

Chapter 9

MASS TRANSFER

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9.1 Mass Transfer Concepts

9.1.1 Introduction

The subject of this chapter is more specific than is implied by the title. Mass transfer occurs whenever fluid flows; that is, some mass is transferred from one place to another. However, the focus here is not on mass transported by this bulk fluid motion, but rather on the transport of one chemical species (or component) within a mixture of chemical species that occurs as a direct result of a concentration gradient and is independent of a pressure gradient. This type of mass transfer is called diffusion.

A familiar example of diffusion mass transfer is the humidification process that occurs when an open container of water is allowed to sit in a room. The gas in the room is a mixture of air (which is itself a mixture of oxygen, nitrogen and other gases) and water vapor. The air-water mixture near the surface of the water is nearly saturated with water vapor and so it has a relatively high concentration of water vapor. The air-water mixture further from the container has a lower concentration of water vapor. Therefore, there is a concentration gradient that drives a mass transfer process causing water to be transported from the liquid surface throughout the room, thereby humidifying it.

This textbook is focused on heat transfer; therefore, it is appropriate to ask why a chapter on mass transfer is included. There are two answers to this question. First, mass transfer, like momentum transfer, plays an important role in many important heat exchange processes and devices. For example, mass transfer is critical to the operation of the cooling coils, cooling towers, and evaporative coolers and condensers that are commonly used in refrigeration and power systems. The energy transfer that occurs as a result of mass transfer can significantly improve the performance of these heat transfer devices. Second, the processes of heat and mass transfer are analogous. The governing equations for heat and mass transfer are similar and therefore many of the relations and solution techniques that have been developed for heat transfer can be directly applied to mass transfer processes as well.

9.1.2 Composition Relationships

Diffusion involves a mixture of two or more distinct chemical species. In this text, we will assume that these species are chemically inert. The methods that are used to quantify the relative amounts of the different species in the mixture are summarized in this section. These relationships can be equivalently expressed on mass or molar bases.

The total mass of the mixture (M) is the sum of the mass of each constituent of the mixture (M_i , where i identifies the component):

$$M = \sum_{i=1}^{N_c} M_i \quad (9-1)$$

where N_s is the number of species that are present in the mixture. The mass fraction of species i (mf_i) is the ratio of the mass of species i to the total mass of the mixture:

$$mf_i = \frac{M_i}{M} \quad (9-2)$$

Since diffusion is a molecular phenomenon, it is often useful to specify composition on a molar basis rather than a mass basis. A mole of species i is defined as the amount of mass that is equal to the molar mass of species i (MW_i). Therefore, the mass units need to be specified. If the mass unit is chosen to be kg, then the corresponding mole is called a kmol (or kgmol). For example, the molar mass of helium is 4 and therefore a kmol of helium has a mass of 4 kg. A pound mole (lbmol) of helium has a mass of 4 pounds. A gram mole (gmol) of helium has a mass of 4 grams, and so on. Often the term “mole” is expressed without reference to mass units; in this case, mole usually refers to a gmol. The moles and mass of species i are related by:

$$N_i = \frac{M_i}{MW_i} \quad (9-3)$$

The total number of moles in a mixture (N) is the sum of the moles of each species (N_i):

$$N = \sum_{i=1}^{N_s} N_i \quad (9-4)$$

The mole fraction of species i (y_i) is the number of moles of that species divided by the total moles of mixture.

$$y_i = \frac{N_i}{N} \quad (9-5)$$

Note that the mass (M) and moles (N) of mixture are related by MW , the molar mass of the mixture, according to:

$$MW = \frac{M}{N} \quad (9-6)$$

where

$$MW = \sum_{i=1}^{N_s} y_i MW_i \quad (9-7)$$

The mass fraction and mole fraction are related according to:

$$MW_i = \frac{M_i}{N_i} = \frac{mf_i M}{y_i N} = \frac{mf_i}{y_i} MW \quad (9-8)$$

The diffusion mass transfer of a component is driven by differences in the concentration of the component. Concentration is the amount of the component present per unit volume and can be expressed on both mass and molar bases using the mass or mole fraction, respectively. On a mass basis, the concentration of species i (c_i) is the mass of species i per unit volume, which can be expressed as:

$$c_i = \frac{M_i}{V} = \frac{mf_i M}{V} = mf_i \rho \quad (9-9)$$

where V is the volume and ρ is the mass density of the mixture. The molar concentration of species i (n_i) is the number of moles of species i per unit volume, which can be expressed as:

$$n_i = \frac{N_i}{V} = \frac{y_i N}{V} = y_i n \quad (9-10)$$

where n is the molar density (N/V) of the mixture.

Ideal Gas Relationships

Diffusing species are often low density gases that obey the ideal gas law. An important example is a mixture of air and water vapor at near atmospheric pressure and temperature; this is an ideal gas mixture that is involved in many evaporative heat and mass transfer processes. When applicable, the ideal gas law may be used to compute the mass or molar concentrations by defining a partial pressure for each component, p_i , according to:

$$p_i = y_i p = mf_i \left(\frac{MW}{MW_i} \right) p \quad (9-11)$$

where p is the total pressure of the gas mixture. Application of the ideal gas law allows the mass and molar concentrations of species to be expressed in terms of the partial pressure of the species.

$$c_i = \frac{M_i}{V} = \frac{p_i}{R_i T} \quad (9-12)$$

$$n_i = \frac{N_i}{V} = \frac{p_i}{R_{univ} T} = \frac{y_i p}{R_{univ} T} \quad (9-13)$$

where R_i is the gas constant for species i (on a mass basis) and R_{univ} is the universal gas constant (8314.3 J/kgmol-K); note that R_{univ} and R_i are related according to:

$$R_i = \frac{R_{univ}}{MW_i} \quad (9-14)$$

9.2 Mass Diffusion and Fick's Law

9.2.1 Development of Fick's Law

Mass diffusion refers to the diffusive transport of a species due to concentration gradients in a mixture. Mass transport by diffusion is analogous to conduction, which is the diffusive transport of energy due to temperature gradients. Therefore, a molecular description of diffusion is similar to the molecular description of conduction that is presented in Section 1.1.2.

Consider the mass diffusion process shown in Figure 9-1 where a pure liquid (species i , e.g., water) is at rest. Molecules from the liquid surface vaporize to form vapor species i that diffuses into the gas (e.g., air) that exists above the liquid. The mole fraction of the species i is highest just above the surface of the liquid (at $x = 0$) where it is at its saturation value ($y_{i,sat}$). The mole fraction of species i decreases with vertical distance from the liquid surface (x), eventually reaching a value of $y_{i,\infty}$ that corresponds to the uniform, and likely small, mole fraction of species i in the gas far removed from the liquid surface. Mass transfer of species i is driven by the gradient in the mole fraction of species i (y_i) that exists in the x -direction. Equivalently, the diffusion mass transfer can be thought of as being driven by the gradient in the molar concentration, mass fraction, mass concentration, or partial pressure of species i .

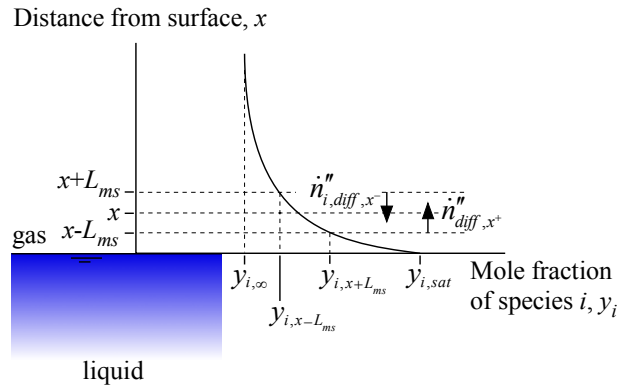


Figure 9-1: Mass diffusion in a mixture.

The flux of molecules of all species that are passing through a horizontal plane at position x is proportional to the molar density of the mixture (n) and the mean velocity of the molecules (v_{ms}) at that location. On average, these molecules experienced their last molecular interaction at $x-L_{ms}$, where L_{ms} is the average distance between molecular interactions (i.e., the mean free path). Therefore, the number of these molecules that are of species i is proportional to the mole fraction of species i at position $x-L_{ms}$. The molar flux of species i passing through a plane located at position x in the positive x -direction due to diffusion ($\dot{n}''_{i,diff,x^+}$) is given approximately by:

$$\dot{n}''_{i,diff,x^+} \approx n v_{ms} y_{i,x-L_{ms}} \quad (9-15)$$

where $y_{i,x-L_{ms}}$ is the mole fraction of species i at position $x-L_{ms}$. Similarly, the molar flux of species i passing through the horizontal plane in the negative x -direction due to diffusion ($\dot{n}''_{i,diff,x^-}$) is given approximately by:

$$\dot{n}''_{i,diff,x^-} \approx n v_{ms} y_{i,x+L_{ms}} \quad (9-16)$$

The net molar flux of species i due to diffusion ($\dot{n}''_{i,diff}$) is the difference between $\dot{n}''_{i,diff,x^+}$ and $\dot{n}''_{i,diff,x^-}$:

$$\dot{n}''_{i,diff} \approx n v_{ms} (y_{i,x-L_{ms}} - y_{i,x+L_{ms}}) \quad (9-17)$$

which can be rearranged to yield:

$$\dot{n}''_{i,diff} \approx -2 n v_{ms} L_{ms} \underbrace{\frac{(y_{i,x+L_{ms}} - y_{i,x-L_{ms}})}{2 L_{ms}}}_{\frac{\partial y_i}{\partial x}} \quad (9-18)$$

Provided that the length between molecular interactions is much smaller than the length scale that characterizes the problem (i.e., the Knudsen number is small), Eq. (9-18) can be written in terms of the gradient in the mole fraction of species i :

$$\dot{n}''_{i,diff} \approx -2 n v_{ms} L_{ms} \frac{\partial y_i}{\partial x} \quad (9-19)$$

Equation (9-19) provides the motivation for Fick's law, which states that diffusive mass transfer is proportional to the gradient in the mole fraction of species i . Fick's law is typically written in terms of the diffusion coefficient for species i through the mixture ($D_{i,m}$):

$$\boxed{\dot{n}''_{i,diff} = -n D_{i,m} \frac{\partial y_i}{\partial x} = -D_{i,m} \frac{\partial n_i}{\partial x}} \quad (9-20)$$

Fick's law can also be written on a mass basis. The diffusive mass (rather than molar) flux of species i ($\dot{m}''_{i,diff}$) is related to the gradient in the mass fraction or mass concentration of species i :

$$\boxed{\dot{m}''_{i,diff} = -\rho D_{i,m} \frac{\partial m f_i}{\partial x} = -D_{i,m} \frac{\partial c_i}{\partial x}} \quad (9-21)$$

Equations (9-20) and (9-21) are statements of Fick's law of diffusion and define the diffusion coefficient for species i in the mixture in the same way that Fourier's law for conduction defines the thermal conductivity of a material:

$$\dot{q}'' = -k \frac{\partial T}{\partial x} \quad (9-22)$$

In both Fick's law and Fourier's law, the diffusive flux of interest is proportional to a gradient and the constant of proportionality (k or $D_{i,m}$) is a property of the substance that reflects its microscopic nature. In Section 1.1.2, it is shown that the material property thermal conductivity reflects the underlying characteristics of the energy carriers in the substance according to:

$$k \approx n_{ms} v_{ms} c_{ms} L_{ms} \quad (9-23)$$

where c_{ms} is the ratio of the amount energy carried by each micro-scale energy carrier to its temperature. The diffusion coefficient reflects the underlying characteristics of the molecules in the mixture. Comparing Eq. (9-19) with Eq. (9-20) leads to:

$$D_{i,m} \approx v_{ms} L_{ms} \quad (9-24)$$

The diffusion coefficient is related to the product of the mean velocity of the molecules and the average distance between molecular interactions. Notice that the diffusion coefficient has units m^2/s as does thermal diffusivity (α) and kinematic viscosity (ν). We will see that the diffusion coefficient plays the same role in diffusion mass transfer processes that α and ν play in heat transfer and momentum transfer processes, respectively.

9.2.2 The Diffusion Coefficient for Binary Mixtures

The diffusion coefficient, $D_{i,m}$, is a transport property that represents the ability of species i to diffuse through a medium m . The medium can be a gas, liquid or solid. In general, diffusion coefficients are largest for gases, lower for liquids and lowest for solids. Gases have a large mean free path which, according to Eq. (9-24), leads to a large diffusion coefficient. The diffusion coefficient is a mixture property that depends, in general, on the properties of all of the interacting species as well as on pressure and temperature. This is also true of the thermal conductivity and viscosity of a gas or liquid mixture and the diffusion coefficient is related to these other transport properties.

Diffusion problems are often concerned with the mass transfer of a single species, such as water vapor, within an otherwise homogeneous phase, such as a pure gas or a homogeneous gas mixture (e.g., air). In this case, the mixture can be treated as a binary (i.e., two component) system. The binary diffusion coefficient for species 1 through another species 2 is termed $D_{1,2}$. It is possible to show that $D_{1,2}$ must be equal to $D_{2,1}$. The remainder of this section describes some methods and computational tools that can be used to estimate the binary diffusion coefficient for various systems. The methods presented in this section are approximate and therefore should only be used to determine approximate or order-of-magnitude values for the diffusion coefficient. Compilations of methods for estimating diffusion coefficients are provided by Perry and Green (1997) and by Poling et al. (2000). Experimental values of diffusion coefficients are provided by Lide and Kehiaian (1994).

Binary Diffusion Coefficients for Gas Mixtures

The diffusion coefficient for a binary gas mixture at low to moderate pressures can be estimated using kinetic theory. A formulation credited to Chapman and Enskog in Poling et al. (2000) results in the following relation:

$$D_{1,2} = 1.883 \times 10^{-22} \frac{T^{3/2}}{p l_{1,2}^2 \Omega_D} \sqrt{\frac{MW_1 + MW_2}{MW_1 MW_2}} \quad (9-25)$$

where $D_{1,2}$ is the binary diffusion coefficient (m^2/s), T is the absolute temperature (K), MW_1 and MW_2 are the molar masses of species 1 and 2, p is the pressure (Pa), $l_{1,2}$ is a characteristic length of the mixture (m), and Ω_D is the dimensionless collision integral for diffusion. The characteristic length ($l_{1,2}$) is estimated according to the average of the Lennard-Jones 12-6 potential characteristic lengths for species 1 and 2.

$$l_{1,2} = \frac{l_1 + l_2}{2} \quad (9-26)$$

Tabulations for the Lennard-Jones characteristic lengths are provided in Perry and Green (1997) and in Poling et al. (2000). They are also provided by the EES function `sigma_LJ` for all of the substances that are available in the EES database. The collision integral for diffusion (Ω_D) is a function of the dimensionless temperature:

$$\tilde{T} = \frac{k_B T}{\varepsilon_{1,2}} \quad (9-27)$$

where k_B is Boltzmann's constant (1.381×10^{-23} J/K), T is the absolute temperature and $\varepsilon_{1,2}$ is a characteristic mixture energy parameter. The mixture energy parameter $\varepsilon_{1,2}$ can be estimated in terms of the Lennard-Jones 12-6 potential characteristic energies for species 1 and 2 (ε_1 and ε_2) according to:

$$\varepsilon_{1,2} = \sqrt{\varepsilon_1 \varepsilon_2} \quad (9-28)$$

Values of the quantity ε_i / k_B (which has units of temperature) for many species are provided in Poling et al. (2000) and are also provided by the EES function `ek_LJ` for all of the substances that are available in the EES database. When values of these parameters are otherwise not available, they can be estimated from the critical temperature (T_{crit}) according to Eq. (9-29), which is adapted from McCabe and Smith (1967):

$$\frac{\varepsilon_i}{k_B} = 0.77 T_{crit,i} \quad (9-29)$$

An analytical approximation for Ω_D provided by Poling et al. (2000) is:

$$\Omega_D = \frac{1.06036}{\tilde{T}^{0.1561}} + \frac{0.1930}{\exp(0.47635\tilde{T})} + \frac{1.03587}{\exp(1.52996\tilde{T})} + \frac{1.76474}{\exp(3.89411\tilde{T})} \quad (9-30)$$

The `D_12_gas` function is provided in EES in order to evaluate the diffusion coefficient for any two gases that are available in the EES database using the technique described in this section.

EXAMPLE 9.2-1: Diffusion Coefficient for Air-Water Vapor Mixtures

Heat and mass transfer applications involving air-water vapor mixtures can often provide significantly higher energy transfer rates than ‘dry’ heat exchange processes; therefore, the diffusion coefficient for water vapor in air is an important quantity in many engineering applications.

- a.) Calculate and plot the diffusion coefficient for water vapor in air as a function of temperature at atmospheric pressure using the technique described in Section 9.2.2.

The solution to this problem is a straightforward application of the method discussed in Section 9.2.2. The input parameters are entered in EES:

"EXAMPLE 9.2-1: Diffusion Coefficient for Air-Water Vapor Mixtures"

\$UnitSystem SI MASS RAD PA K J

\$Tabstops 0.2 0.4 0.6 3.5 in

p=1 [atm]*convert(atm,Pa)

"pressure"

T_C=20 [C]

"temperature, in C"

T=converttemp(C,K,T_C)

"temperature in K"

G1\$='Air'

"species 1"

G2\$='Water'

"species 2"

The Lennard-Jones 12-6 characteristic energies normalized by Boltzmann's constant are obtained for both species (ε_1/k_B and ε_2/k_B) using the `ek_LJ` function in EES and used to estimate the normalized characteristic energy of the mixture according to Eq. (9-28):

$$\frac{\varepsilon_{1,2}}{k_B} = \sqrt{\frac{\varepsilon_1}{k_B} \frac{\varepsilon_2}{k_B}}$$

epsilon\kB_1=ek_LJ(G1\$)

"Lennard-Jones 12-6 potential energy for species 1"

epsilon\kB_2=ek_LJ(G2\$)

"Lennard-Jones 12-6 potential energy for species 2"

epsilon\kB_1_2=sqrt(epsilon\kB_1*epsilon\kB_2)

"characteristic mixture energy"

The dimensionless temperature is computed according to Eq. (9-27):

$$\tilde{T} = T \frac{k_B}{\epsilon_{1,2}}$$

and used to estimate the dimensionless collision integral for diffusion using Eq. (9-30):

$$\Omega_D = \frac{1.06036}{\tilde{T}^{0.1561}} + \frac{0.1930}{\exp(0.47635\tilde{T})} + \frac{1.03587}{\exp(1.52996\tilde{T})} + \frac{1.76474}{\exp(3.89411\tilde{T})}$$

T_bar=T/epsilon\kB_1_2 "dimensionless temperature"
 OMEGA_D=1.06036/T_bar^0.1561+0.1930/exp(0.47635*T_bar)&
 +1.03587/exp(1.52996*T_bar)+1.76474/exp(3.89411*T_bar) "collision integral for diffusion"

The Lennard-Jones 12-6 characteristic lengths for both species (l_1 and l_2) are obtained using the sigma_LJ function in EES and used to estimate the characteristic length of the mixture according to Eq. (9-26):

$$l_{1,2} = \frac{l_1 + l_2}{2}$$

l_1=sigma_LJ(G1\$) "Lennard-Jones 12-6 length for species 1"
 l_2=sigma_LJ(G2\$) "Lennard-Jones 12-6 length for species 2"
 l_1_2=(l_1+l_2)/2 "characteristic mixture length"

The molarmasses of both species (MW_1 and MW_2) are obtained using the MolarMass function in EES and the binary diffusion coefficient is estimated according to Eq. (9-25):

$$D_{1,2} = 1.883 \times 10^{-22} \frac{T^{3/2}}{p l_{1,2}^2 \Omega_D} \sqrt{\frac{MW_1 + MW_2}{MW_1 MW_2}}$$

MW_1=MolarMass(G1\$) "molar mass of species 1"
 MW_2=MolarMass(G2\$) "molar mass of species 2"
 D_1_2=1.883e-22*T^1.5*sqrt((MW_1+MW_2)/(MW_1*MW_2))/(p*l_1_2^2*OMEGA_D)
 "diffusion coefficient"

which leads to $D_{1,2} = 2.07 \times 10^{-5} \text{ m}^2/\text{s}$. Note that these calculations are automated using the D_12_gas function in EES. The D_12_gas function can be accessed by selecting Function Info from the Options menu and selecting EES library routines; scroll down to the Diffusion library and click on it to open the library and display the functions that are contained in the library.

