

CAMBRIDGE

Higher

# MATHEMATICS

GCSE for AQA

Student Book Answers

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# Answers

The answers to past paper questions are the work of the publisher and authors and have been neither seen or verified by AQA.

## 1 Working with integers

### BEFORE YOU START ...

- 1 a  $>$       b  $>$       c  $=$       d  $<$
- 2 a Mistake is just working left to right not doing operations in order. Correct answer is 23.
- b Mistake is just working left to right not doing operations in order. Correct answer is 31.
- c Order of operations is correct, but calculation is wrong. Correct answer is 132.
- 3 a C                      b B                      c B

### LAUNCHPAD

- 1 a 1433      b 1117      c 855      d 18
- 2 a C      b D      c D      d B      e D
- 3  $\frac{128}{4} = 32$  cm
- 4 208
- 5 1350

### EXERCISE 1A

- 1 a i £19.50      ii £1.30
- b 5 packets. We need to know this so we can work out the total cost.
- c Multiplication or repeated addition, because there are 5 packs which each cost £3.90.
- d Divide the cost of one packet by 3 as there are 3 pens in each pack.
- e £1.50 would be a reasonable estimate for the price of one pen, but it is not the correct answer because  $3 \times £1.50 = £4.50$  not £3.90. So £1.50 is too expensive.
- 2 £36.50
- 3 64
- 4 a 4320      b 42 minutes
- 5 56539
- 6 a 222 km      b 6094 km
- 7 4803
- 8  $5678 + 67 = 5745$

### EXERCISE 1B

- 1 a -2      b 2      c 6      d 9      e 29
- 2 a 15      b 11      c 7      d 4      e -16
- 3 a -3      b 25      c 9      d -126      e 0
- 4 a -8      b 8      c -32      d 128      e 0.25
- 5 a -8 and 1 or -6 and 3      b -3, 1 and 3
- c -3 and 1      d -6 and 1
- 6 -26
- 7 This is an investigative task.

The only numbers between 5 and 150 that cannot be made by combining multiples of 5 and 7 are 6, 8, 9, 11, 13, 16, 18 and 23.

### WORK IT OUT 1.2

- 1 Option A is correct. The mistake in option B is doing the addition first.
- 2 Option B is correct. The mistake in option A is not doing the calculation in brackets first.
- 3 Option B is correct. The mistake in option A is calculating  $20 + 1$  not  $20 \times 1$ .
- 4 Option B is correct. The mistake in option A is doing the subtraction before the division.
- 5 Option B is correct. The mistake in option A is not calculating  $18 - 4$  in the numerator.
- 6 Option A is correct. The mistake in option B is doing the addition before the division.

### EXERCISE 1C

- 1 a, c and d are correct.
- b 608      e 368      f 10
- 2 a  $13 - 18 \div 9 = 11$
- b  $8 \div (16 - 14) - 3 = 1$  or  $8 \div (16 - 14) - 1 = 3$
- c  $(4 + 5) - (12 - 9) = 6$  or  
 $(4 + 5) - (6 - 9) = 12$  or  
 $(4 + 9) - (12 - 5) = 6$  or  
 $(4 + 9) - (6 - 5) = 12$  or  
 $(5 + 4) - (12 - 9) = 6$  or  
 $(5 + 4) - (6 - 9) = 12$  or  
 $(5 + 9) - (12 - 4) = 6$  or  
 $(5 + 9) - (6 - 4) = 12$  or  
 $(9 + 4) - (12 - 5) = 6$  or  
 $(9 + 4) - (6 - 5) = 12$  or  
 $(9 + 5) - (12 - 4) = 6$  or  
 $(9 + 5) - (6 - 4) = 12$
- 3 a  $3 \times (4 + 6)$       b  $(25 - 15) \times 9$       c  $(40 - 10) \times 3$
- d  $(14 - 9) \times 2$       e  $(12 + 3) \div 5$       f  $(19 - 9) \times 15$
- g  $(10 + 10) \div (6 - 2)$       h  $(3 + 8) \times (15 - 9)$       i  $(9 - 4) \times (7 + 2)$
- j  $(10 - 4) \times 5$       k  $6 \div (3 + 3) \times 5$
- l no brackets needed      m  $(1 + 4) \times 20 \div 5$
- n  $(8 + 5 - 3) \times 2$       o  $36 \div (3 \times 3 - 3)$       p  $3 \times (4 - 2) \div 6$
- q no brackets needed
- r no brackets needed
- 4 a  $12 \div (28 - 24)$       b  $88 - 10 \times 8$       c  $40 \div 5 \div (7 - 5)$
- d  $9 + 15 \div (3 + 2)$
- 5 a 0.5      b 2      c 0.183      d 0.5
- e 0.333      f 1      g 2      h 0.667
- 6 This is an investigation.  
 Students should find their own methods and explain their thinking.

The formula  $n \frac{a+b}{2}$  will work for any sequence (whether positive or negative), where  $n$  is the number of terms,  $a$  is the lowest number and  $b$  is the highest number in the set because  $\frac{a+b}{2}$  is the mean value of the set.

There are many other methods of finding the answer though, and students could research these online if they are interested.

### EXERCISE 1D

- 1 a 312      b 102      c 400      d -5  
 e -145      f 216 000
- 2 All calculations are correct.
- 3 a  $h = 12$  cm    b  $h = 8$  cm,  $b = 16$  cm
- 4 a -83  
 b Students' own answers.  
 c Students' own reasoning; should suggest that addition and multiplication (of whole numbers) produces higher values.

### CHAPTER REVIEW

- 1 a option B    b option D
- 2 Students' own answers.
- 3 Students' own answers. Some possible solutions are:  
 $-19 + 2$ ;  $-7 + -10$ ;  $-34 + 17$ ;  $51 \div -3$ ;  $17 \times -1$ , and so on.
- 4 4032
- 5 256
- 6

			Sum of rows	
	4	12	10	26
	15	5	6	26
Product of columns	60	60	60	

- 7 -20 and 5
- 8 -4 and 5
- 9 £568
- 10 a 18°C      b 34°C

## 2 Collecting, interpreting and representing data

### BEFORE YOU START ...

- 1 Where  $h$  is height, every 5cm from  $1.35 < h \leq 1.40$ ,  $1.40 < h \leq 1.45$  etc, or every 10 cm  $1.30 < h \leq 1.40$  etc
- 2 a 25      b 0, 25, 50, 75, 100
- 3 a From left to right:  $120^\circ$ ,  $30^\circ$ ,  $17^\circ$   
 b Measure student's drawing

### LAUNCHPAD

- 1 Choose a representative sample, using a random selection method to obtain a mix of male and female students from different years, and small enough so that you have time to ask everyone in your sample. Decide whether you will let the students name their favourite song, or whether you will make them choose from a list of types of music, or both. Survey your sample.
- 2 a Croatia, Greece  
 b Croatia & Greece; Russia & Portugal
- 3 a 31 to 40    b 18 or 19 (depends on accuracy of measurement of angle for sector)
- 4 a The columns are not the same width because the data was surveyed in age categories (classes) that do not each span the same number of years, so the class intervals in the data are not all the same.  
 b  $11 \leq \text{age} < 16$ , i.e. 5 years.  
 c 15 students

### WORK IT OUT 2.1

- a Option D
- b Option A will exclude men, and exclude women who have no children.  
 Option B will, perhaps, tend to over-represent shoppers who are buying something on their way to work.  
 Option C will be restricted to shoppers who have chosen one kind of shop (it will be biased towards book buyers).

### EXERCISE 2A

- 1 D and E are likely to give random results.  
 D - a person's name does not determine any other characteristic of that person  
 E - the chance of picking a given name is the same as picking any other name
- A, B and C are likely to give non-random results.  
 A - the chosen street might be (for example) very wealthy.  
 B - excludes people who work or are out during the day.  
 C - young people are more likely to wear trainers.
- 2 a Any sensible suggestion where parents may be found e.g. soft play area; a park; nursery; baby food section of a supermarket.  
 b Approximately 700.
- 3 a  $\frac{15}{28}$  would mean about 160 machines.  
 b No, because not all the members are likely to be there at once.  
 c Sample the members present at different times of the week.
- 4 a 1.722 million    b 574 000    c 2.296 million

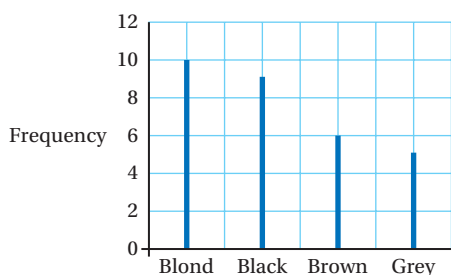
### WORK IT OUT 2.2

Option C is best: Option A wrongly uses a line graph to display discrete data and has no scales; Option B does not show the frequencies, but has one bar per test, and has no scales either.

### EXERCISE 2B

1

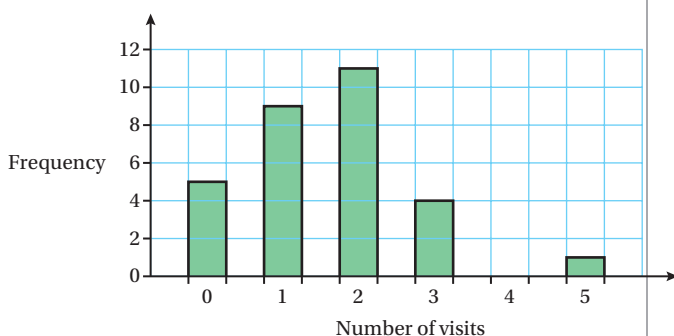
Hair colour



2 a

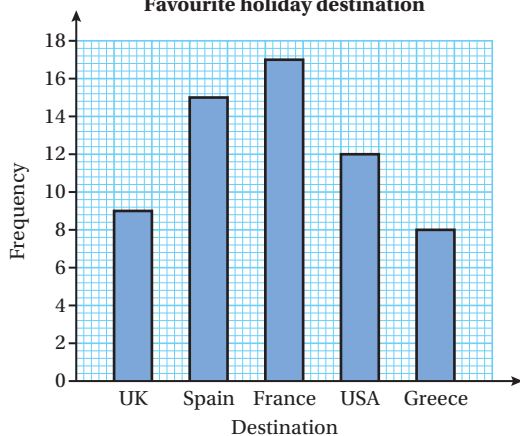
Visits	Frequency
0	5
1	9
2	11
3	4
4	0
5	1

b



3 a

Favourite holiday destination



- b USA has a frequency of 12.  
 25% of 12 is 3.  
 $12 + 25\% = 12 + 3 = 15$ .  
 So Spain is chosen by 25% more people.

4 a C

- b 17%  
 c 7  
 d 4-3

- 5 a April      b 110 mm      c February  
 d Approx 220 mm      e Wetter: 2012 saw over 775 mm of rain
- 6 a Jenny could have spent a few minutes observing the most popular snacks before she began her survey. Then she could have written down a list of those snack names and used tally counting to build up a frequency table based on the list. This would save her from repeatedly writing the same snack names.

b

Snack bought	Frequency
Chocobar	10
Juicebar	7
Cheese puffs	9
NRG drink	10
Gum	6
Apple	3
Crisps	10
Fruit chews	8

c

Snack bought	Frequency
Chocobar	10
Juicebar	7
<b>Total bars:</b>	<b>17</b>
Cheese puffs	9
Crisps	10
<b>Total savoury snacks:</b>	<b>19</b>
NRG drink	10
Gum	6
Apple	3
Fruit chews	8
<b>Total other:</b>	<b>27</b>

### EXERCISE 2C

- 1 a Average time spent watching television; average time spent doing homework  
 b As students move into higher grades they watch less TV.  
 c As TV-watching time decreases, the amount of time spent on homework increases.  
 d i 90 minutes  
    ii 45 minutes
- 2 a True10      b Datalink and G-Commerce  
 c True10      d G-Commerce  
 e G-Commerce  
 f Speedlink, because the costs are greater than the income  
 g £35 000
- 3 a No you cannot, as the values shown are percentages of a company's total sales, and not the actual value of the sales.  
 b C      c C  
 d C      e A third  
 f Just over a quarter of its sales are direct from the shop; nearly 40% are by internet sales; the remaining third are fairly evenly divided between catalogue mail order and through an agent, with the latter accounting for slightly more.

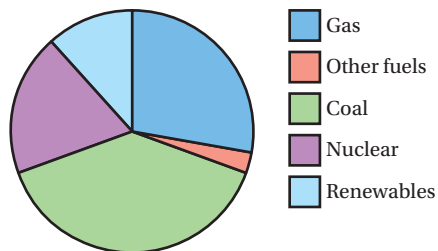
- 4 a Wilhelm      b Susan      c Trevor  
 d Their score on the actual exam was worse than on their mock.

**WORK IT OUT 2.3**

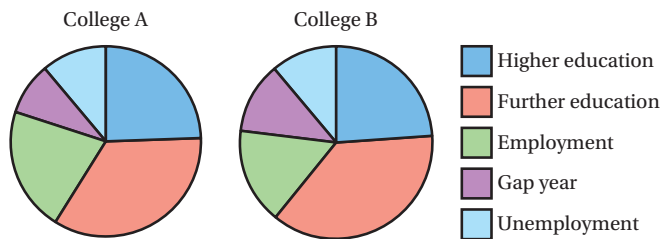
Option B shows the information best. Option A does not show the frequency of each lateness, but the actual minutes. Option C's key has no units and the "Up to" wording is misleading.

**EXERCISE 2D**

1 Electricity generation



- 2 a Any two proportional responses, such as Ireland has a smaller proportion of under 15s than Greece  
 b It isn't possible to say, as we don't know the relative populations  
 c We can't say whether it is true or false - we don't know the numbers, only the proportions
- 3 a Any two of: Proportion of other vehicles has decreased. Proportion of light goods vehicles has increased. Proportion of motorbikes has increased. Proportion of heavy good vehicles has slightly decreased. Proportion of buses and coaches has slightly decreased.  
 b 3.55 million (answers within 0.03 million are acceptable)  
 c About 20%      d Option B
- 4 a 20      b TV      c 30°; 6 students
- 5 College B has a larger proportion of students who go on to further education. A larger proportion of College A students go on to employment. Similar proportions go on to higher education. College B sees more students going on a gap year. (Accept any tenable argument that accurately cites these observations, as there is no clear winner.)

