost"
```

The P_1 and P_2 parameters are computed using EES' internal functions:

```
P_1=P_1f(N, i, d, c, t_bar)                                         "P1 constant"
P_2=P_2f(c, d, DD, i, m, M_s, N, N_L, N_D,  p, R_v, t_bar,V)        "P2 constant"
```

which leads to P_1 = 9.826 and P_2 = 1.317, respectively. Once the P_1 and P_2 parameters are calculated it is relatively easy to determine the life cycle costs of the two alternatives. The fuel cost associated with the first year of operation is provided by:

$$F = \frac{Q\, ng}{\eta}$$

```
F_1=Q*ng/eta_1         "first year fuel cost for low efficiency furnace"
F_2=Q*ng/eta_2         "first year fuel cost for high efficiency furnace"
```

The life cycle cost of each option is computed:

$$LCC = P_1\, F + P_2\, E \qquad \text{(A.5-10)}$$

```
LCC_1=P_1*F_1+P_2*E_1        "life cycle cost of the low efficiency furnace"
LCC_2=P_1*F_2+P_2*E_2        "life cycle cost of the high efficiency furnace"
```

which leads to LCC_1 = 16,750$ and LCC_2 = 14,650$; therefore, the life cycle cost of the high efficiency furnace is substantially less than that of the low efficiency furnace. The life-cycle savings of the pulse-combustion furnace over the conventional furnace is the difference in the life-cycle costs:

$$LCS = LCC_1 - LCC_2 \qquad \text{(A.5-11)}$$

```
LCS=LCC_1-LCC_2              "life cycle savings of the high efficiency furnace"
```

which leads to LCS = $2098. The life-cycle savings of the pulse-combustion furnace alternative is positive indicating that, for the economic parameters assumed in this analysis, it would be a more economical choice.

A.5.4 Economic Figures of Merit

The purpose of an economic evaluation is to obtain a numerical representation of the merit of an alternative system design. A straightforward method of carrying out an economic analysis is to calculate the life-cycle costs of competing designs in order to compare them. However, investment options may not necessarily represent competing designs. For example, suppose the homeowner referenced in EXAMPLE E.3-1 must choose between replacing his or her furnace with a more efficient model or replacing the leaky windows in the home. In this case, a direct comparison of the life-cycle costs is inappropriate. Two common figures of merit which could be used to compare the investment options are the Return On Investment (*ROI*) and the Payback Time (*PT*).

The return on investment is defined as the market discount rate (*d*) which results in a zero life-cycle savings. A value of *ROI* greater than the current market discount rate indicates that purchasing and operating the proposed equipment will achieve a higher earnings than if the first costs were simply invested in the market (and therefore is a wise investment).

The Payback Time (*PT*) provides an indication of how many years will be needed for the investment to pay for itself. This figure of merit can defined in many different ways, depending on how 'pay for itself' is interpreted. A common interpretation is that the cumulative savings in operating costs associated with some equipment must be equal to the additional first costs, which leads to the following expression for the simple payback time.

$$PT = \frac{First\ Costs}{Annual\ Operating\ Cost\ Savings} \qquad \text{(A.5-12)}$$

Equation (A.5-12) has the advantage of being simple to evaluate; however, it does not consider the time value of money. Other definitions for the payback time are 1) the number of years for the yearly cash flow to become positive and 2) the time required for the cumulative savings to equal 0. It is necessary to know the definition of any figure of merit that is used in an economic analysis.

References:

Duffie, J.A., and Beckman, W.A., ***Solar Engineering of Thermal Processes***, Third Edition, Wiley Interscience, New York, (2006)