D_1_2_EES=D_12_gas(G1\$,G2\$,T,P) "EES function for diffusion coefficient"

which leads to $D_{1,2,EES} = 2.07 \times 10^{-5} \text{ m}^2/\text{s}$.

Figure 1 illustrates the calculated diffusion coefficient as a function of temperature. Also shown in Figure 1 are reference values for the diffusion coefficient of water vapor in air, provided by

Bolz and Tuve (1976). Notice that the values calculated using the method presented in Section 9.2.2 are approximately 15% less than the values from Bolz and Tuve (1976). A more accurate function for the diffusion coefficient of water vapor in air is obtained using a regression curve fit to the data from Bolz and Tuve (1976):

$$D_{a,w} = -2.775 \times 10^{-6} \left[\frac{\text{m}^2}{\text{s}} \right] + 4.479 \times 10^{-8} \left[\frac{\text{m}^2}{\text{s-K}} \right] T + 1.656 \times 10^{-10} \left[\frac{\text{m}^2}{\text{s-K}^2} \right] T^2 \quad (1)$$

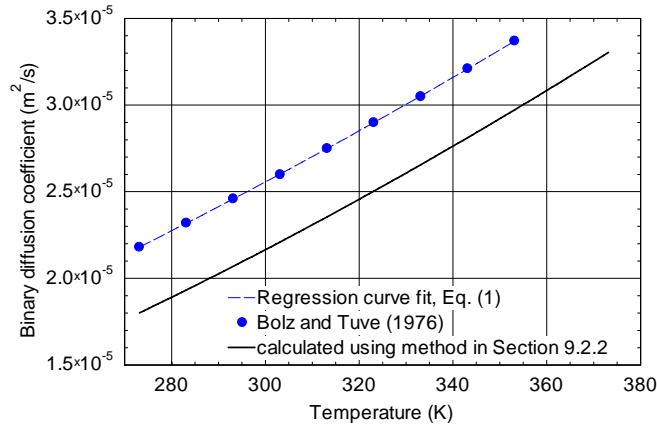


Figure 1: The air-water vapor diffusion coefficient as a function of temperature, estimated using the method described in Section 9.2.2 and provided by Bolz and Tuve (1976).

Infinite Dilution Diffusion Coefficients for Liquids

Diffusion coefficients for a solute within a liquid solvent are much smaller than those associated with the diffusion of one gas species within a gas mixture. This is the result of the smaller value of the distance between molecular collisions, L_{ms} in Eq. (9-24), as well as larger intermolecular forces between adjacent molecules in the liquid; both of these effects tend to reduce the mobility of a diffusing species. The complex force fields governing molecular interactions make it difficult to estimate the diffusion coefficient within a liquid according to any simple theory. The complexity of the problem is reduced in the fairly common situation associated with the diffusion of a solute that has a very low concentration within a liquid solvent. The term ‘infinite solution’ is used to refer to this situation and the diffusion coefficient at infinite dilution is ordinarily identified as $D_{1,2}^o$ where the solute is species 1, the solvent is species 2 and the superscript o indicates infinite dilution.

Correlations for diffusion coefficients at infinite dilution are presented in Poling et al. (2000); the Tyn-Calus correlation is recommended. A modified form of the Tyn-Calus correlation is:

$$D_{1,2}^o = 9.013 \times 10^{-16} \left(\frac{v_1^{0.267}}{v_2^{0.433}} \right) \frac{T}{\mu_2} \left(\frac{\sigma_2}{\sigma_1} \right)^{0.15} \quad (9-31)$$

where $D_{1,2}^o$ is the infinite dilution diffusion coefficient of solute 1 in solvent 2 (m^2/s), v_1 and v_2 are the specific molar volumes of the solute and solvent (m^3/gmol), T is the absolute temperature (K), μ_2 is the viscosity of the solvent ($\text{kg}/\text{m}\cdot\text{s}$), and σ_1 and σ_2 are the surface tensions of the solute and solvent (N/m), respectively, at their normal boiling point temperatures. The function `D_12|o_liquid` in EES uses Eq. (9-31) to estimate the infinite dilution diffusion coefficient.

9.3 Transient Diffusion through a Stationary Medium

Diffusion in a fluid is usually accompanied by fluid motion, as discussed in Section 9.4. However, in this section, we consider the situation where the medium that the species is diffusing through is stationary; for example, the diffusion of a gas through a solid. One purpose of this analysis is to illustrate the similarities that exist between the equations that govern conduction problems and those that govern diffusion problems. Many of the solution techniques discussed in Chapters 1 through 3 for conduction heat transfer can be applied directly to mass transfer problems of this type.

Consider the transient diffusion problem shown in Figure 9-2. A semi-infinite, stationary medium initially has a uniform mass concentration of species 1, $c_{1,ini}$, when the surface (at $x = 0$) is exposed to a fluid that contains species 1. Therefore, the surface of the medium experiences a step change in mass concentration of species 1, from $c_{1,ini}$ to $c_{1,s}$. There are no chemical reactions occurring in the medium.

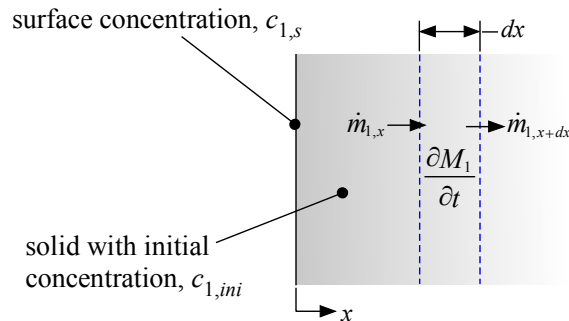


Figure 9-2: Transient diffusion of species 1 through a semi-infinite solid

The governing differential equation for the diffusion of species 1 through the medium can be derived using the same techniques that were used throughout Chapters 1 through 3 for conduction problems; the difference is that here we will balance mass rather than energy. A differential control volume is defined, as shown in Figure 9-2, and a mass balance on species 1 within the control volume leads to:

$$\dot{m}_{1,x} = \dot{m}_{1,x+dx} + \frac{\partial M_1}{\partial t} \quad (9-32)$$

where \dot{m}_1 is the mass flow rate of species 1 and M_1 is the mass of species 1 within the control volume. Expanding the $x+dx$ term in the usual manner leads to:

$$\dot{m}_{1,x} = \dot{m}_{1,x} + \frac{\partial \dot{m}_{1,x}}{\partial x} dx + \frac{\partial M_1}{\partial t} \quad (9-33)$$

The only source of mass flow of species 1 is due to diffusion since the medium is stationary. Therefore, the mass flow rate of species 1 is given by Fick's law, Eq. (9-21):

$$\dot{m}_1 = -A_c D_{1,m} \frac{\partial c_1}{\partial x} \quad (9-34)$$

where A_c is the cross-sectional area for diffusion and $D_{1,m}$ is the diffusion coefficient of species 1 through the medium. The rate of change of the mass of species 1 in the differential control volume can be expressed as:

$$\frac{\partial M_1}{\partial t} = A_c dx \frac{\partial c_1}{\partial t} \quad (9-35)$$

Substituting Eqs. (9-34) and (9-35) into the mass balance, Eq. (9-33), results in:

$$0 = \frac{\partial}{\partial x} \left[-A_c D_{1,m} \frac{\partial c_1}{\partial x} \right] dx + A_c dx \frac{\partial c_1}{\partial t} \quad (9-36)$$

Assuming that the diffusion coefficient and cross-sectional area are constant, Eq. (9-36) can be rearranged to yield the governing differential equation for transient one-dimensional diffusion through a stationary medium.

$$D_{1,m} \frac{\partial^2 c_1}{\partial x^2} = \frac{\partial c_1}{\partial t} \quad (9-37)$$

The boundary conditions for Eq. (9-37) are related to the concentration at the surface,

$$c_{1,x=0} = c_{1,s} \quad (9-38)$$

the initial concentration in the solid,

$$c_{1,t=0} = c_{1,ini} \quad (9-39)$$

and the concentration far from the surface,

$$c_{1,x \rightarrow \infty} = c_{1,ini} \quad (9-40)$$

It is important to recognize the similarity in the governing relations describing heat and mass transfer. Equation (9-37) is analogous to the transient one-dimensional conduction equation that governs the temperature within a semi-infinite body, derived in Section 3.3.3:

$$\alpha \frac{\partial^2 T}{\partial x^2} = \frac{\partial T}{\partial t} \quad (9-41)$$

If temperature (T) in Eq. (9-41) is replaced with concentration of species 1 (c_1) and the thermal diffusivity (α) in Eq. (9-41) is replaced with the diffusion coefficient ($D_{1,s}$) then Eq. (9-41) is identical to Eq. (9-37). Consequently, the solutions and solution techniques for conduction problems that are presented in Chapters 1 through 3 can be directly employed for mass transfer problems. For example, in Section 3.3.3 a semi-infinite solid exposed to a step change in surface temperature is considered; this conduction problem is analogous to the mass transfer problem considered here. The boundary conditions for the conduction problem considered in Section 3.3.3 are:

$$T_{x=0} = T_s \quad (9-42)$$

$$T_{t=0} = T_{ini} \quad (9-43)$$

$$T_{x \rightarrow \infty} = T_{ini} \quad (9-44)$$

Equations (9-38) to (9-40) are identical to Eqs. (9-42) to (9-44) provided that T is replaced with c_1 . Therefore, the solution derived in Section 3.3.3 for the temperature:

$$T = T_s + (T_{ini} - T_s) \operatorname{erf} \left(\frac{x}{2\sqrt{\alpha t}} \right) \quad (9-45)$$

can be used to determine the concentration of species 1, c_1 , provided that α is replaced with $D_{1,m}$ and T is replaced with c_1 :

$$c_1 = c_{1,s} + (c_{1,ini} - c_{1,s}) \operatorname{erf} \left(\frac{x}{2\sqrt{D_{1,m} t}} \right) \quad (9-46)$$

Note from Eq. (9-46) that the diffusion of mass is characterized by a mass diffusion penetration depth, δ_d , just as the diffusion of energy and momentum are characterized by thermal and momentum penetration depths, δ_t and δ_m , respectively. The mass diffusion penetration depth grows approximately according to:

$$\delta_d = 2\sqrt{D_{1,m} t} \quad (9-47)$$

The EES functions and the numerical and analytical solution techniques for transient conduction heat transfer can be applied to mass transfer through a stationary medium.

9.4 Mass Convection

Section 9.3 considered the diffusion of a species through a stationary solid. The diffusion of a species through a fluid is more complex because of the flows (diffusional and otherwise) that may result in the fluid either due to external forces or due to the diffusion mass transfer process itself. In Section 9.4.1, the diffusion of a species through a ‘stationary’ fluid is considered; that is, diffusion through a fluid with no externally imposed velocity (a quiescent fluid). Note that the diffusion process itself will result in some bulk velocity and therefore the fluid is not truly stationary. In Section 9.4.2, mass transfer in the presence of an externally driven fluid is considered.

9.4.1 Diffusion of a Species in a Stationary Fluid (Stefan Flow)

Fick's law predicts the mass transfer of species i due to diffusion only. If there is any bulk motion of the system that species i is diffusing through (which is the usual case in a gas or liquid) then there is also a bulk mass transfer associated with the bulk velocity.

If species i is diffusing in the x -direction through a stationary fluid medium (designated by the subscript m) then the total flux of species i (\dot{n}_i'') is the sum of the transfer due to diffusion ($\dot{n}_{i,diff}''$) and the transfer due to the bulk motion of the fluid:

$$\dot{n}_i'' = \underbrace{\dot{n}_{i,diff}''}_{\text{diffusive transport}} + \underbrace{u n y_i}_{\text{transport due to bulk motion}} \quad (9-48)$$

where u is the total velocity of the mixture in the x -direction, n is the molar density of the mixture, and y_i is the mole fraction of species i . Substituting Fick's law, Eq. (9-20), into Eq. (9-48) leads to:

$$\dot{n}_i'' = -n D_{i,m} \frac{\partial y_i}{\partial x} + u n y_i \quad (9-49)$$

The product of the total velocity of the mixture in the x -direction and the molar density of the mixture is the total molar flux of the mixture, which must be equal to the sum of the mole fluxes associated with each individual component:

$$u n = \sum_{j=1}^{N_s} \dot{n}_j'' \quad (9-50)$$

where N_s is the number of species in the mixture. Substituting Eq. (9-50) into Eq. (9-49) leads to:

$$\dot{n}_i'' = -n D_{i,m} \frac{\partial y_i}{\partial x} + y_i \sum_{j=1}^{N_s} \dot{n}_j'' \quad (9-51)$$

Equations (9-49) and (9-51) can be written on a mass basis as well; the total mass flux of species i is:

$$\dot{m}_i'' = -\rho D_{i,m} \frac{\partial m f_i}{\partial x} + u \rho m f_i \quad (9-52)$$

or

$$\dot{m}_i'' = -\rho D_{i,m} \frac{\partial m f_i}{\partial x} + m f_i \sum_{j=1}^{N_s} \dot{m}_j'' \quad (9-53)$$

Figure 9-3 illustrates the steady-state situation where species 1 is evaporating from the surface of a liquid at the bottom of a tube and diffusing through the gas above the liquid. The gas is a mixture consisting of species 1 vapor and species 2. The liquid is assumed to remain at a constant temperature. The solubility of gas species 2 in liquid 1 is assumed to be zero. Therefore, the liquid surface effectively acts as a barrier to the flux of species 2.

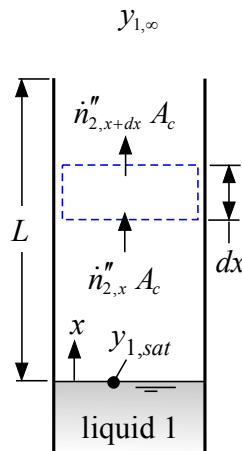


Figure 9-3: Mass diffusion through a stationary fluid

Mass transfer of species 1 is driven from the liquid surface to the top of the container by the gradient in its mole fraction (from $y_{1,sat}$ at the liquid surface to $y_{1,\infty}$ at the top of the tube); eventually all of the liquid will evaporate. It is evident that there must be a net flow of species 1 in the positive x -direction. However, it is less evident that species 2 is diffusing in the opposite direction due to an opposing gradient in its mole fraction.

A differential control volume is defined in Figure 9-3 and a mole balance on species 2 leads to:

$$\dot{n}_{2,x}'' A_c = \dot{n}_{2,x+dx}'' A_c \quad (9-54)$$

where A_c is the cross-sectional area of the tube. Note that Eq. (9-54) does not include any storage of species 2 because we are considering a steady-state situation. Dividing Eq. (9-54) through by A_c and expanding the $x+dx$ term leads to:

$$\dot{n}_{2,x}'' = \dot{n}_{2,x}'' + \frac{d\dot{n}_2''}{dx} dx \quad (9-55)$$

which can be simplified to:

$$\frac{d\dot{n}_2''}{dx} = 0 \quad (9-56)$$

Integrating Eq. (9-56) one time leads to:

$$\dot{n}_2'' = C_1 \quad (9-57)$$

where C_1 is a constant of integration. Equation (9-57) indicates that the net molar flux of species 2 must be constant with position, x , for this steady-state situation. Because since species 2 is insoluble in liquid 1, the boundary condition at the surface of the liquid is:

$$\dot{n}_{2,x=0}'' = 0 \quad (9-58)$$

Substituting Eq. (9-58) into Eq. (9-57) leads to:

$$\dot{n}_2'' = 0 \quad (9-59)$$

which indicates that the net molar flux of species 2 must be zero anywhere along the length of the tube because there is no source of species 2; this situation corresponds to diffusion of species 1 through a stationary fluid.

The net mole flux of species 1 through the stationary fluid is given by Eq. (9-51):

$$\dot{n}_1'' = -n D_{1,2} \frac{dy_1}{dx} + y_1 (\dot{n}_1'' + \dot{n}_2'') \quad (9-60)$$

Substituting Eq. (9-59) into Eq. (9-60) leads to:

$$\dot{n}_1'' = -n D_{1,2} \frac{dy_1}{dx} + y_1 \dot{n}_1'' \quad (9-61)$$

which can be rearranged:

$$\dot{n}_1''(1 - y_1) = -n D_{1,2} \frac{dy_1}{dx} \quad (9-62)$$

Equation (9-62) can be separated and integrated:

$$-\frac{\dot{n}_1''}{n D_{1,2}} \int_0^L dx = \int_{y_{1,sat}}^{y_{1,\infty}} \frac{dy_1}{(1-y_1)} \quad (9-63)$$

An integration variable, w , is defined:

$$w = 1 - y_1 \quad (9-64)$$

and substituted into Eq. (9-63):

$$-\frac{\dot{n}_1''}{n D_{1,2}} \int_0^L dx = \int_{1-y_{1,sat}}^{1-y_{1,\infty}} \frac{-dw}{w} \quad (9-65)$$

Carrying out the integration in Eq. (9-65) leads to:

$$-\frac{\dot{n}_1''}{n D_{1,2}} L = -\ln \left[\frac{1-y_{1,\infty}}{1-y_{1,sat}} \right] \quad (9-66)$$

Solving for the mole flux of species 1:

$$\dot{n}_1'' = \frac{n D_{1,2}}{L} \ln \left[\frac{1-y_{1,\infty}}{1-y_{1,sat}} \right] \quad (9-67)$$

Equation (9-67) is called Stefan's law, which can also be expressed on a mass basis:

$$\dot{m}_1'' = \frac{\rho D_{1,2}}{L} \ln \left[\frac{1-mf_{1,\infty}}{1-mf_{1,sat}} \right] \quad (9-68)$$

When the mole or mass fraction is small (e.g., less than about 0.1), the natural log terms in Eq. (9-67) or Eq. (9-68) can be eliminated by the approximation

$$\ln(1-y) \approx -y \quad \text{for small } y \quad (9-69)$$

With this approximation, Eqs. (9-67) and (9-68) can be written as

$$\dot{n}_1'' \approx n D_{1,2} \frac{(y_{1,sat} - y_{1,\infty})}{L} \quad (9-70)$$

$$\dot{m}_1'' \approx \rho D_{1,2} \frac{(mf_{1,sat} - mf_{1,\infty})}{L} \quad (9-71)$$

Notice the similarity between Eqs. (9-70) and (9-71) to the thermal resistance formula associated with conduction through a plane wall. At low values of concentration, Eqs. (9-70) and (9-71) indicate that the steady-state transfer of mass through a stationary medium is driven by a concentration difference and resisted (on a per unit area basis) by the quantity $L/(n D_{1,2})$ or $L/(\rho D_{1,2})$. At higher values of concentration, the analogy is disrupted by the flows that are induced by the mass transfer process and Eqs. (9-67) and (9-68) must be used. Equation (9-68) provides the basis for the blowing factor that is introduced in Section 9.4.2 in order to approximately consider forced convection mass transfer at high concentration.