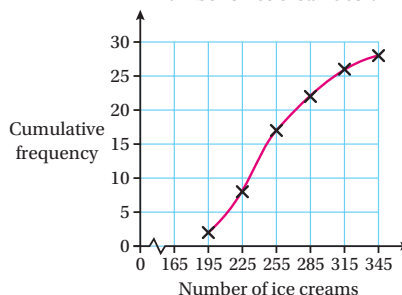


**EXERCISE 2E**

1

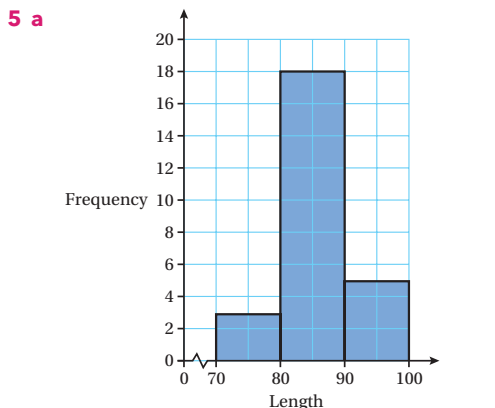
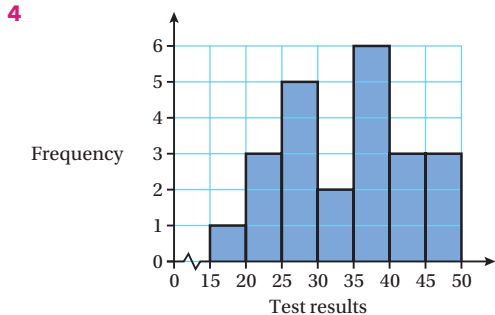
Class interval	Cumulative Frequency
$165 \leq \text{ice creams} < 195$	2
$195 \leq \text{ice creams} < 225$	8
$225 \leq \text{ice creams} < 255$	17
$255 \leq \text{ice creams} < 285$	22
$285 \leq \text{ice creams} < 315$	26
$315 \leq \text{ice creams} < 345$	28

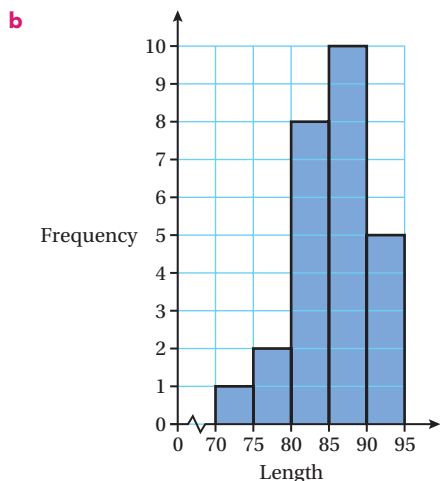
Cumulative frequency curve showing number of ice creams sold



- a 10      b 3
- 2 Cumulative frequency curve showing maximum daily temperature
- 
- | Temperature in °C | Cumulative frequency |
|-------------------|----------------------|
| 13                | 0                    |
| 15                | 2                    |
| 17                | 6                    |
| 19                | 16                   |
| 21                | 20                   |
| 23                | 22                   |
| 25                | 25                   |
| 27                | 25                   |
- a 19      b 4      c 2

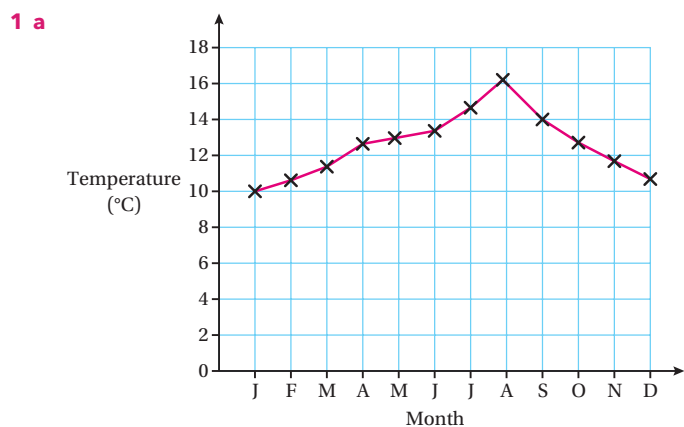
- 3 Cumulative frequency curve showing Time (minutes)
- 
- | Time (minutes) | Cumulative frequency |
|----------------|----------------------|
| 16             | 0                    |
| 18             | 5                    |
| 20             | 15                   |
| 22             | 32                   |
| 24             | 48                   |
| 26             | 60                   |
| 28             | 60                   |
- a 12      b 9      c 6



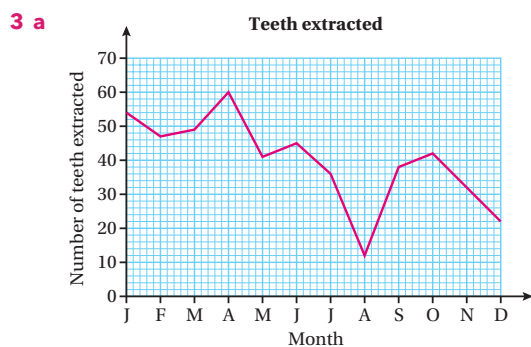
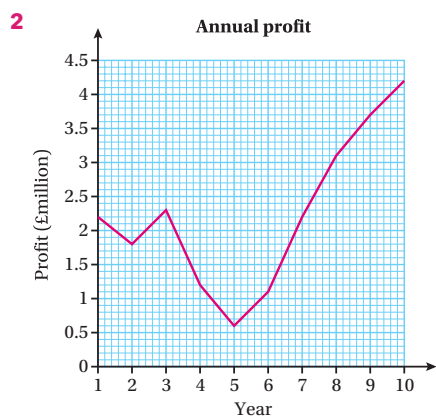


- 6 a** 180      **b** 120      **c**  $16\frac{2}{3}\%$       **d**  $61\frac{1}{9}\%$

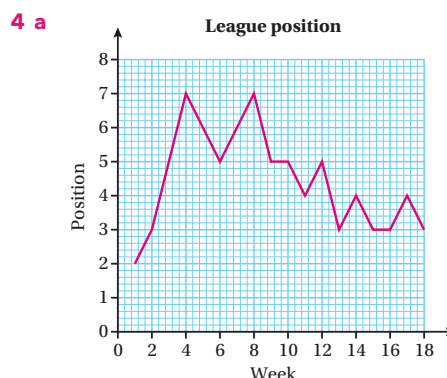
**EXERCISE 2F**



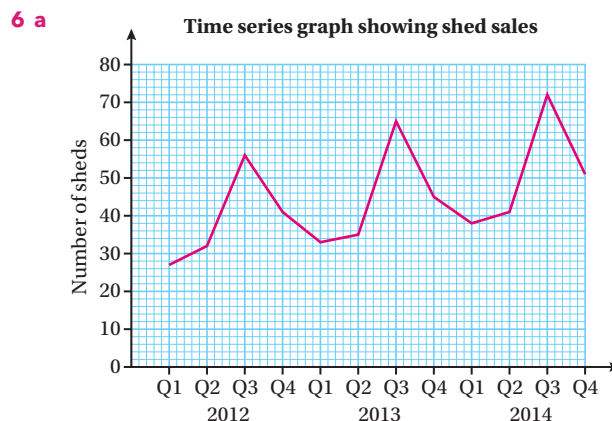
- b** The temperature is mild all year round, rising to the low twenties in summer (June to September) and never falling below an average of  $15^\circ$  in winter.



- b** The number of teeth extracted peaked in April then fell off steadily to a low in August, recovering sharply in the autumn but falling again towards the end of the year.
- c** In August many people are away on holiday and so not able to be at the dentist, or unwilling to have a tooth extracted just before they go away.



- b** The team fell from the number 2 slot at the start of the year to a low of number 7, slowly improving again as the season went on but never quite regaining their former best position.
- 5 a** The third quarter.
- b** The fourth quarter, generally.
- c** In Quarter 1 they were £64 000, Quarter 5 £77 000 and in Quarter 9 £79 000. Given that this is the same quarter in each of three years, it is fair to say that sales are improving.



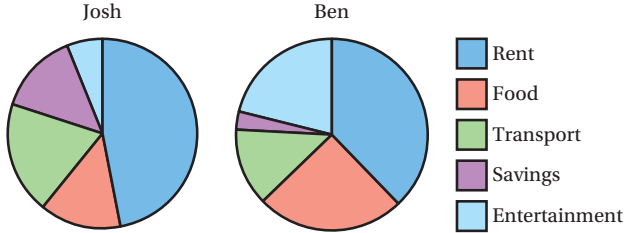
- b** Sales have steadily increased year-on-year.
- c** Yes, shed sales are seasonal: they peak notably in the third quarter and are at their lowest in the first.
- 7 a** The number of light vans has increased steadily till 2007, since when it has remained fairly constant.
- b** The number of vehicles has remained largely the same
- c** The number of heavy goods vehicles might have decreased because they are now larger, and each one carries more goods, or the freight is being carried by the smaller light vans. (Accept any feasible answer.) No, this question cannot be answered by using the graph only.
- 8 a** In June and in August
- b** There might have been some rainfall which caused the level to rise.
- c** Between February and March
- d** 35 mm
- e** 10 mm

**CHAPTER REVIEW**

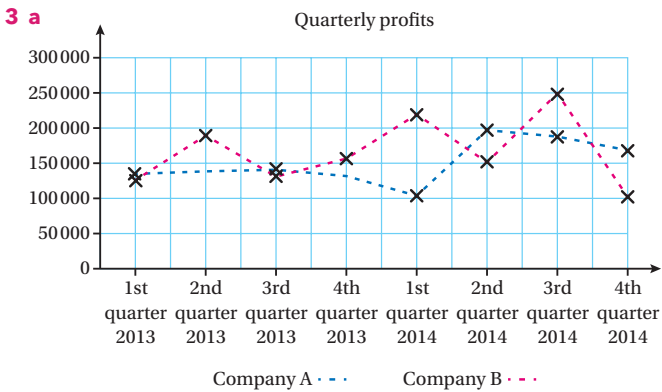
**1 a** Taking the students whose names begin with a certain letter, choosing students whose birthday is in a particular month, or any other suitable method.

**b** 204

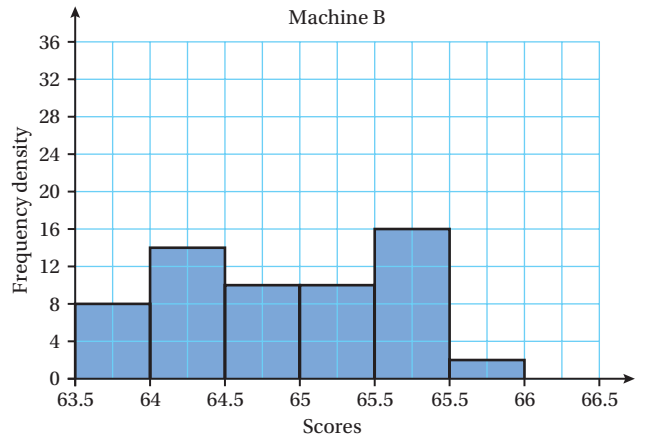
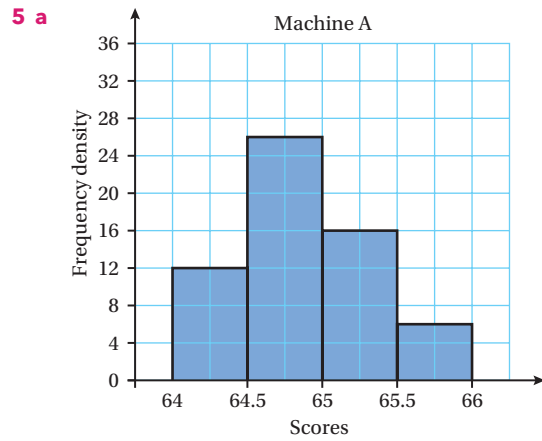
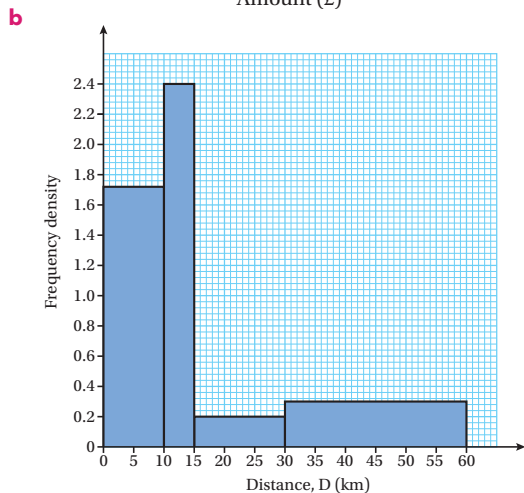
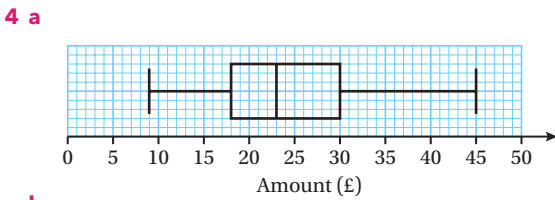
**2 a** Pie charts are best for comparison.



**b** Josh spends a larger proportion of his money on rent.  
 Ben spends a larger proportion of his money on food.  
 Josh saves a greater proportion of his money.  
 Ben spends a greater proportion of his money on entertainment.  
 Josh spends a greater proportion of his money on transport.



**b** Generally Company B makes higher profits and is therefore more successful.  
**c** The drop in profits for company B between the 3rd quarter and 4th quarter of 2014 is the biggest change.  
**d** Option B



**b** Machine A has a smaller range of values. Machine B makes more underweight packets.

**3 Analysing data**

**BEFORE YOU START ...**

- 1 a** 3.3, 3, 2, 6      **b** 35, 35, no mode, 50
- 2**  $A(-1, \frac{1}{2})$ ,  $B(1, -\frac{1}{2})$ ,  $C(2, -1)$
- 3 a**  $-\frac{1}{2}$       **b**  $y = -\frac{1}{2}x$

**LAUNCHPAD**

- 1 a** 0-5 cups   **b** 6 - 10   **c** 10.9 (≈11)   **d** 21
- 2 a** Between March and June the mean mark was unchanged, but the performance of the middle half of all students worsened; however, the lowest and the highest marks both improved.
- b** Not overall, no.
- 3** Scale is misleading as it doesn't begin at zero
- 4 a** A negative correlation
- b** Increasing smoking decreases your life expectancy



### EXERCISE 3A

1 option C

2 a

Days absent ( $d$ )	Frequency	Midpoint	Midpoint $\times$ frequency
$0 \leq d < 5$	15	2.5	37.5
$5 \leq d < 10$	23	7.5	172.5
$10 \leq d < 15$	19	12.5	237.5
$15 \leq d < 20$	12	17.5	210
$20 \leq d < 25$	6	22.5	135
Total	75		792.5

b  $5 \leq d < 10$  c mean: 10.57, median falls in class:  $5 \leq d < 10$ , range: 25

3 a

$0 \leq s < 20$	8
$20 \leq s < 40$	12
$40 \leq s < 60$	9
$60 \leq s < 80$	7
$80 \leq s < 100$	6

b mean: 45.7, median falls in class:  $40 \leq s < 60$ , range: 100

c No, as we have the data and can calculate it

d  $20 \leq s < 40$

4 a

$1.45 \leq h < 1.50$	1
$1.50 \leq h < 1.55$	3
$1.55 \leq h < 1.60$	11
$1.60 \leq h < 1.65$	4
$1.65 \leq h < 1.70$	13
$1.70 \leq h < 1.75$	2
$1.75 \leq h < 1.80$	6
$1.80 \leq h < 1.85$	6
$1.85 \leq h < 1.90$	3

Mean = 1.68, median falls in class:  $1.65 \leq h < 1.70$

b  $1.65 \leq h < 1.70$

c Mean is 1.67, median falls in class  $1.60 \leq h < 1.70$ . The new group intervals lower the estimate of the mean, and the new class holding the median value starts at a lower bottom limit than the previous one.