EXAMPLE 9.4-1: Diffusion Tubes

Perhaps the easiest and least costly way to measure the concentration of an air pollutant is with a diffusion tube. A standard diffusion tube, shown in Figure 1, has a length of $L = 71$ mm and an internal diameter of $D = 11$ mm. A chemical trap that either absorbs or reacts with the pollutant is placed at one end of the tube while the other end of the tube is open to the atmosphere. The tubes are usually mounted with the open end facing down (as shown) so that rain and dirt are not collected in the tube. After exposure to the atmosphere for a known period of time, the tubes are sealed and sent to a laboratory where the amount of pollutant in the trap is measured. Based on this information, the average level of pollutant in the atmosphere can be determined.

In a particular case, a diffusion tube that has been exposed to the environment for $t_{exp} = 30$ days has accumulated a total of $M_{NO_2} = 2$ μg of nitrogen dioxide. The average ambient temperature during the month was $T = 18^\circ\text{C}$. The mass fraction of nitrogen dioxide at the surface of the trap can be assumed to be zero.

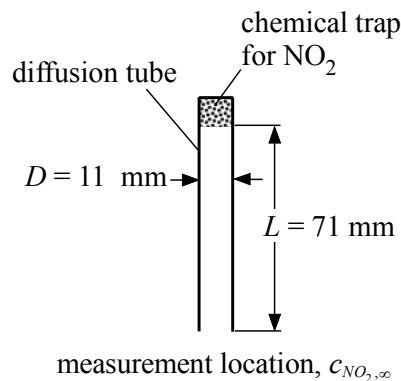


Figure 1: Diffusion tube

- a.) Estimate the average concentration of nitrogen dioxide in the environment during the measurement period ($c_{NO_2,\infty}$ in $\mu\text{g}/\text{m}^3$).

The inputs are entered in EES:

```
"EXAMPLE 9.4-1: Diffusion Tubes"
$UnitSystem SI MASS RAD PA K J
$Tabstops 0.2 0.4 0.6 3.5 in
```

L=71 [mm]*convert(mm,m)	"length of the diffusion tube"
D=11 [mm]*convert(mm,m)	"diameter of the diffusion tube"
M_NO2=2 [microgram]*convert(microgram,kg)	"mass of nitrogen dioxide"
t_exp=30 [day]*convert(day,s)	"time of exposure"
T=converttemp(C,K,18)	"average temperature"
p=1 [atm]*convert(atm,Pa)	"pressure"

The average mass flow rate of nitrogen dioxide from the atmosphere to the sample trap during the measurement period is:

$$\dot{m}_{NO_2} = \frac{M_{NO_2}}{t_{exp}}$$

The associated average mass flux is:

$$\dot{m}''_{NO_2} = \frac{4 \dot{m}_{NO_2}}{\pi D^2}$$

m_dot_NO2=M_NO2/t_exp	"average mass flow rate of NO2"
m``_dot_NO2=m_dot_NO2/(pi*D^2/4)	"average mass flux of NO2"

The diffusion coefficient for nitrogen dioxide in air ($D_{NO_2,air}$) is estimated using the D_12_gas function in EES, discussed in Section 9.2.2. The density of the mixture (ρ) is assumed to be the density of air at atmospheric pressure and the average temperature. (This assumption is valid provided that the mass fraction of nitrogen dioxide is small).

D_NO2_air=D_12_gas('NO2','Air',T,p)	"diffusion coefficient"
rho=density(Air,p=p,T=T)	"density"

Stefan's law for this problem, Eq. (9-68) is:

$$\dot{m}''_{NO_2} = \frac{\rho D_{NO_2,air}}{L} \ln \left[1 - mf_{NO_2,\infty} \right] \quad (1)$$

Equation (1) is solved for the mass fraction of nitrogen dioxide in the atmosphere:

$$mf_{NO_2,\infty} = 1 - \exp \left(\frac{L \dot{m}''_{NO_2}}{\rho D_{NO_2,air}} \right)$$

The concentration of nitrogen dioxide in the atmosphere is therefore:

$$c_{NO_2,\infty} = \rho mf_{NO_2,\infty}$$

mf_NO2_infinity=1-exp(-L*m``_dot_NO2/(rho*D_NO2_air))	"mass fraction of NO2 in the environment"
c_NO2_infinity=mf_NO2_infinity*rho	"concentration of NO2 in the environment"

```
c_NO2_infinity_ugm3=c_NO2_infinity*convert(kg/m^3,microgram/m^3)
    "in microgram/m^3"
```

which leads to $c_{NO_2} = 39.5 \mu\text{g}/\text{m}^3$. This value can be compared to target values of $20 \mu\text{g}/\text{m}^3$ for ecosystem protection and an upper daily average regulatory limit of $100 \mu\text{g}/\text{m}^3$ for human health; these standards have been proposed in many European countries.

9.4.2 Momentum, Energy and Mass Transfer Analogies

The mass transfer rate of species i from a surface to a flowing free stream is driven by a gradient in its composition at the surface. The composition gradient is governed by the presence of a concentration boundary layer (a mass diffusion boundary layer) and therefore the forced convection mass transfer problem resembles the convection problems that are discussed in Chapters 4 through 6. This section discusses methods for approximately considering forced convection mass transfer through the use of momentum, energy, and mass transfer analogies.

Figure 9-4 shows the steady flow of a free stream (e.g., air) over the surface of liquid i (e.g., water) that is evaporating and thus providing a source for the mass transfer of the vapor component of species i . The flow is steady and at a sufficiently low velocity that the boundary layer is laminar. Further, the shear is small enough that there is a minimal effect on the velocity of the liquid at the interface. The mole fraction of species i in the free stream is $y_{i,\infty}$. The mole fraction of species i at the surface of the liquid is $y_{i,sat}$, its saturation value. A concentration or mass diffusion boundary layer thickness (δ_d) can be identified as the distance from the liquid surface over which 99% of the composition change occurs, analogous to the definitions of the thermal and momentum boundary layers that are discussed in Chapter 4.

The difference in the mole fraction at the surface and the mole fraction in the free stream drives a diffusive mass transfer that causes species i to transfer in the y -direction into the gas stream. As the flow proceeds downstream, it has additional time for diffusion to occur so that the mass diffusion penetration depth will increase, as illustrated by the mole fraction profiles at positions x_1 and x_2 in Figure 9-4. The dashed line in Figure 9-4 indicates the thickness of the diffusion boundary layer.

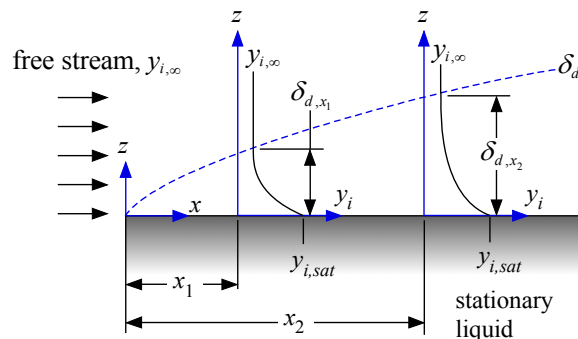


Figure 9-4: Mole fraction profiles of species i for laminar flow over a stationary evaporating liquid.

Figure 9-4 may look familiar, because it is essentially the same illustration that is used in Chapter 4 to discuss the growth of a thermal boundary layer in a laminar external flow over a flat plate. In Section 4.1.2, it is shown that the thickness of the thermal boundary layer for laminar flow is approximately:

$$\delta_t \approx 2\sqrt{\alpha t} \quad (9-72)$$

where α is the thermal diffusivity of the fluid and t is the time that the free stream has been in contact with the surface ($t \approx x/u_\infty$, where u_∞ is the free stream velocity). The transport of momentum by molecular diffusion is analogous to the transport of energy; a “momentum wave” will travel according to:

$$\delta_m \approx 2\sqrt{\nu t} \quad (9-73)$$

where ν is the kinematic viscosity of the fluid. In exactly the same manner, the transport of mass by molecular diffusion of species i will be characterized by a mass diffusion boundary layer thickness given approximately by:

$$\delta_d \approx 2\sqrt{D_{i,m} t} \quad (9-74)$$

Note that α , ν and $D_{i,m}$ all have units of m^2/s and, according to Eqs. (9-72) through (9-74), are analogous fluid transport properties.

The ratio of the kinematic viscosity to the thermal diffusivity is defined as the Prandtl number (Pr), which is a measure of the relative ability of the fluid to transport momentum and energy.

$$Pr = \frac{\nu}{\alpha} \quad (9-75)$$

The analogous ratio of the kinematic viscosity to the diffusion coefficient is the Schmidt number, (Sc) which provides a measure of the relative ability of the fluid to transport momentum and mass of species i :

$$Sc = \frac{\nu}{D_{i,m}} = \frac{\mu}{\rho D_{i,m}} \quad (9-76)$$

The Schmidt number is used in convective mass transfer problems in exactly the same manner that the Prandtl number is used in convective heat transfer problems. The Lewis number (Le) is the ratio of the thermal diffusivity to the diffusion coefficient and provides a measure of the relative ability of a fluid to transport energy and mass under hydrodynamically similar conditions.

$$Le = \frac{\alpha}{D_{i,m}} = \frac{k}{\rho c D_{i,m}} \quad (9-77)$$

Note that the Prandtl, Schmidt, and Lewis numbers are related according to:

$$Le = \frac{Sc}{Pr} \quad (9-78)$$

A mass transfer coefficient (h_D) is defined for convective mass transfer problems in a manner that is analogous to the familiar heat transfer coefficient. The molar flux from the surface of the fluid, (i.e., the transfer rate of species i per unit area, \dot{n}_i'') is given by:

$$\dot{n}_i'' = h_D n (y_{i,sat} - y_{i,\infty}) \quad (9-79)$$

where $y_{i,sat}$ is the mole fraction of species i at the fluid surface ($z = 0$) and $y_{i,\infty}$ is the mole fraction of species i in the free stream. If there is no bulk velocity at the interface, then Fick's law requires that:

$$\dot{n}_i'' = h_D n (y_{i,sat} - y_{i,\infty}) = -n D_{i,m} \left. \frac{\partial y_i}{\partial z} \right|_{z=0} \quad (9-80)$$

The mass transfer coefficient can also be expressed in terms of molar concentrations:

$$\dot{n}_i'' = h_D (n_{i,sat} - n_{i,\infty}) = -D_{i,m} \left. \frac{\partial n_i}{\partial z} \right|_{z=0} \quad (9-81)$$

On a mass basis, the mass transfer coefficient can be expressed in terms of mass fractions:

$$\dot{m}_i'' = h_D \rho (mf_{i,sat} - mf_{i,\infty}) = -\rho D_{i,m} \left. \frac{\partial mf_i}{\partial z} \right|_{z=0} \quad (9-82)$$

or mass concentrations:

$$\dot{m}_i'' = h_D (c_{i,sat} - c_{i,\infty}) = -D_{i,m} \left. \frac{\partial c_i}{\partial z} \right|_{z=0} \quad (9-83)$$

The mass transfer coefficient defined by Eqs. (9-80) through (9-83) has units of length/time (m/s). Notice the similarity of Eqs. (9-80) to (9-83) to Newton's Law of cooling, which defines the local heat transfer coefficient for convective heat transfer:

$$\dot{q}'' = h (T_s - T_\infty) = -k \left. \frac{\partial T}{\partial z} \right|_{z=0} \quad (9-84)$$

Chapter 4 correlates solutions for the heat transfer coefficient for external flow problems in terms of the Nusselt number:

$$Nu = \frac{h L_{char}}{k} \quad (9-85)$$

where L_{char} is a characteristic dimension that depends on the geometry of the problem (e.g., for a flat plate, L_{char} is the distance from the leading edge of the plate, x). An analogous dimensionless quantity called the Sherwood number (Sh) is used to correlate the solutions to mass transfer problems:

$$Sh = \frac{h_D L_{char}}{D_{i,m}} \quad (9-86)$$

The Sherwood number is the dimensionless mass transfer coefficient just as the Nusselt number is the dimensionless heat transfer coefficient. As noted in Section 9.2, the governing differential equations for energy and mass transport are the same if temperature is replaced with concentration and thermal diffusivity is replaced with the diffusion coefficient. Therefore, correlations initially developed for convective heat transfer can, with some limitations, be applied to convective mass transfer problems by replacing the Prandtl number with the Schmidt number and the Nusselt number with the Sherwood number.

The average Sherwood number is defined based on the average mass transfer coefficient integrated over the surface of interest (\bar{h}_D):

$$\bar{Sh} = \frac{\bar{h}_D L_{char}}{D_{i,m}} \quad (9-87)$$

When the boundary conditions are analogous, the correlations provided in Chapters 4 and 5 for the average Nusselt number expressed in terms of the Reynolds and Prandtl numbers, can be also used to estimate the average Sherwood number in terms of the Reynolds and Schmidt numbers.

$$\text{if } \bar{Nu} = \text{function of } (Re, Pr) \text{ then } \bar{Sh} \approx \text{function of } (Re, Sc) \quad (9-88)$$

The analogy between heat and mass transfer allows many mass transfer problems to be solved using correlations that were developed for heat transfer problems. However, there are some limitations to this method. Although the equations describing heat and mass transfer may be mathematically identical, their boundary conditions can be different. For example, heat transfer convection correlations are developed assuming zero velocity at the solid-fluid interface. When mass transfer occurs at the interface, the velocity at the surface will not be zero and will depend on the mass flux of the diffusing species, as discussed in Section 9.4.1; even for a stationary fluid, there will be a velocity in the z -direction at the surface of the fluid shown in Figure 9-4 due

to the diffusion mass transfer. Consequently, the heat transfer correlations can only be applied to convective mass transfer problems that are characterized by low mass fluxes.

One method for correcting, approximately, for the impact of the surface velocity on the mass transfer coefficient is discussed in Lienhard (2005). The mass transfer coefficient computed using the heat/mass transfer analogy embodied by Eq. (9-88) is multiplied by a "blowing factor", BF :

$$BF = \frac{\ln(1+B)}{B} \quad (9-89)$$

where B is related to the mass fractions of species i :

$$B = \frac{mf_{i,\infty} - mf_{i,sat}}{mf_{i,sat} - 1} \quad (9-90)$$

where $mf_{i,\infty}$ and $mf_{i,sat}$ are the mass fractions of species i in the free stream and at the surface, respectively. If both $mf_{i,\infty}$ and $mf_{i,sat}$ are small (as is typically the case), then B approaches 0 and the blowing factor predicted by Eq. (9-89) approaches unity. Notice that the form of the blowing factor is similar to the form of Stefan's law derived in Section 9.4.1 and given in Eqs. (9-67) and (9-68). Indeed, the blowing factor is meant to correct a zero-velocity solution for the presence of diffusional velocities and is related to the ratio of the actual to the zero-velocity solution to diffusion through a stationary medium (i.e., the ratio of Eq. (9-68) to Eq. (9-71)).

EXAMPLE 9.4-2: Evaporation from a Lake

You have been asked to estimate the rate at which water evaporates from the surface of a small lake. The lake is roughly circular with a diameter of $D = 1000$ m. The temperature of the water surface is $T_s = 12^\circ\text{C}$. Wind is blowing over the lake with temperature $T_\infty = 18^\circ\text{C}$ and relative humidity $RH_\infty = 45\%$. The wind velocity is not known; however, a conservative estimate of the wind velocity is $u_\infty = 1$ m/s, which perhaps is low enough to not significantly disturb the lake surface.

a.) Estimate the rate at which water evaporates from the lake surface.

The input information is entered in EES:

"EXAMPLE 9.4-2: Evaporation from a Lake"

\$UnitSystem SI MASS RAD PA K J

\$Tabstops 0.2 0.4 0.6 3.5 in

D = 1000 [m]

"diameter of the lake"

T_s=converttemp(C,K,12 [C])

"temperature of the lake surface"

T_infinity=converttemp(C,K,18 [C])

"temperature of the wind"

RH_infinity=0.45 [-]

"relative humidity of the wind"

u_infinity=1 [m/s]

"estimate of the wind velocity"

p=1 [atm]*convert(atm,Pa)

"pressure"

A heat transfer analogy for flow over an isothermal flat plate will be employed to estimate the rate of evaporation from the lake. The Reynolds number and Schmidt number will be determined and used in the Nusselt number correlation for flow over a flat plate, provided in Section 4.9.2, in order to compute the Sherwood number. The film temperature is computed:

$$T_{film} = \frac{T_s + T_\infty}{2}$$

and used to determine the required air properties (μ , ρ , and ν) using EES' built-in property routine for air. The diffusion coefficient for water vapor in air ($D_{a,w}$) is estimated using the EES function D_12_gas.

T_film=(T_s+T_infinity)/2	"film temperature"
mu=viscosity(Air,T=T_film)	"viscosity"
rho=density(Air,T=T_film,p=p)	"density"
nu=mu/rho	"kinematic viscosity"
D_a_w=D_12_gas('Air','Water',T_film,p)	"diffusion coefficient"

The Schmidt number is computed according to:

$$Sc = \frac{\nu}{D_{a,w}}$$

and the Reynolds number is computed according to:

$$Re = \frac{\rho D u_\infty}{\mu}$$

where the characteristic length of the lake is assumed to be its diameter. The convection correlation for flow over an isothermal plate is accessed using the EES function External_Flow_Plate_ND; note that input Prandtl number is replaced with the Schmidt number and the output is assigned to the average Sherwood number (\overline{Sh}) rather than the average Nusselt number, as indicated by Eq. (9-88).

Sc=nu/D_a_w	"Schmidt number"
Re=rho*D*u_infinity/mu	"Reynolds number"
Call External_Flow_Plate_ND(Re,Sc: Sh_bar,C_f)	
	"obtain Sherwood number using external convection correlation for a flat plate"

The average Sherwood number is used to compute the average mass transfer coefficient:

$$\overline{h}_D = \frac{\overline{Sh} D_{a,w}}{D}$$

h_D_bar=D_a_w*Sh_bar/D	"mass transfer coefficient"
------------------------	-----------------------------

The mass transfer rate is driven by the difference between the concentration of water vapor at the lake surface and in the free stream. The partial pressure of the water vapor at the lake surface is the saturation pressure of water at T_s ($p_{w,sat}$), evaluated using the appropriate EES property routine. The concentration of water vapor at the lake surface ($c_{w,sat}$) is the density of water vapor evaluated at the partial pressure and temperature. The mass fraction of water vapor at the lake surface is:

$$mf_{w,sat} = \frac{c_{w,sat}}{\rho}$$

p_w_sat=pressure(Water,x=1,T=T_s)	"saturation pressure of water vapor at the lake surface"
c_w_sat=density(Water,p=p_w_sat,x=1)	"concentration of water vapor at lake surface"
mf_w_sat=c_w_sat/rho	"mass fraction of water vapor at the lake surface"

The partial pressure of water in the free stream ($p_{w,\infty}$) is the product of the relative humidity and the saturation pressure of water evaluated at T_∞ ($p_{w,sat,\infty}$) evaluated using the appropriate EES property routine.

$$p_{w,\infty} = RH p_{sat,w,\infty}$$

The concentration of water vapor in the free stream ($c_{w,\infty}$) is the density of water evaluated at the partial pressure and temperature. The mass fraction of water in the free stream is:

$$mf_{w,\infty} = \frac{c_{w,\infty}}{\rho}$$

p_w_sat_infinity=pressure(Water,x=1,T=T_infinity)	"sat. pressure of water vapor in the free stream"
p_w_infinity=RH_infinity*p_w_sat_infinity	"partial pressure of water vapor in the free stream"
c_w_infinity=density(Water,p=p_w_infinity,T=T_infinity)	"concentration of water vapor in the free stream"

The blowing factor is calculated using Eqs. (9-89) and (9-90):

$$BF = \frac{\ln(1+B)}{B}$$

$$B = \frac{mf_{w,\infty} - mf_{w,sat}}{mf_{w,sat} - 1}$$

B=(mf_w_infinity-mf_w_sat)/(mf_w_sat-1)	"B for blowing factor"
BF=ln(1+B)/B	"blowing factor"

which leads to $BF = 0.9985$; the mass fraction of water in air is small and therefore the correction associated with the induced velocity at the surface of the lake is negligible. The corrected mass transfer coefficient is:

$$\bar{h}_{D,c} = \bar{h}_D BF$$

$h_D_c_bar=h_D_bar*BF$ "mass transfer coefficient, corrected for blowing"

The mass flow rate of water due to evaporation is calculated according to Eq. (9-83) using the corrected mass transfer coefficient:

$$\dot{m}_w = \bar{h}_{D,c} \frac{\pi D^2}{4} (c_{w,sat} - c_{w,\infty})$$

The volume rate at which liquid water evaporates from the lake is given by:

$$\dot{V}_w = \frac{\dot{m}_w}{\rho_{w,l}}$$

where $\rho_{w,l}$ is the density of liquid water, evaluated using the EES property function.

$m_dot_w=h_D_bar*(pi*D^2/4)*(c_w_sat-c_w_infinity)$ "evaporation mass flow rate"
 $\rho_w_l=density(Water,T=T_s,x=0)$ "density of liquid water"
 $V_dot_w=m_dot_w/\rho_w_l$ "evap. volumetric flow rate of liq. water"
 $V_dot_w_gpd=V_dot_w*convert(m^3/s,gal/day)$ "in gallon per day"

which leads to $\dot{m}_w = 3.58 \text{ kg/s}$ and $\dot{V}_w = 3.6 \times 10^{-3} \text{ m}^3/\text{s}$ (82,000 gal/day). Figure 1 shows the predicted rate of liquid loss as a function of the relative humidity in the air. With the assumed lake and air temperatures, evaporation loss will stop when the relative humidity is 0.7 and condensation on the lake surface will begin to occur at higher relative humidities.

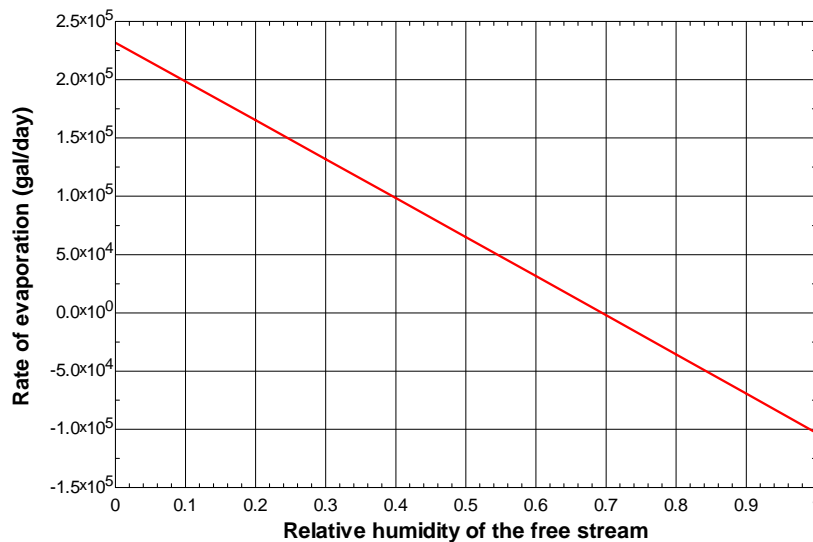


Figure 1: Estimated rate of liquid loss as a function of relative humidity

9.5 Simultaneous Heat and Mass Transfer

Energy is required to evaporate a liquid because the specific internal energy of a vapor is higher than that of a liquid. Therefore, the evaporation of a liquid at an interface will result in a reduction in temperature unless some additional energy is provided. The combined effect of heat and mass transfer can result in significant temperature changes and increased energy transfer rates at a wet surface. Simultaneous heat and mass transfer is used to great advantage in air-water devices such as cooling towers, evaporative condensers, and wet-surface evaporators. Simultaneous heat and mass transfer problems can be considered using the convection heat transfer correlations that are provided in Chapters 4 through 6 together with the heat and mass transfer analogy discussed in Section 9.4.

9.5.1 Wet-bulb Temperature

The concept of wet-bulb temperature is a consequence of simultaneous heat and mass transfer. The amount of water in an air-water vapor mixture can be measured using a device called a psychrometer. A simple psychrometer consists of two ordinary thermometers, as shown in Figure 9-5; the bulb surface of one of the thermometers (the dry-bulb thermometer) is dry while the other thermometer bulb (the wet-bulb thermometer) is wetted by covering it with a wick made of cotton or another material that has been soaked in water. The thermometer bulbs are then exposed to a high velocity flow of ambient air, either by swinging the thermometers or by blowing air over them with a small fan. The dry thermometer measures the dry-bulb temperature and the thermometer exposed to liquid water measures the wet-bulb temperature.

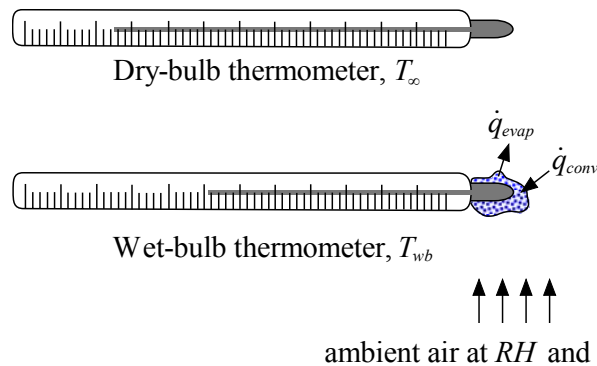


Figure 9-5: Psychrometer consisting of a wet-bulb and dry-bulb thermometer.

Initially, both the wet-bulb and dry-bulb thermometers measure the same temperature. However, liquid water is touching the wet-bulb thermometer and therefore the surface temperature of the wet-bulb thermometer will decrease due to the evaporation that occurs. The temperature reduction will continue until the energy required by evaporation is provided by convection heat transfer from the surrounding air.

Using the ideal gas law, the water vapor concentration in the air at the surface of the wet bulb thermometer is:

$$c_{w,sat} = \frac{p_{sat}(T_{wb})}{R_w T_{wb}} \quad (9-91)$$

where $p_{sat}(T_{wb})$ is the saturation pressure of water at the wet-bulb temperature, T_{wb} , and R_w is the ideal gas constant for water vapor. The concentration of water vapor in the free stream air is:

$$c_{w,\infty} = \frac{RH p_{sat}(T_\infty)}{R_w T_\infty} \quad (9-92)$$

where RH is the relative humidity of the air (a value between 0 and 1) and T_∞ is the free stream air temperature (measured by the dry-bulb thermometer). If the relative humidity is less than 1, then the concentration of water vapor will be lower in the free air stream than it is at the wet-bulb surface; this concentration gradient will lead to a mass transfer of water vapor from the wet-bulb to the surrounding air. The rate of mass transfer is given by:

$$\dot{m}_w = A_s \bar{h}_D (c_{w,sat} - c_{w,\infty}) \quad (9-93)$$

where A_s is the surface area of the wetted bulb and \bar{h}_D is the average mass transfer coefficient. An energy transfer rate, \dot{q}_{evap} , occurs at the surface of the wet-bulb as liquid water evaporates to provide the vapor that is being transferred from the wet-bulb surface:

$$\dot{q}_{evap} = \dot{m}_w \Delta i_{vap} \quad (9-94)$$

where Δi_{vap} is the enthalpy of vaporization of water at the wet-bulb temperature. Substituting Eq. (9-93) into Eq. (9-94) leads to:

$$\dot{q}_{evap} = A_s \bar{h}_D (c_{w,sat,wb} - c_{w,\infty}) \Delta i_{vap} \quad (9-95)$$

The energy needed to evaporate the water will initially be drawn from the thermometer itself and therefore the wet-bulb temperature will fall. As the temperature of the wet-bulb is reduced, a convective heat transfer from the warmer free stream air to the wet-bulb will be induced (\dot{q}_{conv}):

$$\dot{q}_{conv} = A_s \bar{h} (T_\infty - T_{wb}) \quad (9-96)$$

where \bar{h} is the average heat transfer coefficient. After some time, a steady-state condition will be achieved, where the rate at which energy is transferred from the bulb by evaporation will equal the rate at which energy is transferred to the bulb by convection:

$$\dot{q}_{evap} = \dot{q}_{conv} \quad (9-97)$$

At this point, the temperature measured by the wet-bulb thermometer will not change and the measured temperature is referred to as the wet-bulb temperature. The wet-bulb temperature can be evaluated by substituting Eqs. (9-94) and (9-96) into Eq. (9-97):

$$A_s \bar{h}_D (c_{w,sat} - c_{w,\infty}) \Delta i_{vap} = A_s \bar{h} (T_\infty - T_{wb}) \quad (9-98)$$

Solving for the wet-bulb temperature leads to:

$$T_{wb} = T_\infty - \frac{\bar{h}_D}{\bar{h}} \Delta i_{vap} (c_{w,sat} - c_{w,\infty}) \quad (9-99)$$

The average heat transfer coefficient can be written in terms of the average Nusselt number according to:

$$\bar{h} = \frac{k}{L_{char}} \overline{Nu} \quad (9-100)$$

where L_{char} is the characteristic dimension of the thermometer bulb and k is the conductivity of air. According to the discussion in Section 9.4.2, the average mass transfer coefficient can be written in terms of the average Sherwood number:

$$\bar{h}_D = \frac{D_{w,a}}{L_{char}} \overline{Sh} \quad (9-101)$$

where $D_{w,a}$ is the diffusion coefficient for water vapor in air. Substituting Eqs. (9-100) and (9-101) into Eq. (9-99) leads to:

$$T_{wb} = T_\infty - \frac{D_{w,a}}{k} \frac{\overline{Sh}}{\overline{Nu}} \Delta i_{vap} (c_{w,sat} - c_{w,\infty}) \quad (9-102)$$

Substituting the definition of the Lewis number, Eq. (9-77), into Eq. (9-102) leads to:

$$T_{wb} = T_\infty - \frac{\overline{Sh}}{\overline{Nu}} \left(\frac{1}{\rho c Le} \right) \Delta i_{vap} (c_{w,sat} - c_{w,\infty}) \quad (9-103)$$

where ρ is the air density, c is the specific heat of air and Le is the Lewis number. As discussed in Section 9.4.2 and indicated by Eq. (9-88), the Nusselt number is a function of the Reynolds and Prandtl numbers and the Sherwood number is a function of the Reynolds and Schmidt numbers. Equation (9-103) allows the concentration of water vapor in the free air stream (and thus the relative humidity and humidity ratio) to be determined by measuring the wet-bulb and dry-bulb temperatures, provided that the other parameters are known or can be calculated.

EXAMPLE 9.5-1: Wet-bulb and Adiabatic Saturation Temperatures

The wet-bulb temperature and the adiabatic saturation temperature are often used interchangeably; in principle, they are actually quite different quantities. The wet-bulb temperature is the temperature for which the rate of convective heat transfer to the wet-bulb from the surroundings is equal to the rate of energy transfer resulting from evaporation of water at the wetted surface, as discussed in Section 9.5.1. The adiabatic saturation temperature is a thermodynamic property of moist air that is defined as the temperature that the air stream would achieve if it were allowed to become saturated adiabatically. The adiabatic saturation temperature is computed by equating the enthalpy of moist air at a given temperature and relative humidity to the enthalpy of a saturated air-water mixture at the adiabatic saturation temperature (ASHRAE, (1996)).

- a.) Determine the difference between the wet-bulb temperature and the adiabatic saturation temperature as a function of the dry-bulb temperature (T_∞) and relative humidity (RH). Assume that the Reynolds number associated with the flow over the wet-bulb is $Re = 1 \times 10^5$ and then assess the impact of the Reynolds number on your results.

The inputs are entered in EES. Initially, we will carry out the calculation at a dry bulb temperature of $T_\infty = 27^\circ\text{C}$, a relative humidity of $RH = 0.5$, and a Reynolds number of $Re = 1 \times 10^5$. These quantities will be parametrically varied as requested in the problem statement.

"EXAMPLE 9.5-1: Wet-bulb and Adiabatic Saturation Temperatures"

\$UnitSystem SI MASS RAD PA K J

\$Tabstops 0.2 0.4 0.6 3.5 in

"Inputs"

T_infinity_C=27 [C]

"dry-bulb temperature, in C"

T_infinity=converttemp(C,K,T_infinity_C)

"dry-bulb temperature"

RH=0.5 [-]

"relative humidity"

p=1 [atm]*convert(atm,Pa)

"pressure"

Re=1e5 [-]

"assumed Reynolds number"

The air water mixture properties are obtained using the substance AirH2O in EES. In addition to the typical properties that are required to specify the state of a pure fluid, the substance AirH2O requires an additional parameter that indicates the amount of water contained in the air (either the relative humidity or humidity ratio). The properties of the air water vapor mixture (k , ρ , c , and μ) are calculated at the film temperature:

$$T_{film} = \frac{T_\infty + T_{wb}}{2}$$

where T_{wb} is the wet-bulb temperature. A wet-bulb temperature is initially assumed in order to compute the film temperature and proceed with the calculations; the assumed wet-bulb temperature will ultimately be commented out in order to complete the problem.

T_wb=converttemp(C,K,20 [C])

"guess for wet-bulb temperature"

T_film=(T_wb+T_infinity)/2

"film temperature"

k=conductivity(AirH2O,T=T_film,p=p,R=RH)	"thermal conductivity of the air-water vapor mixture"
rho=density(AirH2O,T=T_film,p=p,R=RH)	"density of the air-water vapor mixture"
c=cP(AirH2O,T=T_film,p=p,R=RH)	"specific heat capacity of the air-water vapor mixture"
mu=viscosity(AirH2O,T=T_film,p=p,R=RH)	"viscosity of the air-water vapor mixture"

The kinematic viscosity and thermal diffusivity of the air water mixture are computed according to:

$$\nu = \frac{\mu}{\rho}$$

$$\alpha = \frac{k}{\rho c}$$

nu=mu/rho	"kinematic viscosity"
alpha=k/(rho*c)	"thermal diffusivity"

The diffusion coefficient for water vapor in air ($D_{w,a}$) is obtained using the data provided by Bolz and Tuve (1976), as discussed in EXAMPLE 9.2-1, rather than the less accurate estimate that is available using the EES function D_12_gas.

$$D_{a,w} = -2.775 \times 10^{-6} \left[\frac{\text{m}^2}{\text{s}} \right] + 4.479 \times 10^{-8} \left[\frac{\text{m}^2}{\text{s-K}} \right] T + 1.656 \times 10^{-10} \left[\frac{\text{m}^2}{\text{s-K}^2} \right] T^2$$

D_w_a=-2.775e-6 [m^2/s]+4.479e-8 [m^2/s-K]*T_film+1.656e-10[m^2/s-K^2]*T_film^2	"diffusion coefficient for air-water mixture, according to Bolz and Tuve (1976)"
---	--

The Prandtl number, Schmidt number, and Lewis number are computed:

$$Pr = \frac{\nu}{\alpha}$$

$$Sc = \frac{\nu}{D_{w,a}}$$

$$Le = \frac{\alpha}{D_{w,a}}$$

Pr=nu/alpha	"Prandtl number"
Sc=nu/D_w_a	"Schmidt number"
Le=alpha/D_w_a	"Lewis number"

The latent heat of vaporization of water (Δi_{vap}) is calculated as the difference between the specific enthalpies of saturated vapor (i.e., water at the wet bulb temperature with quality, $x = 1$) and saturated liquid ($x = 0$).

DELTAi_vap=enthalpy(Water,x=1,T=T_wb)-enthalpy(Water,x=0,T=T_wb) "enthalpy of vaporization"

The average Nusselt and Sherwood numbers (\overline{Nu} and \overline{Sh}) are estimated using external flow correlations according to the heat and mass transfer analogy that is discussed in Section 9.4.2. The thermometer bulb is represented approximately as a cylinder and so the EES function External_Flow_Cylinder_ND is used to access the appropriate Nusselt number correlations.