5 a Other groupings are possible but, starting at 2h 30 and using half hour intervals gives six classes.

Time interval	f
$2\text{h } 30 \leq t < 3\text{h } 00$	4
$3\text{h } 00 \leq t < 3\text{h } 30$	5
$3\text{h } 30 \leq t < 4\text{h } 00$	6
$4\text{h } 00 \leq t < 4\text{h } 30$	8
$4\text{h } 30 \leq t < 5\text{h } 00$	4
$5\text{h } 00 \leq t < 5\text{h } 30$	3

b Modal class =  $4\text{h } 00 \leq t < 4\text{h } 30$

c Mean = 3h 57, median falls in class  $4:00 \leq t < 4:30$ , range = 3 hours

6 a 310.29 kg      b  $300 \leq m < 350$       c  $350 \leq m < 400$

### EXERCISE 3B

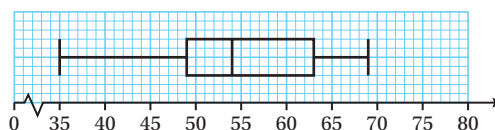
1 median = 45 years; IQR = 20 years

2 median = £2250; IQR = approx. £1000

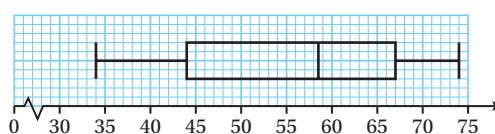
3 median = 93 minutes; IQR = 9 minutes

### EXERCISE 3C

1 a A: Mean 54, Median 54, Mode 35, Range 34.



b B: Mean 55.9, median 58.5, no mode, range 40. The results for test B are slightly higher on average, with a greater spread of results than in A. A's mode indicates a cluster of lower-than-average results.



2 a Ahmed: mean 36.7, median 39.5, mode 27, range 48.

Bill: mean 26.3, median 28, mode 44, range 46.

b Ahmed has had a better season as his mean and median are higher, as is the total number of runs scored

3 Bus 127: mean 19.9, median 20, mode 21, range 6. Bus 362: mean 19.4, median 19, mode 15, range 11. Bus 362 is better on most measures, but it has the greatest range so is potentially less reliable. It does not matter that there is less data as all of these measures enable comparison of two samples even of differing sizes.

4 Means: machine A = 321.2, machine B 328.9. Both have same range (250), median group ( $300 \leq b < 350$ ) and modal group ( $350 \leq b < 400$ ). Machine B is the sensible choice.

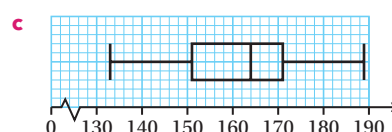
5 a The median would be better than the mean or the mode, as it takes into account all values but reduces distortions caused by extreme values, particularly the high ones.

b Shoe size in my class. (Accept answers in which the data is unlikely to be skewed by a few extremely small or large values; salaries in a firm would not be acceptable, for example.)

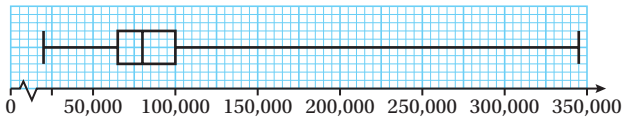
c The most popular make of car sold in the UK last year. The number of bedrooms per house in a housing estate. (Accept examples of discrete data in which (a) non-numeric data necessitates use of the mode; or (b) the most common category is what is of concern, and one is likely to dominate.)

6 a  $Q_1 = 151.5$ , median = 164,  $Q_3 = 171.5$

b 20



- 7 a  $Q_1 = 67\ 000$ , median =  $81\ 500$ ,  $Q_3 = 100\ 000$   
 b 33 000  
 c



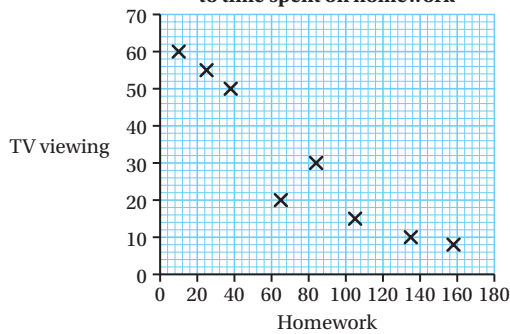
- 8 a 30 (accept close to the number as it's by measurement)  
 b 28 (accept close to the number as it's by measurement)  
 c Team B  
 d Team B  
 e Team A  
 f Team A's mean is higher and their IQR is overall higher.

**EXERCISE 3D**

- 1 There are no values on the vertical axis, and even if there were there is no definition of how the crime rate is measured and the axis should (ideally) be labelled to indicate what the numbers mean.  
 2 The horizontal scale is not consistent. This is probably to make growth from 2000 onwards look better than it is.  
 3 The width of the bars change, exaggerating the later sales  
 4 a Vertical scale does not start at zero  
 b To give an exaggerated impression of the changes  
 5 a They are both the same  
 b The 3D effect makes the scale difficult to read  
 6 Students own answers, but a bar chart or a time-series graph would be best.

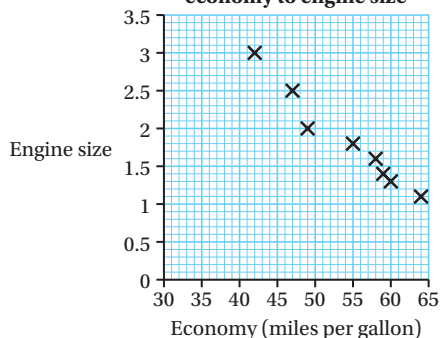
**EXERCISE 3E**

1 Scattergraph comparing time spent viewing TV to time spent on homework

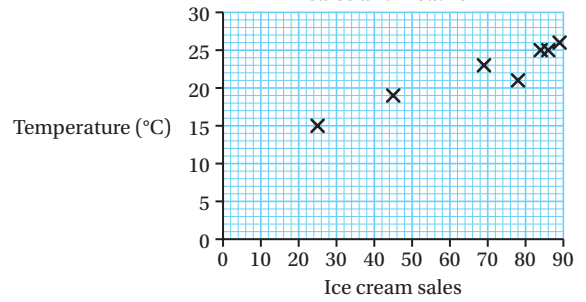


Graph shows a strong negative correlation

2 Scattergraph comparing fuel economy to engine size

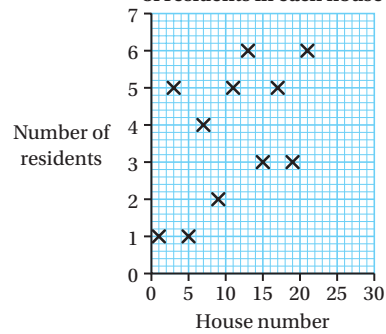


3 a Scattergraph comparing ice cream sales and weather



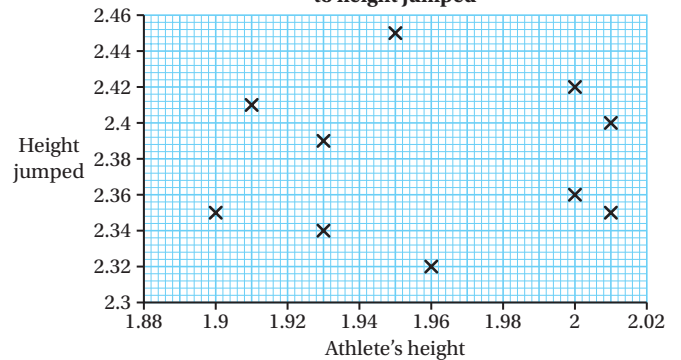
- b Sales might also be affected by location, day of the week or time of day

4 a Scattergraph showing number of residents in each house



- b No correlation

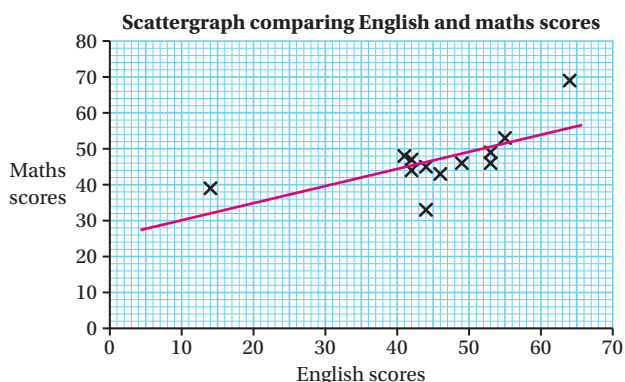
5 a Scattergraph comparing athletes' heights to height jumped



- b No correlation  
 c D

### EXERCISE 3F

- 1 B  
 2 a 48.7    b 49    c 47    d 46    e 46.1  
 3 a, b



- c The point (14, 39)  
 4 a 112.9 minutes  
 b Yes. The time of 156 minutes is an outlier (maybe due to an accident) so the company has calculated the mean from the remaining 19 data to give 110.63 minutes.

### CHAPTER REVIEW

First lake	
Mean	20.2
Median	21
Mode	no mode
Range	40
$Q_1$	11
$Q_3$	28
IQR	17
Second lake	
Mean	16
Median	15.5
Mode	10
Range	15
$Q_1$	11
$Q_3$	19
IQR	8

The first lake is better for experts as the wind speed is more often higher, and if the one day of zero wind is discounted all measures of central tendency are close to preferred speed.

The second lake is better for beginners with a slower wind speed and less variability of wind speed.

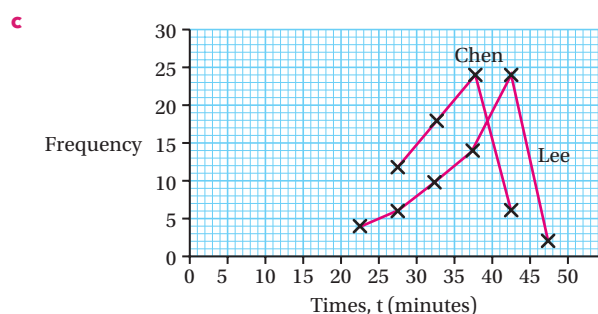
- b C  
 2 a This and every box plot shows the lowest and highest values, the range, the first and third quartiles, the interquartile range, the median.

- b It shows the overall spread of heights in a diagram form which makes comparison with other groups easier.

Time	Frequency	Midpoint	Midpoint $\times$ frequency
$25 \leq t < 30$	12	27.5	330
$30 \leq t < 35$	18	32.5	585
$35 \leq t < 40$	24	37.5	900
$40 \leq t < 45$	6	42.5	255
Total	60		2070

Estimate of Chen's mean journey time is  $2070 \div 60 = 34.5$

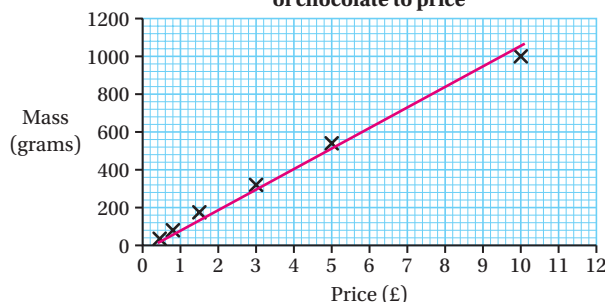
- b It is an estimate because the calculation uses the midpoint of each interval, not individual observed times.



- d Lee's journey generally takes longer than Chen's, and Lee has had the longest individual journey. Lee has a greater range of journey times - in fact, Lee has on a few occasions got to college more quickly than Chen ever has.

- 4 a The vertical scales are different  
 b Graph A, because it makes the growth look bigger by filling up the whole graph with the lines, whereas the other has much empty white space suggesting growth that could have happened but didn't.  
 c An ambitious manager of the company, because he/she might wish to show that the number of subscribers is still not large compared to the potential number of subscribers.

### 5 a Scattergraph comparing mass of chocolate to price



- b A strong positive correlation

## 4 Properties of integers

### BEFORE YOU START ...

- 1 a 25            b 6            c 11  
 2 a 2            b 3            c 5  
 3 a 2, 3, 5, 7, 11, 13, 17, 19    b 1, 4, 9, 16    c 1, 8  
 4 a D            b B            c C            d A

### LAUNCHPAD

- 1 a false    b false    c true    d true    e false    f false  
 2 a multiples of 2; 29 is incorrect  
    b multiples of 11, 56 is incorrect  
    c factors of 12; 8 is incorrect  
    d multiples of 3, 41 is incorrect  
    e factors of 36, 24 is incorrect  
    f multiples of 12; 86 is incorrect  
    g primes to 20, 9 is incorrect  
 3 a B            b A  
 4 a C (24)      b D (360)

### EXERCISE 4A

- 1 a 1, 3, 5, 7, ..., 29                            b 2, 4, 6, 8, ..., 30  
    c 2, 3, 5, 7, 11, 13, 17, 19, 23, 29        d 1, 4, 9, 16, 25  
    e 1, 8, 27                                        f 1, 2, 3, 4, 6, 8, 12, 24  
    g 3, 6, 9, 12, ..., 30                        h 1, 2, 4  
    i 12, 24  
 2 option A  
 3 a 209, 211, 213, 215  
    b various options, for example, 502, 504, 506, 508  
    c 25, 36, 49, 64                                d 1, 23  
    e 17, 19, 23, 29, etc. (selection is technically infinite)  
    f 1, 8, 27, 64, 125, 216, 343, 512, 729, 1000  
    g 8, 16, 24, 32, 40  
    h 1, 2, 3, 4, 6, 9, 12, 18, 36  
 4 a even    b even    c even    d odd    e even    f odd

### EXERCISE 4B

- 1 a 348, 432, 456, 654, 843                    b 606, 607, 660, 670, 706  
    c 123, 231, 312, 1231, 1321                d 12 700, 21 007, 21 700, 71 200  
 2 a 5            b 500            c 50            d 5000  
    e 50 000 000    f 500 000        g 5 000 000    h 50 000  
    i 50 000 000  
 3 option B  
 4 a i 640 ii 406            b i 7531 ii 1357            c i 643 210 ii 102 346

### EXERCISE 4C

- 1 a 2, 3, 5, 7                                    b 53, 59                                    c 97, 101, 103  
 2 option B  
 3 a  $2^2 \times 3^2$             b  $5 \times 13$             c  $2^6$             d  $2^2 \times 3 \times 7$   
    e  $2^4 \times 5$             f  $2^3 \times 5^3$             g  $2 \times 5 \times 127$     h  $13 \times 151$   
 4 a 1080  
    b No. The product of prime factors is unique for each whole number.

5 This is an investigation.

The sieve of Eratosthenes is a grid of numbers on which multiples (of 2, 3, etc.) are crossed out systematically, leaving only primes uncrossed.