Call External_Flow_Cylinder_ND(Re,Pr:Nusselt,C_d1) "Nusselt number"
Call External_Flow_Cylinder_ND(Re,Sc:Sh,C_d2) "Sherwood number"

The partial pressure of water vapor at the surface of the wet-bulb is equal to the saturation pressure of water at the wet-bulb temperature, $p_{w,sat,wb}$. The concentration of water vapor at the wet-bulb surface is computed using the ideal gas law:

$$c_{w,sat} = \frac{p_{w,sat,wb}}{R_w T_{wb}}$$

where R_w is the ideal gas constant for water:

$$R_w = \frac{R_{univ}}{MW_w}$$

R_{univ} is the universal gas constant and MW_w is the molar mass of water.

p_w_sat_wb=pressure(Water,T=T_wb,x=1) "saturation pressure at the wet-bulb temperature"
MW_w=MolarMass(Water) "molar mass of water"
R_w=R#/MW_w "gas constant for water vapor"
c_w_sat=p_w_sat_wb/(R_w*T_wb) "concentration of water vapor at the wet-bulb"

The saturation pressure of water vapor in the free stream ($p_{w,sat,\infty}$) is computed using EES' internal property routine evaluated at T_∞ . The partial pressure of water vapor in the free stream is:

$$p_{w,\infty} = RH p_{w,sat,\infty}$$

The concentration of water vapor in the free stream is calculated using the ideal gas law:

$$c_{w,\infty} = \frac{p_{w,\infty}}{R_w T_\infty}$$

p_w_sat_infinity=pressure(Water,T=T_infinity,x=1) "saturation pressure of water at the dry-bulb temp."
p_w_infinity=RH*p_w_sat_infinity "partial pressure of water at the dry-bulb temperature"
c_w_infinity=p_w_infinity/(R_w*T_infinity) "concentration of water in the free stream"

The wet-bulb temperature is computed using Eq. (9-103):

$$T_{wb} = T_{\infty} - \frac{\overline{Sh}}{\overline{Nu}} \left(\frac{1}{\rho c Le} \right) \Delta i_{vap} (c_{w,sat,wb} - c_{w,\infty}) \quad (1)$$

Before Eq. (1) is entered in EES, a solution is obtained and the guess values are updated; the original guess for the wet-bulb temperature is commented out:

```
{T_wb=converttemp(C,K,20)}           "guess for wet-bulb temperature"
T_wb=T_infinity-Sh*DELTAi_vap*(c_w_sat-c_w_infinity)/(rho*c*Le*Nusselt) "wet-bulb temperature"
T_wb_C=converttemp(K,C,T_wb)       "in C"
```

This leads to $T_{wb} = 292.2$ K (19.1°C). Figure 1 illustrates the predicted wet-bulb temperature as a function of the assumed Reynolds number and shows that the wet-bulb temperature is essentially independent of the Reynolds number. This behavior occurs because the average Sherwood number and average Nusselt number are each approximately proportional to the Reynolds number to some power (depending on which correlation is applicable from those presented in Section 4.9.3) and therefore the Reynolds number dependence in the numerator and denominator of Eq. (1) will cancel.

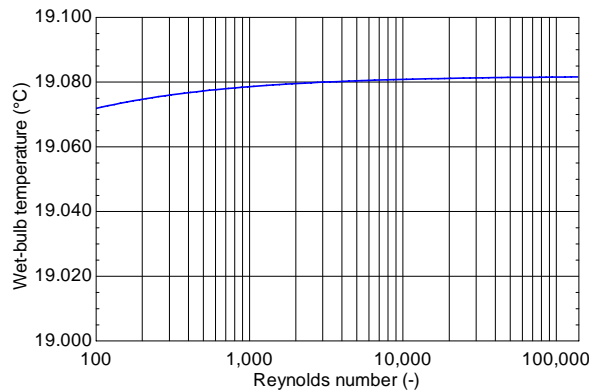


Figure 1: Wet-bulb temperature as a function of the assumed Reynolds number

The adiabatic saturation temperature ($T_{ad,sat}$) is provided by the EES function `Wetbulb` (perhaps it should be renamed!). The difference between the wet-bulb temperature and the adiabatic saturation temperature is computed.

```
T_ad_sat=Wetbulb(AirH2O,T=T_infinity,p=p,R=RH)           "adiabatic saturation temperature"
T_ad_sat_C=converttemp(K,C,T_ad_sat)                     "in C"
err=T_ad_sat-T_wb                                         "discrepancy between wet-bulb and adiabatic saturation temperature"
```

which leads to $T_{ad,sat} = 292.7$ K (19.53°C) and a difference $T_{ad,sat} - T_{wb} = 0.45^{\circ}\text{C}$. The difference is relatively small, which explains why the terms wet-bulb temperature and adiabatic saturation temperature are often confused.

Figure 2 illustrates the discrepancy between the wet-bulb temperature and the adiabatic saturation temperature as a function of the dry-bulb temperature for several values of the relative humidity. Note that differences of more than 1°C may occur at low relative humidity and high

temperature. However, the wet-bulb temperature is fairly closely approximated by the adiabatic saturation temperature over much of the range of interest to heating and air-conditioning applications.

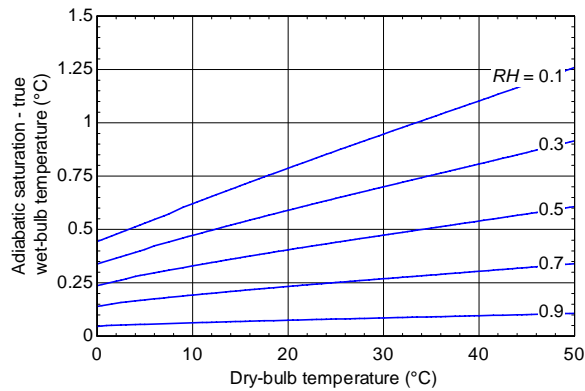


Figure 2: Difference between the wet-bulb temperature and the adiabatic saturation temperature as a function of the dry-bulb temperature for several values of relative humidity.

9.6 Cooling Coil Analysis

9.6.1 Introduction

Cooling coils are cross-flow heat exchangers used in air-conditioning and refrigeration cycles in order to provide both cooling and dehumidification of an air stream that is forced to flow through the coil. Cold fluid flowing through the tubes may be chilled water, brine, or a refrigerant such as ammonia or R134a. The cold fluid is at a temperature that is lower than the dew point of the air stream and therefore water will condense on the air-side of the heat exchanger, as shown in Figure 9-6.

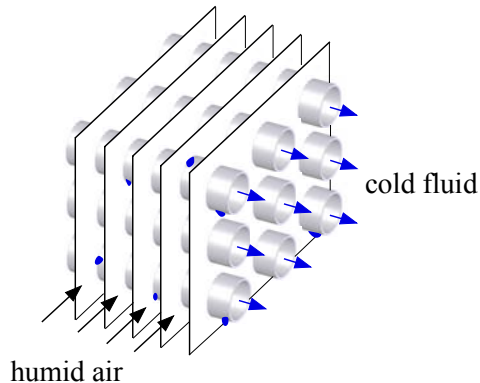


Figure 9-6: Cooling coil with condensation occurring on the outside of the tubes

Heat transfer and fan power considerations typically require that cooling coils are thin (in the flow direction) with a large face area. Air velocities are usually less than 10 m/s and the coil will usually have only two to six rows of tubes perpendicular to the air flow, although more rows may

be used for industrial processes that produce dry air. The geometry of a cooling coil is a typical finned coil heat exchanger in all respects. However, because the cold fluid is at a temperature that is less than the dew point temperature of the entering air, condensate is likely to form on the fin and tube surfaces and the air will exit at a lower humidity ratio. The combined heat and mass transfer processes that occur within the coil do not allow the heat exchanger analysis methods discussed in Chapter 8 to be used directly.

There are two methods that can be used to model a cooling coil. In Section 9.6.2, the dry coil/wet coil approach is discussed. The dry coil/wet coil method divides the cooling coil into a dry coil and a wet coil that are each treated separately; this method is analogous to dividing a heat exchanger with phase change into sub-heat exchangers that are separately undergoing desuperheating, condensing, and subcooling sections, as discussed in Section 8.5.2.

The ideal process associated with the moist air passing through the cooling coil can be represented approximately on a psychrometric diagram, as shown in Figure 9-7.

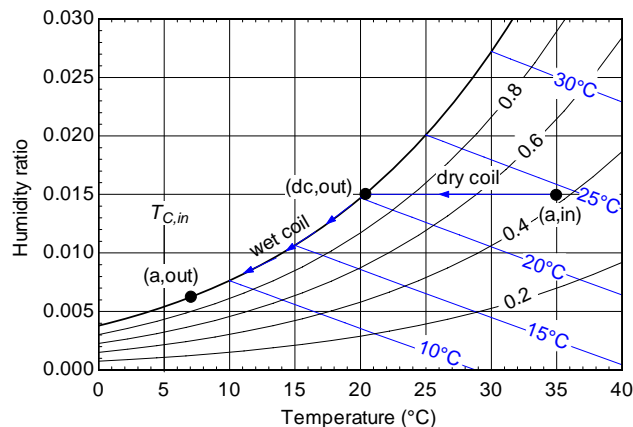


Figure 9-7: The dehumidification process associated with moist air passing through a cooling coil represented approximately on a psychrometric chart.

In Figure 9-7, moist air enters the coil at $T_{a,in} = 35^\circ\text{C}$ and 42% relative humidity ($RH_{in} = 0.42$). The air is exposed to a cold surface (e.g., the cold-side fluid may be refrigerant at 5°C) and therefore the temperature of the air decreases. Initially, the moist air remains above its dewpoint (which is approximately 20°C according to Figure 9-7) and therefore no condensation occurs; this process continues until the air becomes saturated (i.e., the relative humidity reaches 1.0). The coil is "dry" during this process and therefore the associated portion of cooling coil is referred to as the "dry coil". The humidity ratio (ω , the mass ratio of water vapor to air) remains constant in the dry coil and, assuming that the cooling coil is large enough, the air leaves the dry coil at the dew point temperature with 100% relative humidity at state (dc,out).

The saturated air leaving the dry coil enters the "wet coil" region of the cooling coil. In the wet coil, the air remains saturated and its temperature continues to drop. Water condenses out of the air as the humidity ratio drops along the saturation line shown in Figure 9-7. The dehumidification process is modeled using a "saturation" specific heat capacity for the air, which approximately accounts for the latent heat that is removed with the condensing water vapor. Also, the combined heat and mass transfer processes lead to a higher total conductance in the wet

coil than would be expected in the absence of mass transfer. This augmentation of the energy transfer is approximately accounted for using a "wet coil conductance".

An alternative and more empirical analysis of a cooling coil can be carried out using the "enthalpy-based effectiveness" technique that is discussed in Section 9.6.3 and based on Braun et al. (1989). The enthalpy-based effectiveness technique utilizes the concepts of a "saturation specific heat capacity" (due to the condensation process) and augmented "wet coil conductance" (due to the mass transfer) in order to compute an effectiveness using the conventional ε - NTU relationships from Section 8.3. However, the effectiveness definition is cast in terms of the enthalpy of the air-water mixture rather than its temperature in order to account for both the heat and mass transfer. EXAMPLE 9.6-1 analyzes a cooling coil using the dry coil/wet coil technique and EXAMPLE 9.6-2 revisits the problem using the enthalpy-based effectiveness approach. It is shown that the solutions obtained from these methods are consistent for most conditions.

9.6.2 Dry Coil/Wet Coil Analysis

The conventional ε - NTU relations that are presented in Section 8.3 cannot be directly used to simulate a wet coil because of the latent energy release associated with the dehumidification of the air and also because of the simultaneous heat and mass transfer processes that are occurring. However, the dry coil/wet coil analysis technique utilizes the ε - NTU solutions with appropriate modifications to the specific heat capacity of the air (to account for the latent heat release) and the conductance of the coil (to account for the combined heat and mass transfer).

The ε - NTU relations were discussed in Chapter 8 in the context of the solution for a counter-flow heat exchanger operating with two pure fluids and no mass transfer, as shown in Figure 9-8.

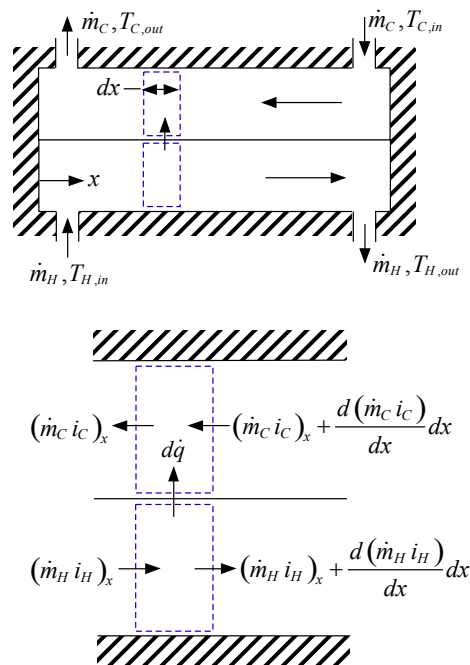


Figure 9-8: A counter-flow heat exchanger with two pure fluids and no mass transfer.

The governing equations are obtained by carrying out energy balances on the hot- and cold-fluids (see Figure 9-8) and introducing the total conductance between the two streams:

$$\boxed{d\dot{q} = -\dot{m}_C c_C \frac{dT_C}{dx} dx} \quad (9-104)$$

$$\boxed{d\dot{q} = -\dot{m}_H c_H \frac{dT_H}{dx} dx} \quad (9-105)$$

$$\boxed{d\dot{q} = (T_H - T_C) UA \frac{dx}{L}} \quad (9-106)$$

The solution to these equations, subject to the boundary conditions associated with the fluid inlet temperatures, is:

$$\ln \left(\frac{T_{H,out} - T_{C,in}}{T_{H,in} - T_{C,out}} \right) = -UA \left(\frac{1}{\dot{m}_H c_H} - \frac{1}{\dot{m}_C c_C} \right) \quad (9-107)$$

Equation (9-107) can be rearranged and expressed in terms of an ε - NTU relationship for a counter-flow heat exchanger:

$$\varepsilon = \begin{cases} \frac{1 - \exp[-NTU(1 - C_R)]}{1 - C_R \exp[-NTU(1 - C_R)]} & \text{for } C_R < 1 \\ \frac{NTU}{1 + NTU} & \text{for } C_R = 1 \end{cases} \quad (9-108)$$

where the effectiveness, ε , is defined according to:

$$\varepsilon = \frac{\dot{m}_H c_H (T_{H,in} - T_{H,out})}{\dot{C}_{min} (T_{H,in} - T_{C,in})} = \frac{\dot{m}_C c_C (T_{C,out} - T_{C,in})}{\dot{C}_{min} (T_{H,in} - T_{C,in})} \quad (9-109)$$

The number of transfer units, NTU , is:

$$NTU = \frac{UA}{\dot{C}_{min}} \quad (9-110)$$

The capacitance ratio, C_R , is:

$$C_R = \frac{\dot{C}_{min}}{\dot{C}_{max}} \quad (9-111)$$

where \dot{C}_{min} and \dot{C}_{max} are the minimum and maximum values, respectively, of the quantities $\dot{m}_C c_C$ and $\dot{m}_H c_H$.

In this subsequent sections, the governing differential equations for the flow of moist air over a dry coil are derived for the situation where the air temperature has not dropped below the dew point temperature and therefore condensation is not occurring (the region labeled dry coil in Figure 9-7). The governing differential equations are placed into the same format as Eqs. (9-104) through (9-106). Then, the governing differential equations for the flow of moist air over a wet coil are derived for the situation where water is condensing onto the coil (the region labeled wet coil in Figure 9-7). With several simplifying assumptions based on a formulation first presented by Braun et al. (1989), these governing equations are also placed into the same format as Eqs. (9-104) through (9-106). Because the governing differential equations for moist air over a dry coil and moist air over a wet coil are both analogous to Eqs. (9-104) through (9-106), the solutions must also be provided by the ε -NTU relation given by Eq. (9-108), using an appropriately defined specific heat capacity and total conductance. The analogy extends beyond a counter-flow configuration and can be used for any heat exchanger configuration, including the cross-flow configuration that is commonly used for a cooling coil.

Effectiveness-NTU Relations for a Dry Coil with Moist Air

The governing equations for the cooling coil with moist air but a dry coil are obtained by considering the differential energy balances shown in Figure 9-9. Because water vapor is not condensing out of the air, the humidity ratio is constant and there is no mass transfer that will augment the sensible heat transfer.

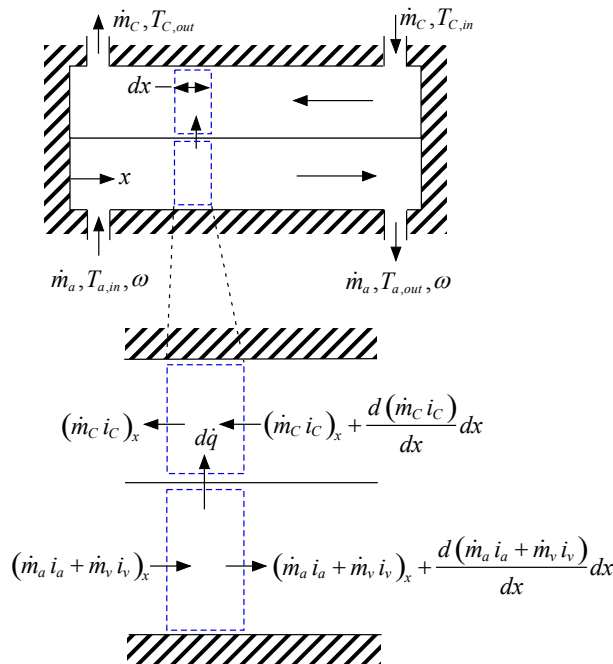


Figure 9-9: A counter-flow heat exchanger with moist air passing through a dry coil.

The differential energy balance on the cold fluid leads to:

$$\boxed{d\dot{q} = -\dot{m}_c c_c \frac{dT_c}{dx} dx} \quad (9-112)$$

A differential energy balance on the air leads to:

$$d\dot{q} = -\frac{d(\dot{m}_a i_a + \dot{m}_v i_v)}{dx} dx \quad (9-113)$$

where \dot{m}_a and \dot{m}_v are the mass flow rates of air and water vapor, respectively, and i_a and i_v are the specific enthalpies of air and water vapor, respectively. The ratio of the amount of water vapor to the amount of dry air is the humidity ratio:

$$\omega = \frac{\dot{m}_v}{\dot{m}_a} \quad (9-114)$$

Substituting Eq. (9-114) into Eq. (9-113) leads to:

$$d\dot{q} = -\frac{d[\dot{m}_a (i_a + \omega i_v)]}{dx} dx \quad (9-115)$$

The mass flow rate of dry air does not change with axial position, therefore Eq. (9-115) can be written as:

$$d\dot{q} = -\dot{m}_a \frac{d \overbrace{(i_a + \omega i_v)}^{i_a''}}{dx} dx \quad (9-116)$$

The term in parentheses in Eq. (9-116) is the enthalpy of the air-water mixture, expressed on a unit mass of dry air basis (i_a''):

$$i_a'' = i_a + \omega i_v \quad (9-117)$$

The enthalpy information provided by most psychrometric charts is typically expressed on a per unit mass of dry air basis. The enthalpy function in EES, when called for an air-water mixture (i.e., for the substance AirH2O) also returns the enthalpy on a per unit mass of dry air basis (i.e., it returns i_a''). Substituting Eq. (9-117) into Eq. (9-116) leads to:

$$d\dot{q} = -\dot{m}_a \frac{di_a''}{dx} dx \quad (9-118)$$

Expanding Eq. (9-118) and recognizing that the humidity ratio is constant in the dry coil section leads to:

$$d\dot{q} = -\dot{m}_a \underbrace{\left(\frac{\partial i_a''}{\partial T} \right)_{p,\omega}}_{c_a''} \frac{dT_a}{dx} dx \quad (9-119)$$

The partial derivative of i_a'' with respect to temperature at constant pressure and constant relative humidity is the specific heat capacity at constant pressure of the air-water mixture, expressed on a unit mass of air basis (c_a''). Examination of Eq. (9-117) shows that:

$$c_a'' = c_a + \omega c_v \quad (9-120)$$

The value of c_a'' is returned by the cp function in EES when the substance AirH2O is provided. The humidity ratio for a typical air-water mixture is small (on the order of 0.02 or less, see Figure 9-7) and therefore c_a'' is approximately equal to c_a . Substituting Eq. (9-120) to Eq. (9-119) leads to:

$$\boxed{d\dot{q} = -\dot{m}_a c_a'' \frac{dT_a}{dx} dx} \quad (9-121)$$

The differential rate of heat transfer is driven by the temperature difference between the air and the cold-side fluid and it can be written in terms of a dry-coil conductance, UA , that does not include the effect of mass transfer:

$$\boxed{d\dot{q} = (T_a - T_c) UA \frac{dx}{L}} \quad (9-122)$$

where UA is calculated using the techniques discussed in Section 8.1.5.

Equations (9-112), (9-121), and (9-122) are identical to Eqs. (9-104), (9-105), and (9-106) if T_H is replaced with T_a and c_H is replaced with c_a'' . For a counterflow configuration, the solution is provided by Eq. (9-108), with these substitutions.

$$\varepsilon = \begin{cases} \frac{1 - \exp[-NTU(1 - C_R)]}{1 - C_R \exp[-NTU(1 - C_R)]} & \text{for } C_R < 1 \\ \frac{NTU}{1 + NTU} & \text{for } C_R = 1 \end{cases} \quad (9-123)$$

where the effectiveness, ε , is:

$$\varepsilon = \frac{\dot{m}_a c_a'' (T_{a,in} - T_{a,out})}{\dot{C}_{min} (T_{a,in} - T_{C,in})} = \frac{\dot{m}_C c_C (T_{C,out} - T_{C,in})}{\dot{C}_{min} (T_{a,in} - T_{C,in})} \quad (9-124)$$

The number of transfer units, NTU , is:

$$NTU = \frac{UA}{\dot{C}_{min}} \quad (9-125)$$

The capacitance ratio, C_R , is:

$$C_R = \frac{\dot{C}_{min}}{\dot{C}_{max}} \quad (9-126)$$

where \dot{C}_{min} and \dot{C}_{max} are the minimum and maximum values, respectively, of the quantities $\dot{m}_C c_C$ and $\dot{m}_a c_a''$. For other heat exchanger configurations (e.g., the cross-flow configurations typically used for cooling coils), the ε - NTU solution, Eq. (9-123), must be replaced by the equivalent expression from Table 8-1, but the definition of the effectiveness, capacity ratio, and number of transfer units given by Eqs. (9-124) through (9-125) are unchanged.

Effectiveness-NTU Relations for a Wet Coil with Moist Air

The governing equations for the cooling coil with moist air and a wet coil are obtained by considering the differential energy balances shown in Figure 9-10. Because water vapor is condensing out of the air, the humidity ratio is changing (decreasing) in the direction of flow and there is a mass transfer process that augments the sensible heat transfer process.

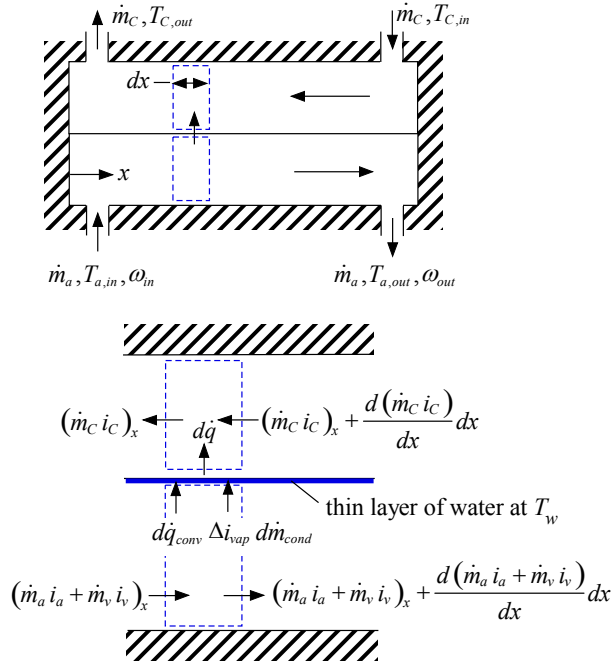


Figure 9-10: A counterflow heat exchanger with moist air passing through a wet coil.

The differential energy balance on the cold fluid is:

$$\boxed{d\dot{q} = -\dot{m}_c c_c \frac{dT_c}{dx} dx} \quad (9-127)$$

A differential energy balance on the air-water mixture is:

$$d\dot{q} = -\frac{d(\dot{m}_a i_a + \dot{m}_v i_v)}{dx} dx \quad (9-128)$$

Equation (9-128) can be expressed in terms of the humidity ratio:

$$d\dot{q} = -\dot{m}_a \frac{d(i_a + \omega i_v)}{dx} dx \quad (9-129)$$

and expanded:

$$d\dot{q} = -\dot{m}_a \left[\frac{\partial(i_a + \omega i_v)}{\partial T} \right]_{p, RH=1} \frac{dT}{dx} dx \quad (9-130)$$

On the air-side, the differential rate of energy transfer from the air to the cold fluid, $d\dot{q}$, is associated with both convection heat transfer from the air-water mixture to the wet coil surface ($d\dot{q}_{conv}$) as well as energy carried by the condensing water. The rate of energy carried by the

condensing water is the product of the mass flow rate of condensation ($d\dot{m}_{cond}$) and the latent heat of vaporization of water (Δi_{vap}):

$$d\dot{q} = d\dot{q}_{conv} + \Delta i_{vap} d\dot{m}_{cond} \quad (9-131)$$

Equation (9-130) does not resemble the corresponding energy balance for a dry heat exchanger, Eq. (9-105). The enthalpy change is not written in terms of a specific heat capacity and the rate of energy transfer, Eq. (9-131), cannot immediately be written in terms of a conductance and a temperature difference. The rate of energy transfer is augmented by mass transfer and the rate that the enthalpy of the air-water mixture is changing is augmented by the latent heat associated with the condensing water. In order to make Eq. (9-130) resemble Eq. (9-105), it is necessary to represent the effect of the latent heat approximately using a saturation specific heat. It is also necessary to represent the combined heat and mass transfer approximately using a wet coil conductance.

Saturation Specific Heat Capacity

In the wet coil section, the humidity ratio is not constant; rather, the air-water vapor mixture is always saturated (see Figure 9-7) and therefore the relative humidity is always 1. Each of the terms in square brackets in Eq. (9-130) is changing with temperature. Equation (9-130) can be expanded:

$$d\dot{q} = -\dot{m}_a \left[\underbrace{\left(\frac{\partial i_a}{\partial T} \right)_p}_{c_a''} + \omega \underbrace{\left(\frac{\partial i_v}{\partial T} \right)_p}_{c_v''} + i_v \left(\frac{\partial \omega}{\partial T} \right)_{p, RH=1} \right] \frac{dT}{dx} dx \quad (9-132)$$

Equation (9-132) can be expressed in terms of c_a and c_v :

$$d\dot{q} = -\dot{m}_a \left[\underbrace{c_a + \omega c_v}_{c_a''} + i_v \left(\frac{\partial \omega}{\partial T} \right)_{p, RH=1} \right] \frac{dT}{dx} dx \quad (9-133)$$

or, by substituting Eq. (9-120) into Eq. (9-133), in terms of c_a'' (the specific heat capacity of the air-water vapor mixture expressed on a per mass of air basis):

$$d\dot{q} = -\dot{m}_a \left[\underbrace{c_a'' + i_v \left(\frac{\partial \omega}{\partial T} \right)_{p, RH=1}}_{c_{a, sat}''} \right] \frac{dT}{dx} dx \quad (9-134)$$

The term within the square brackets in Eq. (9-134) is referred to as the saturation specific heat capacity, $c_{a, sat}''$; it is the partial derivative of the enthalpy of the air-water mixture with respect to

temperature along the saturation line (i.e., along the $RH = 1$ line in Figure 9-7). The second term in brackets will be large and positive (because the partial derivative of the humidity ratio with respect to temperature is positive along the saturation line) and therefore $c''_{a,sat}$ will be substantially greater than c''_a .

$$\boxed{d\dot{q} = -\dot{m}_a c''_{a,sat} \frac{dT}{dx} dx} \quad (9-135)$$

where

$$c''_{a,sat} = \left(\frac{\partial i''_a}{\partial T} \right)_{p,RH=1} \quad (9-136)$$

Notice that the use of the saturation specific heat capacity causes Eq. (9-135) to resemble the corresponding dry heat exchanger energy balance, (9-105). The saturation specific heat capacity can be evaluated numerically according to:

$$c''_{a,sat} = \left(\frac{\partial i''_a}{\partial T} \right)_{p,RH=1} \approx \frac{[i''_a(T + \Delta T, p, RH = 1) - i''_a(T - \Delta T, p, RH = 1)]}{2 \Delta T} \quad (9-137)$$

where ΔT is a small perturbation in temperature. In EES, the saturation specific heat capacity at a particular temperature and pressure can be computed according to:

```
$UnitSystem SI MASS RAD PA K J
$Tabstops 0.2 0.4 0.6 3.5 in
```

```
T_C=20 [C]           "temperature in C"
T=converttemp(C,K,T_C) "temperature"
p=1 [atm]*convert(atm,Pa) "pressure"
DT=0.2 [K]          "perturbation in temperature"
c``_a_sat=(enthalpy(AirH2O,T=T+DT,P=p,R=1)-enthalpy(AirH2O,T=T-DT,P=p,R=1))/(2*DT)
"saturation specific heat capacity"
```

Figure 9-11 illustrates the saturation specific heat capacity as well as the specific heat capacity of dry air (c_a) as a function of temperature. The normal specific heat capacity of moist air on a per unit mass of dry air basis (c''_a) is relatively independent of the humidity ratio and temperature and approximately equal to c_a . However, Figure 9-11 shows that the saturation specific heat capacity ($c''_{a,sat}$) is highly temperature dependent.

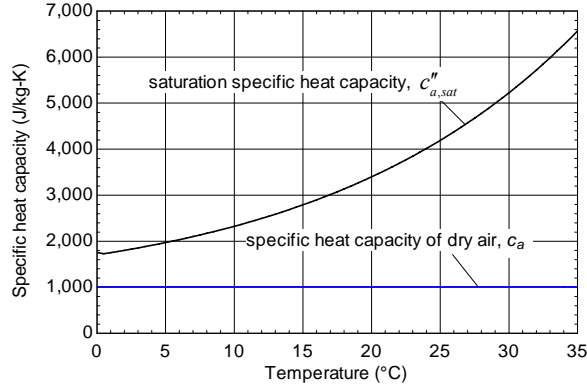


Figure 9-11: Saturation specific heat capacity of an air-water mixture, $c''_{a,sat}$, and the normal specific heat capacity of dry air, c_a .

The saturation specific heat capacity varies substantially with temperature; however, the ε - NTU solutions are derived by assuming that the fluids have constant specific heat capacities. An average saturation specific heat capacity must be used to simulate a wet coil. The average saturation specific heat capacity evaluated between $T_{a,in}$ and $T_{C,in}$ is:

$$c''_{a,sat} \approx \frac{[i''_a(T_{a,in}, p, RH=1) - i''_a(T_{C,in}, p, RH=1)]}{(T_{a,in} - T_{C,in})} \quad (9-138)$$

The average saturation specific heat capacity evaluated between $T_{a,in}$ and $T_{a,out}$ is even more appropriate:

$$c''_{a,sat} \approx \frac{[i''_a(T_{a,in}, p, RH=1) - i''_a(T_{a,out}, p, RH=1)]}{(T_{a,in} - T_{a,out})} \quad (9-139)$$

However, the outlet air temperature, $T_{a,out}$, is not known until the problem is solved and so the use of Eq. (9-139) will require an iterative solution.

Wet Coil Conductance

The rate of energy transfer from the air to the wet coil in the differential control volume in Figure 9-10 ($d\dot{q}$) occurs both due to convective heat transfer ($d\dot{q}_{conv}$) as well as energy carried by the mass transfer ($\Delta i_{vap} d\dot{m}_{cond}$). An energy balance on the water surface (see Figure 9-10) interface leads to:

$$d\dot{q} = d\dot{q}_{conv} + \Delta i_{vap} d\dot{m}_{cond} \quad (9-140)$$

The convective heat transfer is driven by the air to water temperature difference ($T_a - T_w$):

$$d\dot{q}_{conv} = \eta_o A_{tot} h_o (T_a - T_w) \frac{dx}{L} \quad (9-141)$$

where η_o is the overall surface efficiency (for a surface that is finned), A_{tot} is the total air-side surface area (finned and un-finned), and h_o is the local air-side heat transfer coefficient. The differential mass flow rate at which water is condensed ($d\dot{m}_{cond}$) can be expressed in terms of a local mass transfer coefficient (h_D).

$$d\dot{m}_{cond} = \eta_o A_{tot} h_D \rho_a (\omega_{sat,a} - \omega_{sat,w}) \frac{dx}{L} \quad (9-142)$$

Equation (9-142) is equivalent to Eq. (9-83) with the concentration of water vapor, c , replaced by the product of the density of the air-water mixture (ρ_a) and the humidity ratio. The driving force for mass transfer is the difference between the humidity ratio of saturated air at the local air temperature ($\omega_{sat,a}$ evaluated at T_a) and the humidity ratio of saturated air at the water temperature ($\omega_{sat,w}$ evaluated at T_w). Substituting Eqs. (9-141) and (9-142) into Eq. (9-140) leads to:

$$d\dot{q} = \eta_o A_{tot} h_o (T_a - T_w) \frac{dx}{L} + \Delta i_{vap} \eta_o A_{tot} h_D \rho_a (\omega_{sat,a} - \omega_{sat,w}) \frac{dx}{L} \quad (9-143)$$

The mass transfer coefficient, h_D , is related to the convective heat transfer coefficient, h_o , through the definitions of the Sherwood (Sh) and Lewis (Le) numbers, as discussed in Section 9.4.2. The Nusselt number is defined as:

$$Nu = \frac{h_o D_h}{k_a} \quad (9-144)$$

where D_h is the hydraulic diameter of the air flow passages and k_a is the conductivity of the air-water mixture. The Sherwood number (Sh) is defined as:

$$Sh = \frac{h_D D_h}{D_{a,v}} \quad (9-145)$$

where $D_{a,v}$ is the diffusion coefficient for water vapor in air. Combining Eqs. (9-144) and (9-145) leads to:

$$\frac{h_D}{h_o} = \frac{Sh}{Nu} \frac{D_{a,v}}{k_a} \quad (9-146)$$

The Lewis number (Le) relates k_a and $D_{a,v}$ according to:

$$Le = \frac{\alpha_a}{D_{a,v}} = \frac{k_a}{\rho_a c_a'' D_{a,v}} \quad (9-147)$$

Substituting Eq. (9-147) into Eq. (9-146) leads to:

$$\frac{h_D}{h_o} = \frac{Sh}{Nu} \left(\frac{1}{\rho_a c_a'' Le} \right) \quad (9-148)$$

Substituting Eq. (9-148) into Eq. (9-143) leads to:

$$d\dot{q} = \eta_o A_{tot} h_o (T_a - T_w) \frac{dx}{L} + \frac{\Delta i_{vap} \eta_o A_{tot} h_o}{c_a'' Le} \frac{Sh}{Nu} (\omega_{sat,a} - \omega_{sat,w}) \frac{dx}{L} \quad (9-149)$$

or

$$d\dot{q} = \frac{\eta_o A_{tot} h_o}{c_a''} \left[c_a'' (T_a - T_w) + \frac{Sh}{Le Nu} (\omega_{sat,a} - \omega_{sat,w}) \Delta i_{vap} \right] \frac{dx}{L} \quad (9-150)$$

Expanding the latent heat of vaporization in Eq. (9-150) leads to:

$$d\dot{q} = \frac{\eta_o A_{tot} h_o}{c_a''} \left[c_a'' (T_a - T_w) + \frac{Sh}{Le Nu} (\omega_{sat,a} - \omega_{sat,w}) (i_{w,vapor} - i_{w,liquid}) \right] \frac{dx}{L} \quad (9-151)$$

or

$$d\dot{q} = \frac{\eta_o A_{tot} h_o}{c_a''} \left[c_a'' (T_a - T_w) + \frac{Sh}{Le Nu} (\omega_{sat,a} i_{w,vapor} - \omega_{sat,a} i_{w,liquid} - \omega_{sat,w} i_{w,vapor} + \omega_{sat,w} i_{w,liquid}) \right] \frac{dx}{L} \quad (9-152)$$

Rearranging the terms in Eq. (9-152) leads to:

$$d\dot{q} = \frac{\eta_o A_{tot} h_o}{c_a''} \left[\left(c_a'' T_a + \frac{Sh}{Le Nu} \omega_{sat,a} i_{w,vapor} \right) - \left(c_a'' T_w + \frac{Sh}{Le Nu} \omega_{sat,w} i_{w,vapor} \right) + \underbrace{\frac{Sh}{Le Nu} (\omega_{sat,a} - \omega_{sat,w}) i_{w,liquid}}_{\text{negligible}} \right] \frac{dx}{L} \quad (9-153)$$

To proceed, it is necessary to apply a few reasonable assumptions. First, the last term in brackets is negligibly small relative to the other two terms because the difference between the humidity ratios is small and also because the enthalpy of liquid water is much smaller than the enthalpy of water vapor. Therefore, Eq. (9-153) can be written as:

$$d\dot{q} = \frac{\eta_o A_{s,o} h_o}{c_a''} \left[\left(c_a'' T_a + \frac{Sh}{Le Nu} \omega_{sat,a} i_{w,vapor} \right) - \left(c_a'' T_w + \frac{Sh}{Le Nu} \omega_{sat,w} i_{w,vapor} \right) \right] \frac{dx}{L} \quad (9-154)$$

Also, the Lewis number for air-water vapor mixtures near room temperature is approximately 0.9. If Lewis number is assumed to be 1, then the Schmidt number is equal to the Prandtl number based on Eq. (9-78). With this assumption, the discussion in Section 9.4.2 suggests that the Sherwood number must be equal to the Nusselt number and the group $Sh/(Le Nu)$ must be approximately 1 and can therefore be removed from Eq. (9-153).

$$d\dot{q} = \frac{\eta_o A_{tot} h_o}{c_a''} \left[\underbrace{\left(c_a'' T_a + \omega_{sat,a} i_{w,vapor} \right)}_{i_{a,sat,T_a}''} - \underbrace{\left(c_a'' T_w + \omega_{sat,w} i_{w,vapor} \right)}_{i_{a,sat,T_w}''} \right] \frac{dx}{L} \quad (9-155)$$

The two terms within the square brackets in Eq. (9-155) are, approximately, the difference between specific enthalpies of saturated moist air at T_a and T_w , respectively. Therefore, Eq. (9-155) can be written as:

$$d\dot{q} = \frac{\eta_o A_{tot} h_o}{c_a''} \underbrace{\left(i_{a,sat,T_a}'' - i_{a,sat,T_w}'' \right)}_{\approx c_{a,sat}'' (T_a - T_w)} \frac{dx}{L} \quad (9-156)$$

The saturation enthalpy difference in Eq. (9-156) can be replaced with a temperature difference using the saturation specific heat defined in Eq. (9-136):

$$d\dot{q} = \eta_o A_{tot} h_o \underbrace{\frac{c_{a,sat}''}{c_a''}}_{>1, \text{ augmentation due to mass transfer}} (T_a - T_w) \frac{dx}{L} \quad (9-157)$$

Notice that the effect of the mass transfer is to augment the convective heat transfer coefficient by the ratio of the saturation specific heat capacity to the normal specific heat capacity, $c_{a,sat}'' / c_a''$. Examination of Figure 9-11 shows that $c_{a,sat}'' / c_a''$ can be quite large (from 2 to 6, depending on temperature), indicating that the effect of mass transfer is to substantially enhance the energy transfer.