- 6 a They are named after the monk who first suggested them, although his initial thinking about them was proven to be wrong with later discoveries.  
    b GIMPS aims to use the spare processing power of linked computers to 'crunch' greater and greater numbers to see whether they are prime or not.  
 7 a 101, 103, 107 and 109, as well as 191, 193, 197 and 199.  
    b All even numbers are automatically excluded, as are numbers ending with 5 (multiples of 5); that leaves only 4 possible options, numbers ending in 1, 3, 7 and 9.  
    As these could be multiples of 3, 7 or 9, many of these are not prime.

### EXERCISE 4D

- 1 a option A    b option C  
 2 a 18            b 36            c 90            d 24  
    e 36            f 24            g 72            h 96  
 3 Option C  
 4 a 378, 1            b 255, 5            c 864, 3            d 848, 1  
    e 24 264, 2        f 2574, 6            g 35 300, 2        h 17 325, 5  
 5 18m  
 6 120 shoppers  
 7 20 students  
 8 20 minutes  
 9 a 60 days            b 6 times  
 10 6cm  
 11 4.31 pm  
 12 After 420 seconds (7 minutes).  
    Fran 21 laps, Ayuba 5 laps and Claire 4 laps.  
 13 a  $900 \text{ cm}^2$             b 378 tiles  
 14 142 people

### CHAPTER REVIEW

- 1 Yes. You can decide by trial division of prime numbers up to half the size of the number in question.  
 2 option A  
 3

		<sup>a</sup> C																	
	<sup>1</sup> M	U	L	T	I	<sup>b</sup> P	L	E	S										
		B				R													
<sup>2</sup> E	V	E	N		<sup>3</sup> D	I	V	I	S	<sup>c</sup> O	R								
					<sup>d</sup> F	M					D								
	<sup>4e</sup> S	Q	U	A	R	E					D								
	I				<sup>5</sup> T	W	O												
					O														
					<sup>6</sup> P	R	O	D	U	C	T								

- 4** F20: 1, 2, 4, 5, 10, 20  
 F35: 1, 5, 7, 35  
 HCF = 5  
 M20: 20, 40, 60, 80, 100, 120, 140  
 M35: 35, 70, 105, 140  
 LCM = 140

**5 a**  $2^2 \times 3^2$       **b** 9

**6**  $800 = 25 \times 52$

**7 a**  $72 = 2 \times 2 \times 2 \times 3 \times 3$   
 $108 = 2 \times 2 \times 3 \times 3 \times 3$   
 HCF = 36, LCM = 216

**b**  $84 = 2 \times 2 \times 3 \times 7$

$60 = 2 \times 2 \times 3 \times 5$

HCF = 12, LCM = 420

**8** 12th step

**9** 15 March and 27 March

**10** 8 boys and 13 girls per group

## 5 Working with fractions

### BEFORE YOU START ...

**1 a** 18, 27 and 36      **b** 18, 24, 30 and 36

**c** 24 and 36      **d** 18, 24, 30 and 36

**2 a** C      **b** D      **c** B

**3 a** A because you have to do the multiplication first.

**b** B because you have to do the multiplication and division before addition.

**c** B because you have to do the multiplication before subtraction.

### LAUNCHPAD

**1 a**  $\frac{5}{35}$       **b**  $\frac{9}{16}$       **c**  $\frac{22}{10}$

**2 a** The mistake was adding the numerators and denominators.  
 Correct answer:  $\frac{17}{12} = 1\frac{5}{12}$

**b** The mistake was doing  $\frac{9}{10} - \frac{4}{5}$ . Correct answer:  $-\frac{1}{10}$

**c** The mistake was to add the numerators rather than multiplying them. Correct answer:  $\frac{8}{35}$

**d** The mistake was to multiply rather than divide. Correct answer: 60

**3 a**  $\frac{3}{5}$  of 60      **b**  $\frac{7}{10}$  of 300      **c**  $\frac{1}{2}$  of  $\frac{3}{4}$

**4**  $\frac{13}{16}$

**5**  $\frac{1}{40}$

### EXERCISE 5A

**1 a** Option B      **b** Option C      **c** Option C

**2 a**  $\neq$       **b**  $\neq$       **c**  $\neq$       **d**  $\neq$

**e**  $=$       **f**  $=$       **g**  $=$       **h**  $=$

**3 a**  $\frac{8}{32}$       **b**  $\frac{48}{192}$       **c**  $\frac{27}{108}$       **d**  $\frac{13}{52}$

**4** Students' own reasoning, but they should realise that they can make an equation and solve for  $x$  to find the unknown values.

For example,  $3x = 5 \times 18$

**5 a**  $\frac{1}{5}$       **b**  $\frac{2}{3}$       **c**  $\frac{1}{4}$       **d**  $\frac{1}{2}$

**e**  $\frac{1}{3}$       **f**  $-\frac{1}{3}$       **g**  $-1\frac{1}{2}$       **h**  $\frac{3}{5}$

**i**  $\frac{2}{3}$       **j**  $\frac{2}{3}$       **k**  $\frac{5}{7}$       **l**  $\frac{3}{7}$

**6 a**  $\frac{1}{4}, \frac{4}{7}, \frac{3}{5}, 1\frac{3}{4}, \frac{9}{4}$       **b**  $\frac{3}{4}, \frac{19}{24}, \frac{5}{6}, 2\frac{2}{3}, \frac{11}{3}$

**c**  $\frac{1}{7}, \frac{10}{21}, \frac{8}{14}, \frac{7}{7}, \frac{13}{7}, 2\frac{3}{7}$

### EXERCISE 5B

**1 a**  $\frac{4}{9}$       **b**  $\frac{13}{16}$

**2** First find the fraction between  $\frac{1}{3}$  and  $\frac{3}{4}$ ; this is  $\frac{4}{7}$ .

Then apply the rule to find the fraction between  $\frac{1}{3}$  and  $\frac{4}{7}$ , and so on.

Three possible fractions are:  $\frac{5}{10}, \frac{4}{7}$  and  $\frac{7}{11}$ .

**3** Students can place the results on a number line, or cross multiply to show that the fractions are larger/smaller than each other.

**4** Students' own research.

An internet search for mediant fractions will provide several interesting articles and some proofs.

### EXERCISE 5C

**1 a**  $\frac{3}{10}$       **b**  $\frac{1}{45}$       **c**  $\frac{1}{7}$       **d**  $\frac{7}{15}$

**e**  $-\frac{1}{24}$       **f**  $\frac{2}{5}$       **g**  $\frac{4}{33}$       **h**  $\frac{5}{28}$

**i**  $\frac{21}{250}$       **j**  $\frac{3}{88}$       **k**  $\frac{7}{4} = 1\frac{3}{4}$       **l**  $\frac{11}{14}$

**m**  $\frac{3}{4}$       **n**  $\frac{3}{8}$       **o**  $\frac{4}{9}$       **p**  $\frac{87}{12} = 7\frac{1}{4}$

**q**  $\frac{49}{10} = 4\frac{9}{10}$       **r**  $4\frac{7}{20}$       **s**  $\frac{232}{35} = 6\frac{22}{35}$       **t** 1

**u**  $\frac{9}{56}$       **v**  $-\frac{10}{33}$       **w**  $\frac{32}{25} = 1\frac{7}{25}$       **x**  $\frac{15}{22}$

**2 a** Option C      **b** Option B

**3**  $4\frac{3}{4}$

**4**  $8\frac{5}{12}$

**5**  $6\frac{79}{126}$

**6 a**  $4\frac{2}{9}$       **b**  $\frac{4}{5}$       **c**  $\frac{39}{7} = 5\frac{4}{7}$       **d**  $4\frac{3}{4}$

**e**  $\frac{5}{12}$       **f**  $\frac{215}{72} = 2\frac{71}{72}$       **g** 0      **h**  $\frac{11}{170}$

**i**  $\frac{187}{9} = 20\frac{7}{9}$       **j**  $\frac{55}{42} = 1\frac{13}{42}$       **k**  $\frac{16}{35}$       **l**  $6\frac{13}{54}$

### EXERCISE 5D

**1** Option C

**2**  $26\frac{5}{12}$  min

**3**  $\frac{2}{15}$

**4**  $3\frac{5}{12}$  m

**5**  $\frac{7}{18}$  of a minute

- 6  $\frac{7}{24}$   
 7  $42\frac{1}{2}$   
 8 40 meals  
 9 184 m  
 10  $28\frac{20}{27}$  m  
 11  $4\frac{11}{12}$  cm  
 12  $2\frac{5}{14}$  m  
 13 a  $\frac{7}{36}$       b 252 pages

**EXERCISE 5E**

- 1 a 54      b 144      c  $\frac{3}{8}$       d  $\frac{1}{10}$   
 e  $\frac{2}{21}$       f  $\frac{5}{8}$       g  $\frac{7}{4} = 1\frac{3}{4}$       h  $\frac{35}{12} = 2\frac{11}{12}$   
 2 a Option C      b Option D  
 3 a  $\frac{3}{4}$  cup      b  $\frac{7}{4} = 1\frac{3}{4}$  cups      c 1 h      d  $\frac{8}{3} = 2\frac{2}{3}$  h  
 e  $\frac{5}{6}$  h      f  $\frac{15}{4} = 3\frac{3}{4}$  h      g  $\frac{1}{4}$  h      h  $\frac{7}{3} = 2\frac{1}{3}$  min  
 i 12 seconds  
 4 a  $\frac{3}{25}$       b  $\frac{7}{40}$       c  $\frac{1}{25}$       d  $\frac{3}{32}$   
 e  $\frac{1}{12}$       f  $\frac{3}{20}$       g  $\frac{1}{40}$       h  $\frac{23}{160}$   
 5  $\frac{1}{9}$   
 6 24 min  
 7  $\frac{11}{75}$   
 8 General reference 1200, technology 3600, engineering 960, computers 2640.  
 9 Week 1: 250 m, week 2: 900 m and week 3: 350 m.  
 10 10 000 first class, 15 000 business class, 22 500 economy and 12 500 low-cost.

**EXERCISE 5F**

- 1 a  $\frac{1}{2} + \frac{1}{8}$       b  $\frac{1}{2} + \frac{1}{10}$       c  $\frac{1}{4} + \frac{1}{28}$   
 d  $\frac{1}{5} + \frac{1}{45}$       e  $\frac{1}{2} - \frac{1}{5}$   
 2  $\frac{1}{4} + \frac{1}{10} + \frac{1}{20}$   
 3  $\frac{1}{8}$  (factors 1, 2, 8)  
 4 Yes.  
 Students can investigate this and may find the theorem that proves this if they are interested.

**CHAPTER REVIEW**

- 1 Option D  
 2 a  $\frac{1}{6}$       b  $\frac{39}{46}$       c  $4\frac{3}{8}$   
 3 a  $\frac{3}{7}, \frac{4}{5}, \frac{5}{6}, \frac{8}{9}$       b  $1\frac{3}{5}, \frac{16}{9}, 2\frac{2}{5}, \frac{23}{7}$   
 4 a  $\frac{51}{40} = 1\frac{11}{40}$       b  $\frac{41}{40} = 1\frac{1}{40}$       c  $\frac{168}{15} = 11\frac{1}{5}$   
 d  $\frac{464}{85} = 5\frac{39}{85}$       e  $\frac{22}{15} = 1\frac{7}{15}$       f  $\frac{36}{28} = 1\frac{2}{7}$   
 g  $\frac{8}{189}$       h  $\frac{769}{21} = 36\frac{13}{21}$       i  $\frac{1}{24}$

- 5  $16\frac{12}{35}$  m  
 6 a  $\frac{35}{78}$       b  $\frac{119}{3} = 39\frac{2}{3}$       c  $11\frac{107}{180}$       d  $\frac{187}{9} = 20\frac{7}{9}$   
 7  $\frac{17}{100}$   
 8  $16\frac{2}{3}$  bottles (16 full bottles)  
 9 30 plots  
 10  $3\frac{33}{40}$  m deep  
 11  $3\frac{1}{3}$  cups

**6 Working with decimals**

**BEFORE YOU START ...**

- 1 a 300.098      b 0.0398      c 19.308      d 0.98308      e 130.098  
 2 a <      b >      c <      d >      e =  
 3 a  $\frac{4}{16}$       b  $\frac{3}{8}$       c  $\frac{2}{5}$       d  $\frac{3}{4}$       e  $\frac{45}{90}$       f  $\frac{1}{40}$

**LAUNCHPAD**

- 1 There are many possible answers. For example:  
 a 2.155      b 2.1555      c 0.67535  
 2 a  $\frac{9}{100}$       b  $\frac{2}{1000}$       c  $\frac{8}{10}$   
 3 a  $3\frac{1}{4}$       b  $\frac{8}{9}$       c 0.99  
 4 a C      b C      c C      d B      e A      f C  
 5 a  $\frac{2}{9}$       b  $\frac{17}{90}$       c  $1\frac{19}{90}$

**EXERCISE 6A**

- 1 option B  
 2 a  $\frac{3}{5}$       b  $\frac{21}{25}$       c  $1\frac{16}{25}$       d  $\frac{77}{200}$   
 e  $\frac{1}{8}$       f  $1\frac{2}{25}$       g  $\frac{7}{8}$       h  $\frac{1}{125}$   
 i  $3\frac{8}{125}$       j  $\frac{333}{1000}$   
 3 a 0.6      b 0.75      c 0.72      d 0.95  
 e 0.64      f 0.44      g 0.445      h 0.152  
 i 9.25      j 2.9      k 1.8 $\dot{3}$       l 0.375  
 m 2.25      n 0.8      o 2.375  
 4 a repeating digits of the numerator  
 b recurring non-terminating decimals  
 c 0.16, 0.3, 0.5, 0.6, 0.8 $\dot{3}$   
 d 0.0 $\dot{9}$ , 0.1 $\dot{8}$   
 e prediction is 0.2 $\dot{7}$  and 0.3 $\dot{6}$  based on multiples of 9  
 5 a 8.62, 5.29, 5.2, 4.92, 4.09  
 b 7.42, 3.219, 0.76, 0.742, 0.421  
 c 14.89, 14.72, 14.3, 14.07, 14.009  
 d 0.287, 0.273, 0.26, 0.23, 0.206  
 e 0.68,  $\frac{2}{3}, \frac{1}{2}, \frac{5}{11}, 0.45, 0.403$   
 f 0.88,  $\frac{7}{9}, 0.718, 0.625, \frac{3}{8}$   
 6 a <      b <      c <      d =      e >  
 f <      g >      h >      i <

7 There are many possible answers.

a 3.1355    b 0.66455    c 4.9985

8 a Steel Dragon

b California Screaming

c shorter

d Steel Dragon and The Ultimate

e 2.479 km, 2.268 km, 2.243 km, 2.045 km, 2.0 km, 1.851 km

## EXERCISE 6B

1, 2 student discussion

## EXERCISE 6C

1 option B

2 a 1.58    b 1.67    c 1.7

d 13.35    e 22.714    f 34.335

3 a 66.05    b 23.76    c 3.61    d 22.43    e 332.907

f 29.695    g 23.959    h 78.6    i 109 520    j 0.8021

k 205.6158    l 0.03    m 0.0895    n 8.15    o 793

4 a 0.74 s    b 0.44 s    c 9.21 s

d Unlikely. The first runner will start from still so will probably take longer than the other runners who start from a running position.