The wet coil conductance is computed in the same way that the dry coil conductance is determined, but with the convective heat transfer coefficient on the air-side multiplied by the augmentation factor, $c_{a,sat}'' / c_a''$. For a typical cooling coil analysis, the wet coil conductance should be calculated according to:

$$UA_{wet} = \frac{1}{\frac{c_a''}{\eta_o A_{tot} \bar{h}_o c_{a,sat}''} + \frac{R_{f,i}''}{A_{s,i}} + \frac{1}{\bar{h}_i A_{s,i}}} \quad (9-158)$$

where \bar{h}_i and \bar{h}_o are the average heat transfer coefficients on the cold-side and air-side, respectively, $A_{s,i}$ is the surface area on the cold-side, and $R_{f,i}''$ is the area specific fouling resistance on the cold side.

The heat transfer rate within the differential control volume is therefore:

$$\boxed{d\dot{q} = UA_{wet} (T_a - T_C) \frac{dx}{L}} \quad (9-159)$$

Effectiveness-NTU Solution for a Wet Coil

Equations (9-127), (9-135), and (9-159) are identical to Eqs. (9-104), (9-105), and (9-106) if T_H is replaced with T_a , c_H is replaced with $c_{a,sat}''$, and UA is replaced with UA_{wet} . Therefore, the solution is provided by Eq. (9-108) with these substitutions. For a counterflow configuration, the effectiveness is:

$$\varepsilon = \begin{cases} \frac{1 - \exp[-NTU(1 - C_R)]}{1 - C_R \exp[-NTU(1 - C_R)]} & \text{for } C_R < 1 \\ \frac{NTU}{1 + NTU} & \text{for } C_R = 1 \end{cases} \quad (9-160)$$

where the effectiveness, ε , is defined as:

$$\varepsilon = \frac{\dot{m}_a c_{a,sat}'' (T_{a,in} - T_{a,out})}{\dot{C}_{min} (T_{a,in} - T_{C,in})} = \frac{\dot{m}_C c_C (T_{C,out} - T_{C,in})}{\dot{C}_{min} (T_{a,in} - T_{C,in})} \quad (9-161)$$

The number of transfer units, NTU , is:

$$NTU = \frac{UA_{wet}}{\dot{C}_{min}} \quad (9-162)$$

The capacitance ratio, C_R , is:

$$C_R = \frac{\dot{C}_{min}}{\dot{C}_{max}} \quad (9-163)$$

where \dot{C}_{min} and \dot{C}_{max} are the minimum and maximum values, respectively, of the quantities $\dot{m}_c c_c$ and $\dot{m}_a c_{a,sat}$. For other heat exchanger configurations (e.g., the cross-flow configurations typically used for cooling coils), the ε - NTU solution, Eq. (9-160), must be replaced by the equivalent expression from Table 8-1, but the definition of the effectiveness, capacity ratio, and number of transfer units given by Eqs. (9-161) through (9-162) are unchanged.

EXAMPLE 9.6-1: Cooling Coil

The finned, circular tube cross-flow heat exchanger shown in Figure 1 is used as a cooling coil for dehumidification; note that this heat exchanger core was previously examined in EXAMPLE 8.1-1 and EXAMPLE 8.1-2. The width and height of the front face of the heat exchanger are $W = 0.20$ m and $H = 0.26$ m, respectively. The fins are made of copper with a thickness $th_{fin} = 0.33$ mm and a fin pitch $p_{fin} = 3.18$ mm. Ten rows of tubes ($N_{t,row} = 10$) in two columns ($N_{t,col} = 2$) are connected in series. The vertical and horizontal spacing between adjacent tubes is $s_v = 25.4$ mm and $s_h = 22$ mm, respectively. The length of the heat exchanger in the direction of the air flow is $L = 0.06$ m. The tubes are made of copper with an outer diameter $D_{out} = 1.02$ cm and a wall thickness $th = 0.9$ mm. The roughness of the inner surface of the tube is $e = 1.0$ μ m. Assume for this problem that the efficiency of the fins is 100%.

Treated water enters the tube with mass flow rate $\dot{m}_w = 0.06$ kg/s. The inlet temperature is $T_{w,in} = 5^\circ\text{C}$ and the pressure of the water is $p_w = 2$ atm. Clean moist air is forced to flow through the heat exchanger perpendicular to the tubes (i.e., in cross-flow) with a volumetric flow rate $\dot{V}_a = 0.06$ m³/s. The inlet temperature of the air is $T_{a,in} = 27^\circ\text{C}$ and the inlet relative humidity is $RH_{a,in} = 0.75$. The air is at atmospheric pressure.

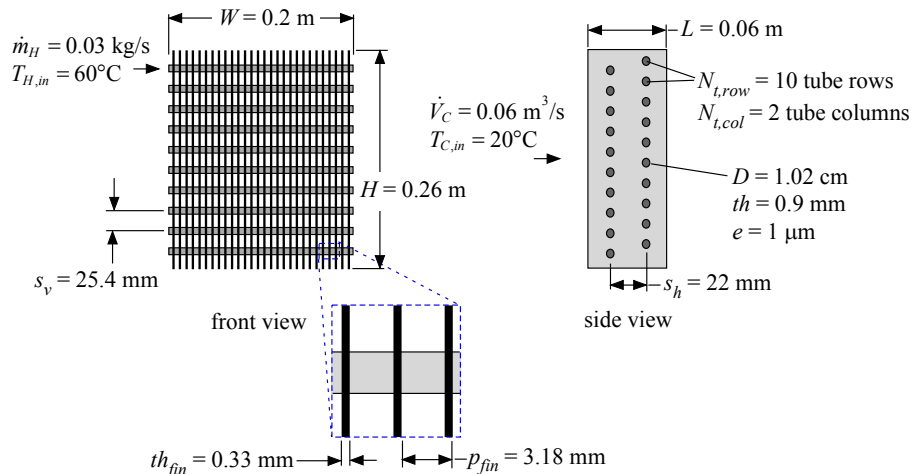


Figure 1: Schematic of a plate fin heat exchanger used as a cooling coil.

- a.) Use the dry coil/wet coil analysis technique to compute the heat transfer in the cooling coil and the rate at which water condenses out of the air.

The inputs are entered in EES:

```
"EXAMPLE 9.6-1: Cooling Coil"
$UnitSystem SI MASS RAD PA K J
$Tabstops 0.2 0.4 0.6 3.5 in
```

```
"Inputs"
```

```
D_out=1.02 [cm]*convert(cm,m)           "outer diameter of tube"
th = 0.9 [mm]*convert(mm,m)            "tube wall thickness"
N_t_row=10 [-]                          "number of tube rows"
N_t_col=2 [-]                            "number of tube columns"
H=0.26 [m]                               "height of heat exchanger face"
W=0.20 [m]                               "width of heat exchanger face"
L=0.06 [m]                               "length of heat exchanger in air flow direction"
V_dot_a=0.06 [m^3/s]                    "volumetric flow rate of air"
p_a=1 [atm]*convert(atm,Pa)             "atmospheric pressure"
T_a_in=convertTemp(C,K,27 [C])          "inlet air temperature"
RH_a_in=0.75 [-]                        "inlet air relative humidity"
T_w_in=convertTemp(C,K,5 [C])           "inlet water temperature"
p_w=2 [atm]*convert(atm,Pa)             "pressure of water"
m_dot_w=0.06 [kg/s]                     "water flow rate"
s_v=25.4 [mm]*convert(mm,m)             "vertical separation distance between tubes"
s_h=22 [mm]*convert(mm,m)              "horizontal separation distance between tubes"
th_fin=0.33 [mm]*convert(mm,m)          "fin thickness"
p_fin=3.18 [mm]*convert(mm,m)           "fin pitch"
e=1.0 [micron]*convert(micron,m)        "roughness of tube internal surface"
```

The inlet humidity ratio ($\omega_{a,in}$) and the dewpoint associated with the incoming air (T_{dp} , the temperature at which the relative humidity reaches 100% and therefore water will begin to condense) are computed:

```
omega_a_in=HumRat(AirH2O,P=p_a,T=T_a_in,R=RH_a_in)  "humidity ratio at inlet"
T_dp=Temperature(AirH2O,P=p_a,w=omega_a_in,R=1)     "dewpoint temperature of air"
T_dp_C=converttemp(K,C,T_dp)                       "in C"
```

which leads to $T_{dp} = 295.3 \text{ K}$ (22.19°C). Because the entering water temperature is less than T_{dp} , water will condense onto the coil surface provided that the cooling coil is sufficiently large.

The water-side thermal resistance is evaluated. The inner diameter of the tube is:

$$D_{in} = D_{out} - 2th$$

and the total length of tube is:

$$L_{tube} = N_{t,row} N_{t,col} W$$

The average water-side heat transfer coefficient (\bar{h}_w) is computed using the PipeFlow function in EES at the average temperature in the heat exchanger. The resistance to heat transfer between the water and the tube inner surface is:

$$R_{in} = \frac{1}{\pi D_{in} L_{tube} \bar{h}_w}$$

"Internal flow through the tube"

D_in=D_out-2*th "tube inner diameter"
 L_tube=N_t_row*N_t_col*W "total tube length"
 T_avg=(T_a_in+T_w_in)/2 "average temperature"
 call PipeFlow('Water',T_avg,p_w,m_dot_w,D_in,L_tube,e/D_in:&
 h_bar_w, h_bar_w_H ,DELTA_p_w, Nusselt_bar_w_T, f_w, Re_w)
 "access correlations for internal flow"
 R_in=1/(pi*D_in*L_tube*h_bar_w) "resistance to convection on water-side"

The area-specific fouling resistance for closed-loop treated water ($R''_{f,in}$) is obtained using the FoulingFactor function in EES and used to compute the fouling resistance on the water side:

$$R_{f,in} = \frac{R''_{f,in}}{\pi D_{in} L_{tube}}$$

"Fouling resistance"

R``_f_in=FoulingFactor('Closed-loop treated water') "fouling factor on inner surface of tube"
 R_f_in=R``_f_in/(pi*D_in*L_tube) "fouling resistance on inner surface of tube"

The conductivity of copper (k_m) is obtained at the average temperature using the internal property routine in EES and used to compute the conduction resistance through the tube:

$$R_{cond} = \frac{\ln\left(\frac{D_{out}}{D_{in}}\right)}{2\pi k_m L_{tube}}$$

"Conduction resistance"

k_m=k_('Copper',T_avg) "tube conductivity"
 R_cond=ln(D_out/D_in)/(2*pi*k_m*L_tube) "tube resistance"

The mass flow rate of air is computed according to:

$$\dot{m}_a = \rho_{a,in} \dot{V}_a$$

where $\rho_{a,in}$ is the density of the air at the inlet. The heat exchanger geometry corresponds to compact heat exchanger core fc_tubes_s80-38T. The compact heat exchanger function CHX_h_finned_tube is used to evaluate the dry heat transfer coefficient on the air side (\bar{h}_a):

"Compact heat exchanger correlation"

rho_a_in=density(Air,T=T_a_in,P=P_a) "density of air at inlet"
 m_dot_a=V_dot_a*rho_a_in "mass flow rate of air"
 TypeHX\$='fc_tubes_s80-38T' "heat exchanger identifier name"
 Call CHX_h_finned_tube(TypeHX\$, m_dot_a, W*H, 'Air',T_avg, P_a:h_bar_a)

"access compact heat exchanger procedure"

The total surface area of the fins is:

$$A_{s,fin,tot} = 2 \left(\frac{W}{P_{fin}} \right) \left(H L - N_{t,row} N_{t,col} \frac{\pi D_{out}^2}{4} \right)$$

and the total surface area of the unfinned tube is:

$$A_{s,unfin} = \pi D_{out} L_{tube} \left(1 - \frac{th_{fin}}{P_{fin}} \right)$$

The total air-side surface area is:

$$A_{tot} = A_{s,fin,tot} + A_{s,unfin}$$

$$A_{s,fin,tot} = 2 * (W/p_{fin}) * (H * L - N_{t,row} * N_{t,col} * \pi * D_{out}^2 / 4)$$

"total fin area"

$$A_{s,unfin} = \pi * D_{out} * L_{tube} * (1 - th_{fin}/p_{fin})$$

"total un-finned area"

$$A_{tot} = A_{s,fin,tot} + A_{s,unfin}$$

"total air-side surface area"

The dry coil/wet coil analysis divides the heat exchanger into a separate dry coil and wet coil. In the dry coil, the temperature of the moist air is reduced to the dew point temperature; the exit temperature from the dry coil ($T_{a,dc,out}$) is therefore the dew point temperature.

$$T_{a,dc,out} = T_{dp}$$

"temperature of air at exit of dry coil"

The fraction of the total heat exchange surface required by the dry coil section is determined. The specific heat capacity of the moist-air on a per mass of dry air basis (c''_a) is calculated using EES' internal psychrometric function.

$$c''_a = cP(\text{AirH}_2\text{O}, P = p_a, T = T_{a,in}, R = RH_{a,in})$$

"specific heat capacity of moist air in dry coil"

The specific heat capacity of the cooling water (c_w) is calculated using EES' internal property routine:

$$c_w = cP(\text{Water}, P = p_w, T = T_{avg})$$

"specific heat capacity of water"

The capacitance rate of the air and the water in the dry-coil are:

$$\dot{C}_{a,dc} = \dot{m}_a c''_a$$

$$\dot{C}_w = \dot{m}_w c_w$$

$$C_{dot,a,dc} = m_{dot,a} * c''_a$$

"capacity rate of air"

$C_{dot_w} = m_{dot_w} * c_w$

"capacity rate of water"

Provided that the cooling coil is sufficiently large so that the dew point temperature is reached, the rate of heat transfer in the dry coil section of the heat exchanger is given by:

$$\dot{q}_{dc} = \dot{C}_{dc} (T_{a,in} - T_{a,dc,out})$$

The minimum capacitance rate in the dry coil section ($\dot{C}_{min,dc}$) is obtained using the Min function and used to compute the maximum possible heat transfer in the dry coil:

$$\dot{q}_{dc,max} = \dot{C}_{min,dc} (T_{a,in} - T_{w,in})$$

The effectiveness of the dry coil section is the ratio of the actual to the maximum possible heat transfer rate:

$$\varepsilon_{dc} = \frac{\dot{q}_{dc}}{\dot{q}_{dc,max}}$$

$q_{dot_dc} = C_{dot_a_dc} * (T_{a,in} - T_{a_dc_out})$

"heat transfer rate in dry coil"

$C_{dot_min_dc} = \text{Min}(C_{dot_a_dc}, C_{dot_w})$

"minimum heat capacity rate in dry coil"

$q_{dot_dc_max} = C_{dot_min_dc} * (T_{a,in} - T_{w,in})$

"maximum possible heat transfer rate in dry coil"

$eff_dc = q_{dot_dc} / q_{dot_dc_max}$

"effectiveness of dry coil"

The number of transfer units in the dry coil section (NTU_{dc}) is obtained using the ε - NTU solution for a cross-flow heat exchanger (with the air unmixed and the water mixed) accessed using the HX function in EES. The conductance required in the dry-coil section is:

$$UA_{dc} = NTU_{dc} \dot{C}_{min,dc}$$

$NTU_{dc} = \text{HX}(\text{'crossflow_one_unmixed'}, eff_dc, C_{dot_a_dc}, C_{dot_w}, \text{'NTU'})$

"number of transfer units in dry coil"

$UA_{dc} = NTU_{dc} * C_{dot_min_dc}$

"conductance in dry coil"

The total resistance to heat transfer in the dry coil is the inverse of the conductance:

$$R_{dc} = \frac{1}{UA_{dc}}$$

and is made up of the resistance to convection on the water side, a fouling resistance, resistance to conduction through the tube, and a convection resistance on the air side (assuming that the fin efficiency is 100%):

$$R_{dc} = \frac{R_{in} + R_{f,in} + R_{cond}}{F_{dc}} + \frac{1}{\bar{h}_a A_{tot} F_{dc}}$$

where F_{dc} is the fraction of the total heat exchanger required by the dry coil.

$$R_{dc}=1/UA_{dc} \quad \text{"thermal resistance in dry coil"}$$

$$R_{dc}=(R_{in}+R_{f,in}+R_{cond})/F_{dc}+1/(\bar{h}_a A_{tot} F_{dc}) \quad \text{"determine } F_{dc}\text{"}$$

which leads to $F_{dc} = 0.329$ (i.e., 33% of the coil is dry, according to this analysis). The remainder of the heat exchanger coil is wet:

$$F_{wc} = 1 - F_{dc}$$

"Wet coil analysis"

$$F_{wc}=1-F_{dc} \quad \text{"fraction of heat exchanger that is wet"}$$

The wet coil is analyzed using the concepts of a saturation specific heat capacity and wet coil conductance. The saturation specific heat capacity is used to account for the latent heat associated with the condensing water vapor and is calculated using Eq. (9-139):

$$c''_{a,sat} = \frac{\left[i''_a(T_{dc,out}, p, RH=1) - i''_a(T_{a,out}, p, RH=1) \right]}{(T_{dc,out} - T_{a,out})}$$

Initially, a value for $T_{a,out}$ is assumed; this value is subsequently commented out in order to close the solution:

$$T_{a,out}=T_{w,in} \quad \text{"an initial assumption for the saturation specific heat capacity"}$$

$$c''_{a,sat}=(\text{enthalpy}(\text{AirH2O},T=T_{a,dc,out},R=1,P=p_a)\&$$

$$\text{enthalpy}(\text{AirH2O},T=T_{a,out},R=1,P=p_a))/(T_{a,dc,out}-T_{a,out})$$

$$\text{"saturation specific heat capacity"}$$

The capacitance rate of the air in the wet coil is:

$$\dot{C}_{a,wc} = \dot{m}_a c''_{a,sat}$$

and the minimum capacitance rate in the wet coil ($\dot{C}_{min,wc}$) is computed using the Min function:

$$C_{dot_a_wc}=\dot{m}_{dot_a}c''_{a_sat} \quad \text{"capacity rate of air in wet coil"}$$

$$C_{dot_min_wc}=\text{Min}(C_{dot_a_wc},C_{dot_w}) \quad \text{"minimum heat capacity rate in wet coil"}$$

The conductance of the wet coil is computed by augmenting the air side heat transfer coefficient by the ratio $c''_{a,sat} / c''_a$ in order to account for the enhancement of the energy transfer rate due to mass transfer. The thermal resistance of the wet coil section is:

$$R_{wc} = \frac{R_{in} + R_{f,in} + R_{cond}}{F_{wc}} + \frac{c''_a}{\bar{h}_a c''_{a,sat} A_{tot} F_{wc}}$$

and the conductance of the wet coil is:

$$UA_{wc} = \frac{1}{R_{wc}}$$

$R_{wc}=(R_{in}+R_{f_{in}}+R_{cond})/F_{wc}+c''_a/(A_{tot}*h_{bar}_a*c''_a_{sat}*F_{wc})$ "wet coil resistance"
 $UA_{wet}=1/R_{wet}$ "wet coil conductance"

The number of transfer units in the wet coil is:

$$NTU_{wc} = \frac{UA_{wc}}{\dot{C}_{min,wc}}$$

The effectiveness of the wet coil (ε_{wc}) is obtained using the ε - NTU solution for a cross-flow heat exchanger, obtained using the HX function in EES:

$NTU_{wc}=UA_{wc}/C_{dot_min_wc}$ "number of transfer units in the wet coil"
 $eff_{wc}=HX('crossflow_one_unmixed', NTU_{wc}, C_{dot_a_wc}, C_{dot_w}, 'epsilon')$ "effectiveness of wet coil"

The heat transfer rate in the wet coil section is obtained from the effectiveness:

$$\dot{q}_{wc} = \varepsilon_{wc} \dot{C}_{min,wc} (T_{dc,out} - T_{w,in})$$

$q_{dot_wc}=eff_{wc}*C_{dot_min_wc}*(T_{a_dc_out}-T_{w_in})$ "heat transfer rate in wet coil"

The temperature of the air leaving the wet coil is obtained using an energy balance on the air-side:

$$T_{a,out} = T_{dc,out} - \frac{\dot{q}_{wc}}{\dot{C}_{a,wc}}$$

Before calculating the value of $T_{a,out}$, the guess values for the problem are updated (select Update Guesses from the Calculate menu) and the initial guess for $T_{a,out}$ is commented out:

$\{T_{a,out}=T_{w,in}\}$ "an initial assumption for the saturation specific heat capacity"
 $T_{a,out}=T_{a_dc_out}-q_{dot_wc}/C_{dot_a_wc}$ "temperature of air leaving wet coil"
 $T_{a,out_C}=converttemp(K,C,T_{a,out})$ "in C"

which leads to $T_{a,out} = 290.5$ K (17.3°C).

The wet coil model assumes that the air leaves saturated (i.e., at 100% relative humidity) and therefore the humidity ratio of the air leaving the heat exchanger ($\omega_{a,out}$) can be obtained using EES' psychrometric functions.

$omega_{a,out}=HumRat(AirH2O,T=T_{a,out},R=1,P=p_a)$ "humidity ratio at exit"

The total heat transfer in the cooling coil predicted by the wet coil/dry coil method is the sum of the heat transferred in the wet and dry sections:

$$\dot{q}_{wc/dc} = \dot{q}_{dc} + \dot{q}_{wc}$$

and the mass flow rate of condensate is:

$$\dot{m}_{cond,wc/dc} = \dot{m}_a (\omega_{a,in} - \omega_{a,out})$$

"coil performance"

$q_dot_wcdc=q_dot_dc+q_dot_wc$ "total heat transfer rate using wet coil/dry coil method"

$m_dot_cond_wcdc=m_dot_a*(\omega_{a,in}-\omega_{a,out})$ "mass flow rate of condensate"

$m_dot_cond_wcdc_kgph=m_dot_cond_wcdc*convert(kg/s,kg/hr)$ "in kg/hr"

which leads to $\dot{q}_{wc/dc} = 1503 \text{ W}$ and $\dot{m}_{cond,wc/dc} = 0.0003146 \text{ kg/s}$ (1.132 kg/hr).

9.6.3 Enthalpy-Based Effectiveness Analysis

Section 9.6.2 provides a model for a partially wet coil that is based on the idea that the air passing through the coil will follow the ideal process in Figure 9-7 where it is cooled at constant humidity until it encounters the saturation (i.e., 100% relative humidity) line, at which point both dehumidification and cooling occur simultaneously. However, the air likely follows a path that is more similar to the one shown in Figure 9-12. Air that is flowing close to the finned or tube surfaces experiences different conditions than the air that is flowing towards the center of the channel and some air can bypass the coil altogether. Consequently, condensation can occur at the finned surface even though the mean air temperature is somewhat greater than the dew point temperature.

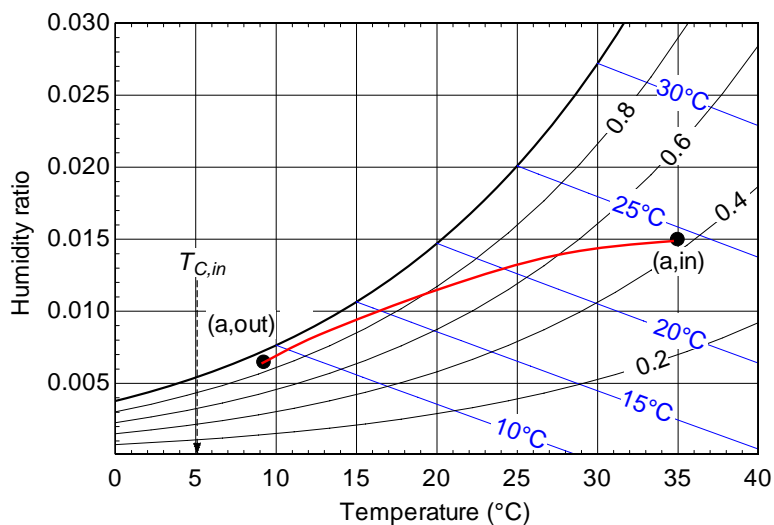


Figure 9-12: Actual process followed by the air passing through a cooling coil.

This observation has led to the semi-empirical, enthalpy-based effectiveness model of a cooling coil. The entire coil is assumed to be "wet". Therefore, the capacitance of the air-side is computed using the saturation specific heat capacity ($c_{a,sat}''$) and the energy transfer due to the combined heat and mass transfer is characterized using the wet coil conductance (UA_{wet}). The effectiveness of the coil (ε) is computed using the appropriate ε - NTU solution for the heat exchanger configuration. The effectiveness is defined based on the enthalpy and the humidity ratio of the moist air stream, rather than its temperature:

$$\varepsilon = \frac{i_{a,in}'' - i_{a,out}''}{i_{a,in}'' - i_{a,out,min}''} \quad (9-164)$$

$$\varepsilon = \frac{\omega_{a,in} - \omega_{a,out}}{\omega_{a,in} - \omega_{a,out,min}} \quad (9-165)$$

where the quantities $i_{a,in}''$ and $\omega_{a,in}$ are the enthalpy and humidity ratio, respectively, of the inlet air and the quantities $i_{a,out,min}''$ and $\omega_{a,out,min}$ are the minimum possible values of the enthalpy and humidity ratio of the air, respectively. The minimum possible values of enthalpy and humidity ratio are consistent with the air leaving saturated and at the inlet temperature of the cold fluid, as shown in Figure 9-13.

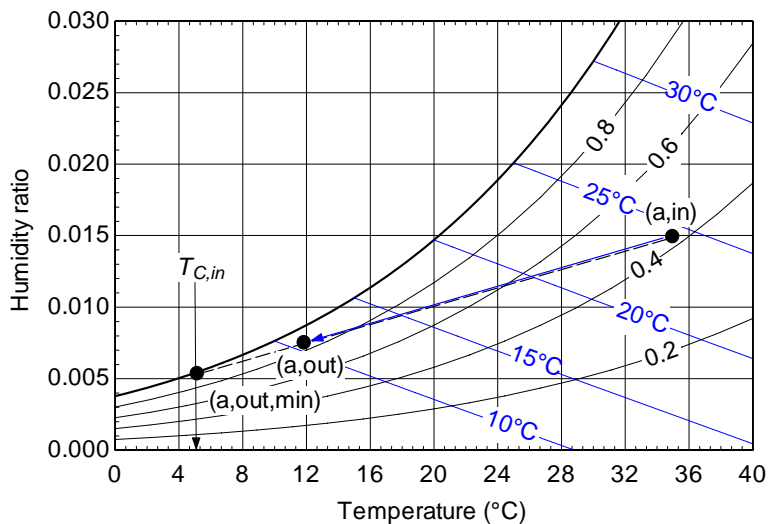


Figure 9-13: Enthalpy-based effectiveness model of a cooling coil.

The exit enthalpy and relative humidity of the air leaving the cooling coil can be computed using Eqs. (9-164) and (9-165). An energy balance on the air provides the heat transfer rate and a mass balance on the water vapor provides the rate of condensation.

EXAMPLE 9.6-2: Cooling Coil (revisited)

- a.) Use the enthalpy-based effectiveness technique to compute the performance of the cooling coil that was investigated in EXAMPLE 9.6-1.

The EES code required to investigate the cooling coil using the enthalpy-based effectiveness technique is appended to the EES code developed in EXAMPLE 9.6-1.

The saturation specific heat capacity of the air is computed according to:

$$c''_{a,sat,ebe} = \frac{[i''_a(T_{a,in}, p, RH = 1) - i''_a(T_{a,out,ebe}, p, RH = 1)]}{(T_{a,in} - T_{a,out,ebe})}$$

where $T_{a,out,ebe}$ is the air outlet temperature predicted using the enthalpy-based effectiveness method. As in EXAMPLE 9.6-1, it is necessary to assume a value for $T_{a,out,ebe}$ in order to compute $c''_{a,sat}$; this assumed value will be commented out in order to complete the solution:

"enthalpy-based effectiveness approach"

```
T_a_out_ebe=T_w_in           "initial guess for T_a_out_ebe"
c``_a_sat_ebe=(enthalpy(AirH2O,T=T_a_in,R=1,P=P_a)-&
  enthalpy(AirH2O,T=T_a_out_ebe,R=1,P=P_a))/(T_a_in-T_a_out_ebe)
  "saturation specific heat capacity if entire coil is wet"
```

The capacitance rate of the air is:

$$\dot{C}_{a,ebe} = c''_{a,sat} \dot{m}_a$$

The minimum capacitance rate ($\dot{C}_{min,ebe}$) is determined using the Min function.

```
C_dot_a_ebe=c``_a_sat_ebe*m_dot_a           "capacitance rate of the air"
C_dot_min_ebe=Min(C_dot_a_ebe,C_dot_w)     "minimum capacitance rate"
```

The enthalpy-based effectiveness based effectiveness technique assumes that the total coil is wet. Therefore, the thermal resistance of the coil is:

$$R_{ebe} = R_{in} + R_{f,in} + R_{cond} + \frac{c''_a}{\bar{h}_a c''_{a,sat,ebe} A_{tot}}$$

and the conductance of the wet coil is:

$$UA_{ebe} = \frac{1}{R_{ebe}}$$

```
R_ebe=R_in+R_f_in+R_cond+c``_a/(A_tot*h_bar_a*c``_a_sat_ebe)
  "total resistance of coil if it is completely wet"
UA_ebe=1/R_ebe           "total conductance of coil if it is completely wet"
```

The number of transfer units is:

$$NTU_{ebe} = \frac{UA_{ebe}}{\dot{C}_{min,ebe}}$$

and the effectiveness of the coil (\mathcal{E}_{ebe}) is computed using the ε - NTU solution for a cross-flow heat exchanger, accessed using the HX function in EES:

```
NTU_ebe=UA_ebe/C_dot_min_ebe "number of transfer units if all coil is wet"
eff_ebe=HX('crossflow_one_unmixed', NTU_ebe, C_dot_a_ebe, C_dot_w, 'epsilon') "effectiveness"
```

The enthalpy of the inlet air ($i''_{a,in}$) and the minimum possible enthalpy and relative humidity of the outlet air ($i''_{a,out,min}$ and $\omega_{out,min}$) are computed using EES' psychrometric functions. The effectiveness is used to predict the actual outlet air enthalpy ($i''_{a,out,ebe}$) and relative humidity ($\omega_{a,out,ebe}$) according to Eqs. (9-164) and (9-165):

$$\mathcal{E}_{ebe} = \frac{i''_{a,in} - i''_{a,out,ebe}}{i''_{a,in} - i''_{a,out,min}}$$

$$\mathcal{E}_{ebe} = \frac{\omega_{a,in} - \omega_{a,out,ebe}}{\omega_{a,in} - \omega_{a,out,min}}$$

```
i``_a_in=enthalpy(AirH2O,T=T_a_in,R=RH_a_in,P=P_a) "inlet enthalpy"
i``_a_out_min=enthalpy(AirH2O,T=T_w_in,R=1,P=P_a) "minimum possible air exit enthalpy"
omega_a_out_min=HumRat(AirH2O,T=T_w_in,R=1,P=p_a) "minimum possible air exit humidity ratio"
eff_ebe=(i``_a_in-i``_a_out_ebe)/(i``_a_in-i``_a_out_min) "enthalpy-based effectiveness"
eff_ebe=(omega_a_in-omega_a_out_ebe)/(omega_a_in-omega_a_out_min) "humidity ratio of exit air"
```

The outlet temperature of the air ($T_{a,out,ebe}$) can be predicted using the enthalpy and humidity ratio with EES' psychrometric function. The guess values are updated and the initial, assumed value for $T_{a,out,ebe}$ is commented out:

```
{T_a_out_ebe=T_w_in} "initial guess for T_a_out_ebe"
T_a_out_ebe=temperature(AirH2O,h=i``_a_out_ebe,w=omega_a_out_ebe,P=p_a)
"outlet temperature of air"
T_a_out_ebe_C=converttemp(K,C,T_a_out_ebe) "in C"
```

which leads to $T_{a,out,ebe} = 292.3 \text{ K}$ (19.18°C).

The rate of heat transfer in the cooling coil predicted using the enthalpy-based effectiveness approach is:

$$\dot{q}_{ebe} = \dot{m}_a (i''_{a,in} - i''_{a,out,ebe}) \quad (9-166)$$

and the rate of condensation is:

$$\dot{m}_{cond,ebe} = \dot{m}_a (\omega_{a,in} - \omega_{a,out,ebe}) \quad (9-167)$$

`q_dot_ebe=m_dot_a*(i``_a_in-i``_a_out_ebe)`

"heat transfer rate using enthalpy-based effectiveness method"

`m_dot_cond_ebe=m_dot_a*(omega_a_in-omega_a_out_ebe)`

"mass flow rate of condensate using enthalpy-based effectiveness method"

`m_dot_cond_ebe_kgph=m_dot_cond_ebe*convert(kg/s,kg/hr)` "in kg/hr"

which leads to $\dot{q}_{ebe} = 1312 \text{ W}$ and $\dot{m}_{cond,ebe} = 0.0002914 \text{ kg/s}$ (1.049 kg/hr). These values are within 13% and 7%, respectively, of the solution obtained using the dry coil/wet coil technique.

Figure 1 illustrates the predicted rate of heat transfer using the dry coil/wet coil and enthalpy-based effectiveness approach as a function of the inlet air relative humidity for various values of the inlet air temperature. The two approaches yield consistent answers over a wide range of temperature and relative humidity. At very low relative humidity, the dry coil/wet coil technique predicts that the entire coil surface is dry (i.e., $F_{dc} > 1$) and therefore the EES code from EXAMPLE 9.6-1 should be altered in order to provide a meaningful solution.

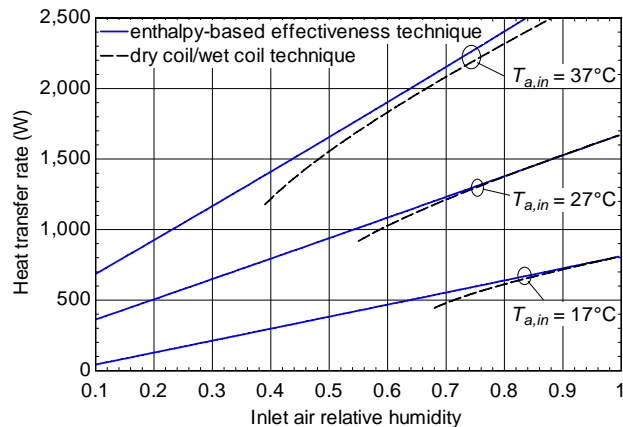


Figure 1: Rate of heat transfer predicted by the enthalpy-based effectiveness and dry coil/wet coil techniques as a function of the inlet air relative humidity for several values of the inlet air temperature.

References

- ASHRAE, Technical Committee TC1.1, Psychrometrics: Theory and Practice, Principle Investigators Olivieri, J. and Singh, T., American Society of Heating Refrigeration and Air-Conditioning Engineers, Inc, Atlanta, GA, (1996).
- Bolz, R.E. and G.L. Tuve, *Handbook of Tables for Applied Engineering Science*, 2nd edition, CRC Press, (1976).
- Braun, J.E., S.A. Klein, and J.W. Mitchell, "Effectiveness Models for Cooling Towers and Cooling Coils," *ASHRAE Transactions*, Vol. 95, No. 2, pp. 164-174 (1989).

- Lide, D.R. and H.V. Kehiaian, *CRC Handbook of Thermophysical and Thermochemical Data*, CRC Press, U.S., 1994 ISBN 0-8493-0197-1, (1999).
- Lienhard, J.H. IV and J.H. Lienhard V, *A Heat Transfer Textbook*, 3rd Edition, Phlogiston Press, Cambridge, MA (2005).
- McCabe, W.L. and J.C. Smith, *Unit Operations of Chemical Engineering*, 2nd edition, McGraw-Hill, New York, (1967)
- Perry, R.H. and D.W. Green, *Perry's Chemical Engineers' Handbook*, 7th Edition, McGraw-Hill, New York, (1997)
- Poling, B.E., J.M. Prausnitz, and J. O'Connell, *The Properties of Gases and Liquids*, 5th Edition, McGraw-Hill, New York, (2000), ISBN 0070116822 / 9780070116825.