5 yes, Nadia has 1.54 litres

6 166.67 mg vitamin C, 7.8 mg boron and 36.85 mg calcium

7 a 180.25 kWh    b 9398.75 kWh

## EXERCISE 6D

1 option A

2 185.9 km

3 42 cups (with some juice left over)

4 1800

5 72 posts (there are approximately 71 'gaps' of 0.84m but there will be a post at each end)

6 £8.58

7 a £78.44 per day    b £392.20 per week

c £20394.40. Daily rate  $\neq$  weekly rate because there are 73 five-day weeks in a year (as opposed to 52 seven-day weeks).

## EXERCISE 6E

1 Nazeem's hypothesis is correct as long as the original fraction is in its simplest terms.

2  $\frac{3}{6}$  is not in its simplest terms; when reduced to  $\frac{1}{2}$  it fits Nazeem's original hypothesis.

## EXERCISE 6F

1 a 0.375    b 0.3125    c 0.4 $\dot{5}$     d 0.4

e 2.571428 $\dot{8}$     f 0.4 $\dot{6}$     g 4.571428 $\dot{8}$     h 1.571428 $\dot{8}$

2 a  $\frac{8}{9}$     b  $\frac{25}{9}$     c  $\frac{812}{999}$     d  $3\frac{5}{11}$

e 2    f  $\frac{65}{99}$     g  $\frac{5}{18}$     h  $2\frac{1}{2}$

3 a  $\frac{4}{9}$     b  $\frac{74}{99}$     c  $\frac{79}{90}$     d  $\frac{103}{900}$

e  $\frac{283}{300}$     f  $\frac{619}{3333}$     g  $\frac{169}{37}$     h  $\frac{17}{150}$

4  $6\frac{1}{3}$  cm

## EXERCISE 6G

1 a  $\frac{3}{10}$  and  $\frac{3}{9}$     b  $\frac{17}{100}$  and  $\frac{17}{99}$     c  $\frac{173}{1000}$  and  $\frac{173}{999}$

2 Students should realise that the recurring version of the terminating fraction has a denominator of  $n - 1$ .

3 Students' own reasoning and discussion.

## CHAPTER REVIEW

1 a 4.08, 4.2, 4.22, 4.8, 4.97

b 2.12,  $2\frac{9}{25}$ ,  $2\frac{46}{50}$ , 2.955, 2.96

c  $\frac{3}{4}$ , 0.78,  $\frac{4}{5}$ ,  $\frac{5}{6}$ , 0.86, 0.91

2 a A    b 6.45 cm    c A + B + C

3 a >    b >    c >

4 a  $\frac{22}{25}$     b  $2\frac{3}{4}$     c  $\frac{1}{125}$

5 a 3.15    b 69.67    c 32    d 0.32

6 a 7.816    b 1.092    c 876    d 0.01807

e 2.884    f 13.6

7 0.2

8 Kate, she has £2.39 more.

9 a  $\frac{2}{9}$     b  $\frac{6}{11}$     c  $4\frac{77}{90}$     d  $\frac{1207}{495}$

## 7 Basic Algebra

### BEFORE YOU START ...

1 a B    b A    c A

2 a i 19 and -11    ii 27 and -3

b 6

3 a 6y    b 5x

4 a C    b B    c D    d A

### LAUNCHPAD

1 a  $3n + 4$     b  $3(n - 4)$     c  $\frac{4n^2 + 3}{2}$

2 a  $5a + b$     b  $6x + 4$     c  $-6a^2 + 3ab$

3 a  $mn - mp$     b  $7x + 23$     c  $z^2 + 3z$

4 a  $3x + 12 = [3](x + 4)$

b  $5x + 10y = [5](x + 2y)$

c  $x^2 - 3x = [x](x - 3)$

d  $ab - ac = a([b] - [c])$

e  $-x + 7x^2 = -x([1] [-] [7x])$

5 a  $2(x + 2y)$     b  $-3(x + 3)$     c  $5(x + y)$

### EXERCISE 7A

1 a  $3x + 7y$     b  $5(x^2 - 4)$     c  $\frac{(x^3 + y^2)}{4}$

d  $4(x + 6) - y$     e  $\frac{x^2}{2}$  or  $\frac{1}{2}x^2$     f  $\frac{(3x - 25)}{5}$

2 Option C

3 a 6a    b 20b    c -9d    d 12ab

e 10cd    f -12mn    g 6pq    h  $a^2$

i  $m^2$     j  $8a^2$     k  $-15a^2$     l  $8m^2$

m  $56a^2b$     n  $12cd^2e$     o  $8a^3$

- 4 a B            b G            c I            d H  
 e C            f D            g F            h A  
 i E

- 5 a  $3x$             b  $9y$             c  $3a^2$             d  $5p$   
 e  $x$             f  $\frac{3}{2}y$             g  $-2x^2$             h  $4a^2$   
 i  $\frac{5}{n^2}$

- 6 a  $x^{13}$             b  $y^{13}$             c  $15a^9$             d  $10x^9$   
 e  $a^{-2}$             f  $2b^5$             g  $2p^{-1}$             h  $x^{12}$   
 i  $8a^{21}$             j  $10x^{-2}y^4$             k 5

7 Option C

- 8 a Perimeter:  $2x + 16$ , area:  $2x + 12$   
 b Perimeter:  $4x + 16$ , area:  $x^2 + 8x + 16$   
 c Perimeter:  $2x + 15$ , area:  $\frac{x}{2} + \frac{7}{2}$  or  $\frac{(x+7)}{2}$   
 d Perimeter:  $4x + 2y$ , area:  $x^2 + 2y$   
 9 a  $x + 10$             b  $x - 10$             c  $\frac{x}{3}$   
 10 a  $\pounds(C - 5)$             b  $\pounds C/3$             c  $\pounds 15$

EXERCISE 7B

- 1 a 24            b 21            c 54            d 15  
 e 24            f -15            g 36            h 9  
 2 a 60            b 24            c 2            d -200  
 e -6            f 9            g 22            h -8  
 i -12            j 3004            k 30            l  $-\frac{12}{5}$   
 3 e.g.  $5x + 9y = 37$  and  $-2x - 3y = 37$   
 a if  $x = 2$  and  $y = 3$ ,  $5 \times 2 + 3 \times 9 = 37$   
 b if  $x = -5$  and  $y = -9$ ,  $-2 \times -5 - 3 \times -9 = 37$

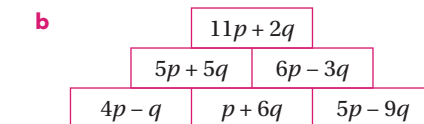
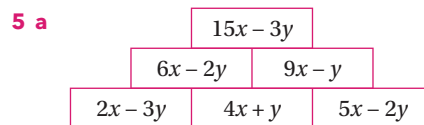
EXERCISE 7C

- 1 a unlike            b like            c like            d like  
 e unlike            f unlike            g like            h like  
 i unlike            j unlike            k like            l like  
 2 a  $6x + 9y$             b  $2d - 3c$   
 c  $-3xy - y^2$             d  $2a^2 + 2ab^2 + 2ab$   
 e  $-f + 2g$             f  $6a^2b$   
 g  $12mn^3$   
 3 a  $2a + [5a] = 7a$             b  $5b - [3b] = 2b$   
 c  $8mn + [4mn] = 12mn$             d  $11pq - [5pq] = 6pq$   
 e  $4x^2 + [3x^2] = 7x^2$             f  $6m^2 - [5m^2] = m^2$   
 g  $8ab - [10ab] = -2ab$             h  $-3st + [8st] = 5st$   
 4 a option B            b option D            c option D            d Option A  
 5 a  $8a \times [2] = 16a$             b  $9b \times [2] = 18b$   
 c  $8a \times [2b] = 16ab$             d  $5m \times [3n] = 15mn$   
 e  $3a \times [4a] = 12a^2$             f  $6p \times [5p] = 30p^2$   
 g  $-5b \times [-2b] = 10b^2$             h  $4m \times [3mn] = 12m^2n$   
 6 a -28x            b  $16xyz$             c  $10a^2$   
 d  $ab^2c^2d$             e  $24x^2y$             f  $\frac{-y^2}{x}$   
 g -3x            h  $\frac{-12y}{x}$             i  $3x^2$

- 7 a  $\frac{2x}{3}$             b  $\frac{a}{3}$             c  $\frac{-2m}{3}$             d  $\frac{2p}{3}$   
 e  $\frac{2x^2}{3}$             f  $\frac{3xy}{4}$             g  $12b$             h  $\frac{1}{3}$

EXERCISE 7D

- 1 option C  
 2 a incorrect;  $4a + 4b$             b incorrect;  $5a + a$   
 c correct            d incorrect;  $-3p + 15$   
 e incorrect;  $a^2 + ab$             f correct  
 g incorrect;  $-6x + 30$             h incorrect;  $12a^2 - 21a$   
 i correct            j incorrect;  $6x^2 - 21xy$   
 3 a  $2c + 5$             b  $a + 9$             c  $5b + 25$   
 d  $2e + 5$             e  $3f - 18$             f  $8a^2 + 13a$   
 g  $10b^2 - 9b$             h  $15a^2 + 6a$             i  $2b^2 - 15b$   
 4 a  $5y + 14$             b  $16b - 9$             c  $a + 1$   
 d  $b - 22$             e  $x^2 + x - 6$             f  $2p^2 - 3p - 5$   
 g  $10z$             h  $4y^2 - 16y$



- 6 a Let  $a = 1$ :  $1 + 1 = 2$  and  $1 \times 1 = 1$  (note if you use 2, it will be equal)  
 b Let  $x = 1$ :  $3(1) + 4 - 1 + 2 = 8$  and  $2(1) + 2 = 2 + 2 = 4$   
 c Let  $m = 1$ :  $(1 + 2)^2 = 9$  and  $1^2 + 4 = 5$   
 d Let  $x = 1$ :  $\frac{1+3}{3} = \frac{4}{3}$  and  $1 + 1 = 2$

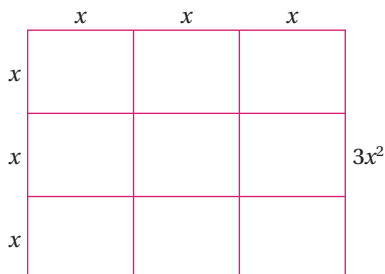
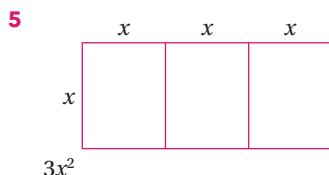
EXERCISE 7E

- 1 a  $2(x + 2)$             b  $6(2m - 3n)$             c  $3(a - b - 2)$             d  $x(y - z)$   
 e  $5xy(1 - 3z)$             f  $7b(2a - 3c)$             g  $p(q - r)$             h  $x(x - 1)$   
 i  $6ac(3b - 2)$             j  $2x(x - 2y)$             k  $2xy(x - 2y)$             l  $-6(a + 2)$   
 m  $-3(a + 3)$             n  $-x(y + 5)$             o  $-x(x - 6)$   
 2 a  $x(7 - y + x)$             b  $2x(y + 2z + 5)$             c  $5(2x - y + 3z)$   
 d  $(x - 2)(x + 5)$             e  $(a - 7)(a - 1)$             f  $-2(x - 3)$   
 g  $3xy(x + 2y)$             h  $\frac{1}{4}x^2(144 - x^2)$             i  $-a(x^2 + y^2)$

EXERCISE 7F

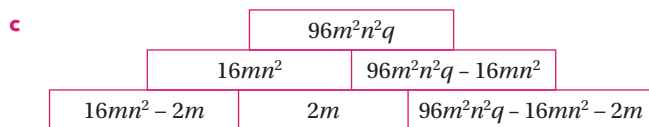
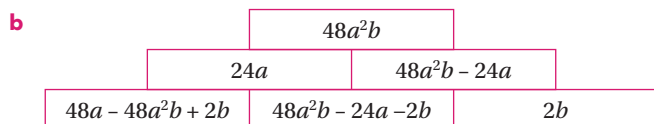
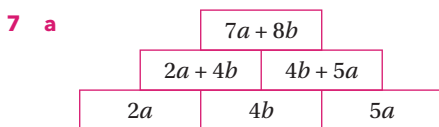
- 1 a false            b true            c true            d false            e true  
 2 all total  $3m$  so, yes, it is a magic square.  
 3 a  $(5a + 4) - (2a + 3) = 3a + 1$   
 $3b + 1 - b = 2b + 1$   
 b  $P = 10a + 6b + 10$   
 c  $210.25 \text{ units}^2$   
 4 a dimensions  $2x$  by  $(x + 2)$   
 b Students' own answers. Must sum to  $x^2 + 2$  (two sides = half the perimeter).





6  $A = 2xy + b(a - x) = 2xy + ab - bx$

$A = 2ay + (a - x)(b - 2y) = 2ay + ab - 2ay - bx + 2xy = ab - bx + 2xy$



8 a Let the number be  $x$ .

Double is  $2x$ .

Add six gives  $2x + 6$ .

Halve it gives  $x + 3$ .

Subtract the original number ( $x$ ) gives 3.

b Students' own ideas.

9  $a = 2, b = 17$

10 a  $13g + 10$  b, c are fully simplified d  $6x^2$  e  $x^8$

11 a  $\frac{13x^2}{3y^3}$  b  $\frac{(6a+5)}{2b}$  c  $-20x + 80$  d  $\frac{(2-a)}{5}$

12 a  $10x^2y^2 - 5x^3$  b  $7pq^2 + 3p^2 - 2p^4$

### CHAPTER REVIEW

1 a  $2b^2 - 15b$  b  $-8x^2 + 50x$  c  $\frac{(5p+4)}{4}$  d  $y^2 + 2y$

2 e  $\frac{8xy}{16} \neq \frac{xy}{4}, \frac{8xy}{16} = \frac{xy}{2}$

g  $\frac{15}{2x} \times \frac{2}{3x} \neq \frac{10}{x^2}, \frac{15}{2x} \times \frac{2}{3x} = \frac{5}{x^2}$

3 a Let  $x = 1$ .

$5(1 + 3) = 20$  and  $5(1) + 3 = 8$ , so this is not an identity.

b Let  $m = 1$ .

$-3(1 - 2) = 3$  and  $-3(1) - 6 = -9$ , so this is not an identity.

c Let  $y = 1$ .

$4(1 - 3) + 2(1 + 4) = 2$  and  $6(1) - 4 = 2$ , so this suggests an identity.

Left hand side =  $4(y - 3) + 2(y + 4) = 4y - 12 + 2y + 8 = 6y - 4 =$  right hand side.

4  $1/t + 1/w = (w + t)/tw$

5  $n + (n + 1) = 2n + 1$ .

$2n + 1$  is always an odd number since  $2n$  is always an even number.

6  $(n + 1)^2 - n^2 = n^2 + 2n + 1 - n^2$  (the difference between squares of two consecutive numbers)

$= 2n + 1$

$= n + (n + 1)$  (the sum of two consecutive numbers).

7 If the sides are equal then  $7x - 19 = 3(x + 3)$  leading to  $x = 7$  and  $6(x - 2) = 4x + 2$  also leading to  $x = 7$ . Since these values of  $x$  are the same, the sides of the quadrilateral could all be 30.

## 8 Properties of polygons and 3D objects

### BEFORE YOU START ...

- 1 a Point b Vertex c Edge d Face  
e Right angle f Acute angle g Base h Height

2 a Rectangle ABCE; triangle ADE; trapezium ABCD

b Octahedron

### LAUNCHPAD

1 a Parallelogram

b Lines are parallel

c  $PQ \parallel SR$

d PSR (or RSP)

2 Two lines of reflective symmetry and rotational symmetry of order 2

3 a Right-angled

b Isosceles

c Equilateral

d Obtuse-angled isosceles

4 a Parallelogram, rectangle, square, rhombus




b Rhombus, square

c Parallelogram, rectangle, square, rhombus

d Rectangle, square

e Trapezium

f Quadrilateral

Solid	Mathematical name	Number of faces	Number of edges	Number of vertices
	cuboid	6	12	8
	cube	6	12	8
	square-based pyramid	5	8	5

### EXERCISE 8A

1 a Equilateral triangle

b Regular pentagon

c Regular hexagon

d Regular octagon

2 Many examples could be provided

a Stop sign

b Playing dice

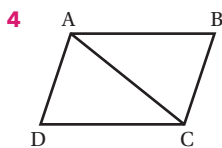
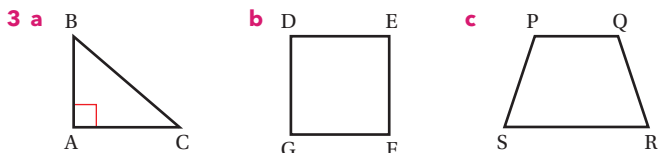
c Sheet of paper

d Shape of the side of a house

**EXERCISE 8B**

1	Column A	Column B
a	A shape that has two fewer sides than an octagon.	hexagon
b	A shape that has two sides more than a triangle.	pentagon
c	A shape with four sides.	quadrilateral
d	An 8-sided shape.	octagon
e	A figure that has length and height.	two-dimensional
f	A closed plane shape with all sides $x$ cm long and all angles the same size.	regular polygon
g	A 10-sided figure'	decagon
h	Another name for a regular 4-sided polygon.	square
i	The more common name for a regular 3-sided polygon.	equilateral triangle

- 2 a true      b true      c true      d true  
 e false      f false      g true      h true



$AB \parallel DC$  and is intersected by  $CA$ .

$DA \parallel CB$  and is intersected by  $CA$ .

As alternate interior angles are equal  $\angle ACD = \angle CAB$  and  $\angle DAC = \angle ACB$ .

This means  $\triangle DAC$  and  $\triangle BCA$  have two angles equal and the side  $CA$  in common, so the triangles are congruent, and hence  $AB = DC$  and  $DA = CB$ .

**EXERCISE 8C**

- 1 a none                                      b all lines ( $AB, CD, EF, GH$ )  
 c  $CD, HG$                                       d  $AB$

2 option B

3 Shape	Number of lines of symmetry	Order of rotational symmetry
square	4	4
rectangle	2	2
isosceles triangle	1	0
equilateral triangle	3	3
parallelogram	0	2
regular hexagon	6	6
regular octagon	8	8
regular decagon	10	10

4 A propeller with three blades.

5 H

6 Lines of symmetry: vertical and horizontal axes through the centre of the image, and both diagonal axes through the image; rotational symmetry of order 2.

7 Students' own answers.

8 Students' own answers.

**EXERCISE 8D**

- 1 Equilateral triangle.  
 2 Obtuse-angled isosceles triangle.  
 3 (a This is possible.)  
 b The angles would sum to more than  $180^\circ$ .  
 c Angles do not sum to  $180^\circ$ .  
 d Angles of an equilateral triangle are all  $60^\circ$ .  
 e Isosceles triangle has two sides of equal length.

4 Option A

5 a  $38^\circ$                                       b Isosceles

6  $a = 54^\circ, b = 66^\circ, c = 60^\circ, d = 115^\circ, e = 115^\circ, f = 16^\circ, g = 104^\circ, h = 76^\circ, i = 70^\circ$

7 a 17.5mm      b 19.30mm      c 20mm

**WORK IT OUT 8.1**

Option B is correct.

In option A,  $x$  and 78 do not meet at the same point on a straight line.

In option C, the second shape is not a rhombus.

**EXERCISE 8E**

1 Option A

- 2 a Rectangle, square  
 b Rectangle, square  
 c Parallelogram, rectangle, square, rhombus  
 d Quadrilateral  
 e Square  
 f parallelogram, rectangle, square, rhombus

3 Shape	Diagonals are equal in length	Diagonals bisect each other	Diagonals are perpendicular
rhombus		✓	✓
parallelogram		✓	
square	✓	✓	✓
kite			✓
rectangle	✓	✓	

4 Interior angles of a rhombus do not all equal  $90^\circ$ .

5 No, it could be a rhombus.

6 Two possibilities:  $90^\circ$  and  $90^\circ$ , or  $47^\circ$  and  $133^\circ$ .

- 7 a Always true      b Sometimes true      c Sometimes true  
 d Always true      e Sometimes true

8 a True                                      b False                                      c False                                      d True

9  $x = 20^\circ$

10 20 mm

11  $AD = BC$  and  $AD \parallel BC$  (as  $ABCD$  is a parallelogram).

$AD = FE$  and  $AD \parallel FE$  (as  $ADEF$  is a parallelogram).

Therefore,  $FE = BC$  and  $FE \parallel BC$ .

So,  $FECB$  is a parallelogram (as one pair of opposite sides are equal and parallel).

12 No, it could be an isosceles trapezium.

### EXERCISE 8F

1 Answers here are for (a) a small cylinder mounted on the flat face of the larger cylinder, (b) pyramid fully on top of a cube, (c) pentagonal pyramids joined at base and (d) triangular prism fully on top of one of the flat surfaces of the cuboid. (Students may have other correct answers; check against their sketches.)

3D Shape	Polyhedron	Faces	Vertices	Edges
a Large and small cylinder		5	0	0
b Cube and square pyramid	✓	9	9	16
c Two identical pentagonal pyramids	✓	10	7	15
d Triangular prism and cuboid	✓	9	10	17

2

3D shape	Faces	Vertices	Edges
cube	6	8	12
cuboid	6	8	12
triangular pyramid	4	4	6
square pyramid	5	5	8
triangular prism	5	6	9
hexagonal prism	8	12	18

- a  $E = F + V - 2$   
 b 20 faces  
 c It works for both objects  
 Truncated pyramid:  $F = 6, E = 12, V = 8; F + V - 2 = 12 = E$   
 Pyramid on top of cuboid:  $F = 9, E = 16, V = 9; F + V - 2 = 16 = E$   
 d No.  $F + V - 2 = 53, E = 45, 53 \neq 45$   
 3 a 86 cm      b 170.4 cm      c 96 cm

### CHAPTER REVIEW

- 1 a False      b True      c False  
 d True      e True  
 2 Hexagon has 6 lines of symmetry (lines connecting opposite vertices and lines connecting midpoints of opposite sides) and rotational symmetry of order 6.  
 3  $b = 117^\circ, c = 117^\circ, d = 63^\circ$   
 4 No, the missing angle is  $95^\circ$  because angles in a quadrilateral total  $360^\circ$ .  
 5 a  $x = 70^\circ, y = 110^\circ$   
 b  $a = 97^\circ, b = 83^\circ, c = 97^\circ$   
 6 Yes because both pairs of opposite sides are parallel.  
 7 No, the diagonals of both a rhombus and a kite intersect at right angles  
 8 a 8 edges  
 b 5 faces, 6 vertices  
 c 6 faces, 12 edges  
 d 1 face, 0 edges, 0 vertices

- 9 a parallel      b perpendicular  
 c trapezium      d right angle  
 e obtuse angle      f reflective symmetry  
 10 a CB//DE      b ED, DC and DC, CB      c 1      d 0  
 11 a 2      b 2  
 c  $90^\circ$       d  $180^\circ$   
 12 a 11      b  $1.25 \text{ g cm}^{-3}$

## 9 Angles

### BEFORE YOU START ...

- 1 a  $84^\circ$       b  $64^\circ$       c  $187^\circ$       d  $208^\circ$   
 2 a right-angled isosceles triangle      b rhombus  
 c rectangle  
 3  $x = y$  as the triangle formed by the diagonals is isosceles and base angles of an isosceles triangle are equal  
 4 a  $35^\circ$       b  $120^\circ$

### LAUNCHPAD

- 1 a  $72^\circ$ ; angles on a straight line sum to  $180^\circ$ .  
 b  $60^\circ$ ; angles round a point sum to  $360^\circ$ .  
 c  $108^\circ$ ; angles round a point sum to  $360^\circ$ .  
 2 no, angles on a line are  $180^\circ$   
 3 various combinations eg  $a$  and  $c$ ,  $c$  and  $g$ ,  $b$  and  $f$   
 4  $c = f$  only if the red line is perpendicular to the two parallel lines, ie  $c$  and  $f$  are  $90^\circ$   
 5  $44^\circ$ ; angle  $BCA = 68^\circ$ ; angle  $CAB = \text{angle } BCA = 68^\circ$ , so  $x = 180^\circ - 68^\circ - 68^\circ = 44^\circ$

### EXERCISE 9A

- 1 option A  
 2 option B  
 3 a  $188^\circ$       b  $110^\circ$       c  $95^\circ$   
 d reflex; obtuse; obtuse  
 4 a  $x = 49^\circ, y = 80^\circ$  ( $41 + x = 90$  and  $10 + y = 90$ )  
 b  $x = 50^\circ$  (the angles are on a straight line)  
 c  $p = 60^\circ$  (the angles are round a point)  
 5 angle  $ABE = 50^\circ + 65^\circ + 59^\circ = 174^\circ$ ; if  $AE$  is a straight line, angle  $ABE$  must be  $180^\circ$   
 6 a  $x = 137^\circ$  (straight line),  $y = 43^\circ$  (vertically opposite),  $z = 137^\circ$  (vertically opposite)  
 b  $x = 49^\circ$  (straight line),  $y = 49^\circ$  (vertically opposite),  $w = 84^\circ$  (vertically opposite),  $z = 47^\circ$  (vertically opposite)  
 7  $z = 200^\circ$   
 8 a  $x = 69^\circ, y = 69^\circ, a = 111^\circ, b = 111^\circ$   
 b  $x = 90^\circ, y = 90^\circ, a = 90^\circ, b = 90^\circ$   
 c  $x = 124^\circ, y = 124^\circ, a = 56^\circ, b = 56^\circ$   
 d  $x = 60^\circ, y = 60^\circ, a = 120^\circ, b = 120^\circ$   
 9 a  $30^\circ$       b  $192^\circ$       c  $138^\circ$   
 10 a read the wrong scale      b  $138^\circ$   
 11 a  $x = 50^\circ$  (vertically opposite)  
 b  $x = 68^\circ$  (straight line),  $y = 26^\circ$  (angles in a triangle),  $z = 94^\circ$  (straight line)  
 c  $x = 20^\circ$  (angles round a point)

### EXERCISE 9B

- option C
- $a = 137^\circ, b = 137^\circ, c = 137^\circ, d = 43^\circ$
- option A
- $x = 80^\circ$
- $a = 125^\circ, b = 55^\circ, c = 35^\circ$
- $x = 135^\circ, y = 135^\circ$
- $a = 138^\circ, b = 42^\circ, c = 138^\circ, d = 138^\circ, e = 42^\circ, f = 138^\circ, g = 42^\circ$
- $\angle CEG = 32^\circ$
- $\angle DCF = 54^\circ$
- a**  $x = 30^\circ$       **b**  $x = 22^\circ$
- a** AB is parallel to DC (alternate angles)  
**b** AB is not parallel to DC (co-interior angles do not sum to  $180^\circ$ )  
**c** AB is parallel to DC (co-interior angles sum to  $180^\circ$ )

### EXERCISE 9C

- option B
- a**  $x = 55^\circ$       **b**  $c = 70^\circ$
- $x = 75^\circ, y = 105^\circ$
- $a = 40^\circ, b = 50^\circ$
- $x = 130^\circ, y = 145^\circ, z = 85^\circ$
- $a = 48^\circ$  (base angles isosceles triangle),  $b = 84^\circ$  (angles in a triangle),  $c = 96^\circ$  (straight line)
- $x = 18\frac{1}{3}^\circ$
- Either two angles of  $53^\circ$  and one of  $74^\circ$ ; or two angles of  $74^\circ$  and one of  $32^\circ$
- $\angle ECD = y$  (corresponding angles),  $\angle ACE = x$  (alternate angles)  
So  $\angle ACD = x + y$
- a**  $x = 75^\circ$  (angles in a triangle),  $y = 75^\circ$  (corresponding angles),  $z = 35^\circ$  (corresponding angles)  
**b**  $z = 70^\circ$  (angles in a triangle)  
**c**  $x = 20^\circ$  ( $\angle ACB = 50^\circ$  (isosceles triangle)  
 $\angle BCE = 50^\circ$  (alternate)  
 $\angle ACD = 80^\circ$  (straight line)  
so,  $x = 20^\circ$  (angles in triangle))  
**d**  $x = 85^\circ$  (angles in a triangle)  
**e**  $x = 50^\circ$  (angles in a triangle)
- option A

### EXERCISE 9D

- Students' drawings
- 8 triangles; 18 triangles

<b>3</b>	<b>Number of sides in polygon</b>	3	4	5	6	7	8	10	20
	<b>Number of triangles</b>	1	2	3	4	5	6	8	18
	<b>Angle sum of interior angles</b>	$180^\circ$	$360^\circ$	$540^\circ$	$720^\circ$	$900^\circ$	$1080^\circ$	$1440^\circ$	$3240^\circ$

- number of triangles is two less than the number of sides
- $n - 2$
- a** 180 times two less than  $n$       **b**  $180(n - 2)$
- $1800^\circ$
- interior angle = sum of interior angles  $\div 12 = 1800^\circ \div 12 = 150^\circ$

### WORK IT OUT 9.1

Option A is correct. In the other solutions the pentagon has not been divided into triangles correctly.

### EXERCISE 9E

- option C
- a**  $1260^\circ$       **b**  $1800^\circ$       **c**  $4140^\circ$
- a**  $2340^\circ$       **b**  $360^\circ$       **c**  $156^\circ$       **d**  $24^\circ$
- a**  $45^\circ$       **b**  $135^\circ$       **c** octagon
- $x = 37^\circ, y = 97^\circ, z = 92^\circ$
- $137^\circ$
- $55^\circ, 110^\circ$
- No. The sum of its exterior angles must equal  $360^\circ$ , so  $55^\circ \times n = 360^\circ$ . This gives  $n = 6.55$ ,  $n$  is not an integer so there is no regular polygon with interior angles of  $125^\circ$ .
- a** 11      **b** 19
- $1260^\circ$

### CHAPTER REVIEW

- a**  $a = 270^\circ$       **b**  $b = 120^\circ$       **c**  $c = 61^\circ$       **d**  $d = 55^\circ$   
**e**  $e = 92^\circ$       **f**  $f = 162^\circ$
- a**  $108^\circ$  (corresponding)      **b**  $63^\circ$  (co-interior)
- pentagon
- $360^\circ$
- $x = 64^\circ, y = 116^\circ$
- $z = 105^\circ$
- a** 10      **b** decagon
- $36^\circ$
- $I =$  sum of interior angles,  $E =$  sum of exterior angles  
We know  $I = 180(n - 2)$  (formula for sum of interior angles)  
 $I + E = 180n$  the sum of each interior and exterior angle is  $180^\circ$  and there are  $n$  angles  
So  $E = 180n - I = 180n - 180(n - 2) = 180n - 180n + 360 = 360^\circ$
- $x = 85^\circ, y = 15^\circ$
- $x = 137^\circ$
- Sum of interior angles =  $360^\circ$ , adding all angles gives  $18x = 360^\circ$ , so  $x = 20^\circ$   
 $\angle ABC = 18^\circ, \angle BCD = 162^\circ$   
These are co-interior angles and they sum to  $180^\circ$ , so AB and DC are parallel; ABCD is therefore a trapezium.
- a** 22      **b**  $x = 109.09^\circ$

## 10 Perimeter

### BEFORE YOU START...

- a** Pentagon      **b** Hexagon      **c** Octagon
- a** 5000 m      **b** 1 200 000 cm  
**c** 8.5 m      **d** 4800 mm
- a** True      **b** False      **c** False
- $l = \frac{p}{2} - w$
- $r = \frac{p}{p + 2}$

**LAUNCHPAD**

- 1 290 m  
 2 a 116 mm    b 317 mm  
 3 42.25 mm  
 4 35 mm  
 5 a 38.33 cm    b 43.98 cm  
 6 12 cm  
 7 8.23 m (to 2 dp)

**EXERCISE 10A**

- 1 30 cm  
 2 29.96 m  
 3 Option D  
 4 a  $3a$     b  $2x + 2y$  (or  $2x + y$ )    c  $8z$   
 5 Option C  
 6 120 cm  
 7 144 cm  
 8 360 cm  
 9 a All involve the product of a base  $\times$  a perpendicular height. For the rectangle and square, the perpendicular height happens to be on of the sides of the shape.  
 b Because there is no general relationship between the lengths of the four sides.  
 10 a 520 m    b 51 posts needed with the 4 extras  
 c £1047.49  
 11 a Team A: 2780 m    Team B: 3045 m    Team C: 3210 m  
 b 12.84 km/hr

**WORK IT OUT 10.1**

Option A is correct because there are 16 sides to the perimeter each of 6.5 cm.

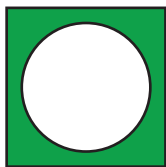
**EXERCISE 10B**

- 1 rectangle width 44 mm  
 parallelogram length 45 mm  
 rhombus side length and width 3.0625 cm  
 square length and width 11.82 cm  
 2 a 66 m    b 37.5 cm    c 4.8 cm  
 d 44.6 cm    e 280 mm  
 3 a 11 cm    b 28 cm    c 35 cm  
 d 19 cm    e 17.5 cm    f 34 cm  
 4 Option C  
 5 5.7 m

**EXERCISE 10C**

- 1 a 62.83 mm    b 43.98 cm    c 5.65 m  
 d 6.79 m    e 16.76 cm  
 2 Option C  
 3 131.95 cm  
 4 6220.35 mm

5 a



- b 25.13 cm    c 26.08 cm  
 d i 59.13 cm    ii 60.08 cm  
 6 a 6.37 mm    b 4.84 cm  
 7 5.8 cm  
 8 A square plate with side lengths 24.51 cm  
 9 47 cm

**EXERCISE 10D**

- 1 a 21.99 cm    b 17.17 cm    c 29.6 cm    d 9.69 m  
 2 a 16.19 cm    b 22.28 cm    c 7.24 cm    d 16.54 m  
 e 44.2 cm    f 54.99 m    g 1.88 cm    h 24.56 cm  
 i 62.46 cm    j 43.42 cm

**WORK IT OUT 10.2**

Option B is correct.

Option A is wrong because the whole circumference of each circle was calculated rather than just  $\frac{3}{4}$ .

Option C is wrong because the wrong formula for working out the circumference was used.

**EXERCISE 10E**

- 1 a 208.76 m    b 234.85 m  
 2 a Discus    b 7.85 m    c 7.34 m outer and 6.71 m  
 3 314.61 m  
 4 Line A is 10.12 m long. Line B is 20.25 m long.  
 5 10.47 m and 26.10 m  
 6 40074.78 km  
 7 Option B  
 8 35.1 cm  
 9 1.714 m  
 10 a 383.27 m    b 11.98 m    c 325.27 m  
 d 0.213 m/s    e 2938.95 km  
 f i 3.57 million people per annum    ii £104.30 million

**CHAPTER REVIEW**

- 1 Option B  
 2 7 cm  
 3 Yes. 31.4 cm is the circumference and as the ribbon is 50 cm, she has plenty spare to tie the bow.  
 4 18 cm  
 5 18.5 m  
 6 No. Cycled 0.043175 km  
 7 335.5 mm  
 8 91.06 m  
 9 112 cm  
 10 111.4 cm  
 11 a Main 18.85 m, stage 2 9.42 m, stage 3 4.71 m  
 b Main 18.85 m, stage 2 15.42 m, stage 3 10.71 m; Total 44.98 m  
 12 46.99 cm

## 11 Area

### BEFORE YOU START ...

- 1 a** Parallelogram    **b** Trapezium    **c** Rectangle  
**2 a** 25    **b** 200    **c** 25  
**3 a** 12    **b** 100    **c** 0.5  
**4 a**  $5\text{ m}^2 = 50\,000\text{ cm}^2$   
**b**  $870\text{ cm}^2 = 87\,000\text{ mm}^2$   
**c**  $4\text{ km}^2 = 4\,000\,000\text{ m}^2$

### LAUNCHPAD

- 1 a**  $A = \frac{4 \times 3}{2}$     **b**  $A = 4 \times 3$     **c**  $A = 4 \times 3$   
**d**  $A = 4 \times 4$     **e**  $A = 4 \times 4$   
**2 a** Area (A)    **b** coefficient    **c** radius  
**3**  $4.02\text{ m}^2$  (2 dp)  
**4**  $105\text{ cm}^2$   
**5**  $786\text{ m}^2$  (to 2 dp)

### EXERCISE 11A

- 1 a** Option C    **b** Option C  
**2 a**  $0.495\text{ m}^2$     **b**  $2437.5\text{ mm}^2$   
**3** 12 m  
**4**  $22.5\text{ cm}^2$   
**5**  $0.9\text{ m}^2$   
**6** 100 cm  
**7**  $4.32\text{ m}^2$   
**8 a**  $6688.5\text{ m}^2$   
**b** red triangle:  $0.36\text{ m}^2$ , green and blue triangles:  $0.18\text{ m}^2$   
**c**  $0.42\text{ m}^2$   
**d i**  $15\,592.5\text{ mm}^2$   
**ii**  $3500\text{ mm}^2$  based on white triangle being approximately  $\frac{1}{3}$  of height of large middle triangle.  
**e i**  $27\,000\text{ mm}^2$     **ii**  $30\,375\text{ mm}^2$     **iii**  $33\,750\text{ mm}^2$   
**f** Red 36.38%, white 41.75%, blue 21.97%

### WORK IT OUT 11.1

Option C is correct.

In Option A both dimensions are incorrect; in Option B the height is incorrect.

### WORK IT OUT 11.2

Option A is correct.

In Option B the area should be for a rectangle not a triangle.

In Option C there are  $10\,000\text{ cm}^2$  in  $1\text{ m}^2$ .

### EXERCISE 11B

- 1 a**  $60\text{ cm}^2$     **b**  $703\text{ mm}^2$     **c**  $308\text{ cm}^2$     **d**  $3.78\text{ m}^2$   
**2 a**  $412.5\text{ mm}^2$     **b**  $22.5\text{ cm}^2$     **c**  $64\text{ cm}^2$     **d**  $10.5\text{ cm}^2$   
**3** 12 cm  
**4** Option B  
**5 a** 6 cm    **b** 17 cm    **c** 10 cm    **d** 5 cm    **e** 5 cm

- 6 a**  $308\text{ m}^2$     **b** 7700 kg of soil, 3080 kg of compost  
**c** 78 m    **d i** 3.85    **ii** 3  
**7 a**  $x(x+7)$     **b**  $8x^2 + 8x - 6$     **c**  $\frac{15x^2}{2} + 12x$   
**d** 28    **e**  $12x^2 + 26x$     **f**  $\frac{x}{2}$

### EXERCISE 11C

- 1** Option A  
**2 a**  $254.47\text{ cm}^2$     **b**  $514.72\text{ cm}^2$     **c**  $153.94\text{ cm}^2$     **d**  $356.33\text{ cm}^2$   
**3 a**  $149.85\text{ mm}^2$     **b**  $3.67\text{ cm}^2$     **c**  $3.91\text{ m}^2$     **d**  $384.34\text{ mm}^2$   
**4**  $0.17\text{ m}^2$   
**5 a**  $153.94\text{ cm}^2$     **b**  $201.06\text{ cm}^2$   
**6 a**  $64.34\text{ cm}^2$     **b** 20.11 cm  
**7 a**  $1950.3\text{ m}^2$     **b**  $4.91\text{ m}^2$   
**8** Circumference =  $2\pi r = 75.398\text{ mm}$ . So  $r = 12\text{ mm}$ .  
 Area =  $\pi r^2 = 452.39\text{ mm}^2$   
**9 a**  $599\text{ mm}^2$   
**b** diameter 19.56 mm, circumference 61.43 mm  
**c** diameter 12.22 mm, circumference 38.4 mm

### EXERCISE 11D

- 1 a**  $8 \times 5 + 2 \times 5 = 50\text{ m}^2$   
**b**  $7.2 \times 4.5 + 5.1 \times (7.2 - 1.2 - 2.1) = 52.29\text{ m}^2$   
**c**  $7.2 \times 7.8 - 5.4 \times 3.4 = 37.8\text{ cm}^2$   
**d**  $12 \times 2.4 + 1.2 \times 6 = 36\text{ cm}^2$   
**e**  $2 \times 19.1 \times 3.8 = 145.16\text{ cm}^2$   
**f**  $8.53 \times 7.84 - 0.5 \times 3.71 \times (7.84 - 5.82) = 63.13\text{ cm}^2$   
**g**  $0.5 \times \pi \times 4.3^2 + 0.5 \times \pi \times 2.15^2 = 36.31\text{ cm}^2$   
**h**  $\frac{124}{360} \times 15^2 = 243.47\text{ cm}^2$   
**2 a**  $250.47\text{ cm}^2$     **b**  $13.73\text{ cm}^2$     **c**  $153.96\text{ cm}^2$   
**d**  $149.1\text{ cm}^2$     **e**  $30.18\text{ cm}^2$     **f**  $77.43\text{ cm}^2$   
**g**  $15.14\text{ cm}^2$     **h**  $69.53\text{ m}^2$   
**3** Option B  
**4 a** perimeter = 39.24 m, area =  $46.91\text{ m}^2$   
**b** perimeter = 7.069 cm, area =  $3.63\text{ cm}^2$   
**c** perimeter = 26.57 cm, area =  $32.57\text{ cm}^2$

### EXERCISE 11E

- 1** 183 (assuming can cut to fit)  
**2**  $200.84\text{ cm}^2$   
**3 a**  $706.95\text{ cm}^2$     **b**  $678.67\text{ cm}^2$   
**4**  $103.87\text{ cm}^2$   
**5**  $19.24\text{ m}^2$   
**6**  $113.1\text{ cm}^2$   
**7** 6.93 cm  
**8** £56.70  
**9 a**  $152.25\text{ cm}$     **b**  $1413.72\text{ cm}^2$   
**11 a** possible dimensions: rectangles  $3.4 \times 6.0\text{ m}$ , parallelograms  $3.4 \times 7.0\text{ m}$   
**b** no as area = base  $\times$  height

- c** Possible answers are:  
Parallelograms could be better for car parks that do not have edges at right angles. They could also be useful if space is tight as, since vehicles will not have to turn through  $90^\circ$  when reversing, it may be possible to reduce the gap between rows of parking bays.
- d** Students' sketches and reasoning

## CHAPTER REVIEW

- 15.38 m
- $68 \text{ cm}^2$
- Option C
- $13.5 \text{ m}^2$
- $660.5 \text{ m}^2$
- $372.53 \text{ cm}^2$
- $211.25 \text{ cm}^2$
- $324 \text{ cm}^2$

## 12 Rounding and estimation

### BEFORE YOU START ...

- 1 a** Correct    **b** Incorrect    **c** Incorrect
- 2 a** True    **b** True    **c** False    **d** True
- 3 a** 4.0    **b** 3.55    **c** 0.045

### LAUNCHPAD

- 1 a** 90    **b** 2000    **c** 134.12  
**d** 20.0    **e** 1000    **f** 235 000 000
- 2** 0.9 cm
- 3 a** £5.15    **b** £5.16
- 4** Around £15
- 5** Around 11 litres
- 6 a** 14    **b** 4    **c** 4
- 7**  $9.5 \text{ m} \leq 10 \text{ m} < 10.5 \text{ m}$
- 8 a**  $17.05 \leq (a+b) < 18.15$     **b**  $47.925 \leq (ab) < 52.925$   
**c**  $4.67 \leq \frac{a+b}{a} < 5.11$

### EXERCISE 12A

- 1 a** A    **b** A    **c** B    **d** A    **e** B
- 2 a i** 55    **ii** 11    **iii** 10    **iv** 12  
**b i** 30    **ii** 60    **iii** 110    **iv** 35 810  
**c i** 500    **ii** 5700    **iii** 2400    **iv** 35 800  
**d i** 3000    **ii** 0    **iii** 36 000    **iv** 67 000  
**e i** 100 000    **ii** 1 200 000    **iii** 12 400 000    **iv** 123 500 000  
**f i** 1 000 000    **ii** 1 000 000    **iii** 14 000 000    **iv** 546 000 000
- 3 a** £28    **b** 30    **c** £200  
**d** 2 m    **e** no, it is 63.8 million to the nearest 100 000
- 4** 4 and 5 (or -4 and -5)

### EXERCISE 12B

- 1** Option C
- 2 a i** 4.5    **ii** 4.53    **iii** 4.526  
**b i** 25.3    **ii** 25.26    **iii** 25.256

- c i** 125.6    **ii** 125.62    **iii** 125.617  
**d i** 0.5    **ii** 0.54    **iii** 0.538  
**e i** 32.4    **ii** 32.40    **iii** 32.397
- 3 a** 19.87    **b** 302.04    **c** 0.29  
**d** 0.21    **e** 21 245.84    **f** 0.00  
**g** 0.10    **h** 1.00    **i** 100.00
- 4** There will be a variety of justifications for answers.  
**a** 24.49 kg    **b** 3.14    **c** 14 km per litre    **d** £14.10
- 5**  $\pm 4.6$
- 6 a** Rounded to the nearest million  
**b** The 2000 value could have been as low as 500 000 as this rounds up to 1 million and the 2013 value could have been just short of 2 500 000, as values 'just' below this would round down to 2 million.

### EXERCISE 12C

- 1 a i** 800    **ii** 4000    **iii** 70 000    **iv** 0.05  
**b i** 790    **ii** 3100    **iii** 0.0033    **iv** 0.00075  
**c i** 789    **ii** 46 700    **iii** 0.00421    **iv** 753 000  
**d i** 38    **ii** -4,100    **iii** 3.0    **iv** 2 000 000  
**e i** 37.7    **ii** -4,130    **iii** 3.04    **iv** 2 000 000
- 2** Rounding 0.000 134 567 to two decimal places will give 0.00 which doesn't tell us anything. Rounding to two significant figures is a more accurate way to round very small numbers.
- 3 a** Option B    **b**  $1.2 \text{ kg/m}^3$     **c** 300 000 000 m/s  
**d**  $9.81 \text{ m/s}^2$
- 4 a**  $\pm 4.6$   
**b** Rounding to two decimal places is more accurate than rounding to two significant figures. Rounding in two different ways would mean that the answer is less accurate than if you always rounded to two decimal places.

### EXERCISE 12D

- 1 a** 37.67    **b** -4.12    **c** 3.03    **d** 0.99
- 2 a** 4.52    **b** 25.2    **c** 125  
**d** 0.537    **e** 32.3    **f** 200
- 3** £20.67 Rounding to two decimal places is the most useful way to approximate as this is to the nearest penny. (Rounding up would also always ensure there is enough money to cover the bill.)

### WORK IT OUT 12.1

Estimate A is the closest estimate to the actual cost, but students may have justification for choosing a different estimate. (For example, always rounding up so you have an overestimate may be good for budgeting.)

### EXERCISE 12E

- 1 a**  $100 \times 4 = 400$     **b**  $400 \times 1 = 400$     **c**  $1 \times 20 = 20$   
**d**  $10 \times 0.5 = 5$     **e**  $3 \times 5 \times 5 = 75$     **f**  $5 \times 10 = 50$   
**g**  $\frac{200}{20} = 10$     **h**  $\frac{60}{0.5} = 120$
- 2** Option D
- 3 a** Answer C,  $190 \times 10 = 1900$   
**b** Answer B,  $16 \div 8 = 2$

- 4 a  $\frac{80 \times 0.5}{40 \times 3} = \frac{40}{120} \approx 0.3$       b  $\frac{20 + 3}{20 - 6} = \frac{23}{14} \approx 1.5$   
 c  $\frac{900 \div 40}{2 \times 0.2} = \frac{22.5}{0.4} = \frac{225}{4} \approx 56$   
 d  $\frac{1000 \div 500}{20 \div 40} = \frac{2}{0.5} = 4$   
 5 a  $\sqrt{\frac{3 \times 4}{0.4 \times 0.3}} = 10$       b  $\sqrt{\frac{4 \times 12}{8 \times 0.3}} = \sqrt{20} \approx 4.5$   
 6 a 2160 m      b 250 seconds  
 7 a  $3 \times 3 \times 25 = 225$  Not sensible      b  $5 \times 9 = 45$  Not sensible  
 c  $50 \times 9 = 450$  Sensible      d  $3 \times 200 = 600$  Not sensible  
 e  $\frac{9}{3 \times 20} = 0.15$  Sensible      f  $\frac{0.03}{0.5} = 0.06$  Not sensible  
 8 6.9 cm  
 9  $\pm 7.1$   
 10 0.68087  
 11 3

### EXERCISE 12F

- 1 a  $95.5 \leq n < 96.5$       b  $95.95 \leq n < 96.05$   
 c  $95.995 \leq n < 96.005$       d  $0.55 \leq n < 0.65$   
 e  $0.055 \leq n < 0.065$       f  $0.595 \leq n < 0.605$   
 g  $3.1415 \leq n < 3.1425$       h  $9.85 \leq n < 9.95$   
 i  $3.065 \leq n < 3.075$   
 2 a  $4.85 \text{ cm} \leq 1 < 4.95 \text{ cm}$       b  $12.5195 \text{ m} \leq 1 < 12.5205 \text{ m}$   
 c  $42.95 \text{ cm} \leq 1 < 43.05 \text{ cm}$       d  $28.5 \text{ mm} \leq 1 < 29.5 \text{ mm}$   
 3 a Option B  
 b No.  $1.395 \text{ m} \leq \text{length of wood} < 1.405 \text{ m}$   
 c Least weight 43.35 kg; greatest weight 43.4499999 kg (weight  $< 43.45$ )  
 4 a £197.21 and £619.01, £192.41 and £603.95, £191.93 and £602.45  
 b In the example, there is nearly a £6 saving for gold and a £17 saving for platinum.  
 5 a Lower bound: 99.5 m      Upper bound: 100.5 m  
 b Lower bound: 15.25 seconds      Upper bound: 15.35 seconds  
 6  $4.45 \text{ m} \leq L < 4.55 \text{ m}$

### EXERCISE 12G

- 1 Upper bound =  $50.5 \text{ kg} - 11.5 \text{ kg} = 39 \text{ kg}$   
 Lower bound =  $49.5 \text{ kg} - 12.5 \text{ kg} = 37 \text{ kg}$   
 2 a LB: 3.605 cm, UB: 3.615 cm and LB: 2.565 cm, UB 2.575 cm  
 b LB:  $9.246825 \text{ cm}^2$ , UB:  $9.308625 \text{ cm}^2$   
 c LB:  $9.25 \text{ cm}^2$ , UB:  $9.31 \text{ cm}^2$   
 3 a  $11.955 \text{ km}^2 \leq \text{area} < 13.075 \text{ km}^2$   
 b 4.52% (2 dp)  
 c This error represents about half a square km of land and if land prices are high, this could be quite significant.  
 4 £99.5 - £30.5 = £69  
 5 a i 24 cm      ii 7.11 cm (3 sf)      b  $26.25 \text{ cm}^2$   
 6  $3.16 \leq \pi < 3.52$  (3 sf)  
 7  $8.114 \text{ m/s} \leq s < 8.120 \text{ m/s}$  (3 dp)  
 8 5 g  
 9  $0.76885 \text{ m/s} \leq v < 0.9095 \text{ m/s}$

## CHAPTER REVIEW

- 1  $\frac{49}{160} \approx 0.3$   
 2  $\frac{7000}{500} = 14\text{p}$   
 3 5210  
 4  $\frac{1}{3}$   
 5 445 and 544  
 6 Option C  
 7 No, a bar could weigh 54.9 g and this is an error of 9.8% which exceeds the tolerance.  
 8 16.4 ohms  
 9 LB: 45.18%, UB: 46.67% (2 dp)

## 13 Percentages

### BEFORE YOU START ...

- 1 a 2.1      b 0.21      c 24      d 2.4  
 2  $\frac{16}{36} = \frac{4}{9}$ ,  $\frac{15}{35} = \frac{3}{7}$ ,  $\frac{30}{36} = \frac{5}{6}$ ,  $\frac{9}{36} = \frac{1}{4}$ ,  $\frac{39}{52} = \frac{3}{4}$ ,  $\frac{13}{39} = \frac{1}{3}$   
 3 a false      b false      c false      d true      e false

### LAUNCHPAD

- 1 a  $\frac{17}{50}$       b  $1\frac{3}{20}$   
 2 a 12%, 0.125,  $\frac{7}{50}$ , 19%,  $\frac{5}{12}$   
 b 2.5%, 12.5%, 1.08, 1.25, 200%,  $2\frac{3}{4}$   
 3 76%  
 4 64  
 5 27.2%  
 6 3.33%  
 7 £21.90  
 8 £2590  
 9 £3.99

### EXERCISE 13A

- 1 a Option D      b Option D      c Option B  
 2 a 5%      b 54%      c 44%      d 85%  
 e 50%      f 66.7%      g 62.5%      h 184%  
 i 30%      j 4%      k 47%      l 112%  
 m 207%      n 225%      o 3.5%      p 0.7%  
 3 a  $\frac{1}{4}$       b  $\frac{4}{4}$       c  $\frac{9}{10}$       d  $\frac{1}{8}$   
 e  $\frac{1}{2}$       f  $\frac{49}{50}$       g  $\frac{3}{5}$       h  $\frac{11}{50}$   
 4 a 0.82      b 0.97      c 0.45      d 0.28  
 e 0.0005      f 0.0008      g 0.00006      h 0.000 007  
 i 1.25      j 3      k 0.0728      l 0.09007  
 5 a 6.5%      b 33.3%      c 67.5%  
 6 24%  
 7 a  $0.8\%$ ,  $\frac{1}{20}$ , 0.1, 30%,  $\frac{3}{5}$       b 0.15,  $\frac{1}{4}$ , 57%, 0.75, 0.88  
 c 0.25, 60%,  $\frac{2}{3}$ , 0.75,  $\frac{9}{10}$       d  $0.395$ ,  $\frac{3}{7}$ ,  $0.43$ ,  $\frac{4}{9}$ , 45%  
 e 49.3%, 55%,  $\frac{19}{25}$ , 80%,  $\frac{5}{6}$



$$8 \frac{167}{200}$$

$$9 \ 6\%$$

$$10 \ 5\%$$

$$11 \ a \ 27 \text{ out of } 30 \quad b \ 84.7\%$$

### WORK IT OUT 13.1

A is correct.

B - Answer is too big.  $\frac{400}{9}$  is not a percentage, it is a mixed number ( $44\frac{4}{9}$ )

C - 0.009 is  $\frac{9}{1000}$  which is only 0.9%.

D - The % sign is missing from the calculation.

E -  $\frac{9}{400}$  is not 9%, it is only 2.25%.

### EXERCISE 13B

- |           |          |             |
|-----------|----------|-------------|
| 1 a 12.5  | b 36     | c 24        |
| d 2925    | e 9      | f 270       |
| g 16      | h 66     | i 135       |
| 2 a £9.50 | b 42 kg  | c 15.75 cm  |
| d 26.1 kg | e £14    | f 2.08 min  |
| g £74     | h 6.84 m | i 58 litres |

$$3 \ 68 \text{ out of } 80$$

$$4 \ a \ \text{option D} \quad b \ \text{option C}$$

$$5 \ 5.4, \text{ so } 5 \text{ phones}$$

$$6 \ a \ 1127 \quad b \ 1323$$

$$7 \ 201$$

$$8 \ £22.48$$

$$9 \ a \ 46.5 \text{ m}^2 \quad b \ 573.5 \text{ m}^2$$

$$10 \ 3164$$

$$11 \ a \ 9 \text{ ct} = 37.5\% \text{ gold and } 18 \text{ ct} = 75\% \text{ gold}$$

$$b \ 5.475 \text{ g}$$

$$c \ 6.1125 \text{ g}$$

d Student's own reasoning, but research will show that even if 9 carat is only 37.5% pure gold, gold remains the largest component of the alloy.

### EXERCISE 13C

$$1 \ a \ \text{The village}$$

b Students' own reasoning, but given the numbers, the city is probably still the most risky in terms of crime.

### WORK IT OUT 13.2

Student B is correct. Student A is wrong. Part of a race cannot be greater than the whole (100%), so 300% can't be right.

### EXERCISE 13D

- |          |          |          |          |
|----------|----------|----------|----------|
| 1 a 8%   | b 1.5%   | c 15%    | d 4%     |
| e 35%    | f 6.33%  | g 6.25%  | h 18.86% |
| i 33.3%  | j 27.78% | k 32.61% | l 53.33% |
| m 57.14% | n 7.14%  | o 26.67% | p 16.8%  |
| q 25%    | r 64.29% | s 17.5%  | t 1%     |

$$2 \ \text{option C}$$

$$3 \ \text{Angelika}$$

$$4 \ 60\%$$

$$5 \ 6\%$$

$$6 \ 20\%$$

$$7 \ 29.63\%$$

$$8 \ 81.39\%$$

$$9 \ a \ 33.33\% \quad b \ 0.31\%$$

$$10 \ 250\%$$

$$11 \ 33\frac{1}{3}$$

### WORK IT OUT 13.3

Answers here will vary from student to student.

### EXERCISE 13E

$$1 \ a \ £54.72 \quad b \ £945 \quad c \ £32.28$$

$$d \ £40 \ 236 \quad e \ £98.55 \quad f \ £99.68$$

$$2 \ \text{option D}$$

$$3 \ a \ £58.48 \quad b \ £520 \quad c \ £83.16$$

$$d \ £19882 \quad e \ £76.93 \quad f \ £45.24$$

$$4 \ \text{option D}$$

$$5 \ £129375$$

$$6 \ £3244$$

$$7 \ £7$$

$$8 \ 358$$

$$9 \ £2393.75, £2429.66$$

$$10 \ £42430$$

$$11 \ a \ £12 \quad b \ 27739 \quad c \ £114840$$

12 It means that compared with the average amount of rainfall (in mm) over the past period, the rainfall increased by almost  $\frac{1}{4}$ . You'd need to know the average rainfall so you could work out how much more rain actually fell in 2014.

$$13 \ 6.7$$

### EXERCISE 13F

$$1 \ a \ £120 \quad b \ 1500 \text{ g} \quad c \ 666.67 \text{ kg} \quad d \ £1739.13$$

$$2 \ \text{option C}$$

$$3 \ a \ £1000 \quad b \ £121.25 \quad c \ £720.83 \quad d \ £45.83$$

$$4 \ £50$$

$$5 \ \text{option A}$$

$$6 \ a \ 1200 \text{ students} \quad b \ 960 \text{ students}$$

$$7 \ £150$$

$$8 \ 260 \text{ g}$$

$$9 \ 61.05 \text{ kg}$$

$$10 \ 500 \text{ runners}$$

### CHAPTER REVIEW

$$1 \ a \ \text{option D} \quad b \ \text{option B}$$

$$2 \ a \ \frac{1}{4} \quad b \ \frac{3}{10} \quad c \ \frac{7}{200}$$

$$3 \ a \ 5\% \quad b \ 12.5\% \quad c \ 53.33\%$$

$$d \ 50\% \quad e \ 125\% \quad f \ 0.5\%$$

- 4 4% increase
- 5 223 435
- 6 57.3 million
- 7 33.6 hours (33 h 36 min)
- 8 a 12.5%                      b 37.5%
- 9 £460
- 10 £850
- 11 £873.44 to nearest pence

## 14 Powers and roots

### BEFORE YOU START ...

- 1 a <                      b =                      c >                      d =
- 2 a B                      b C                      c C
- 3 a  $\frac{4}{3}$                       b  $\frac{1}{12}$                       c  $\frac{5}{7}$

### LAUNCHPAD

- 1 a  $4^5$                       b  $8^3$                       c  $5^2$
- d  $9^7$                       e  $\left(\frac{1}{3}\right)^4$                       f  $\left(\frac{1}{2}\right)^3$
- 2 a  $3^5 = 243$                       b  $4^2 = 16$                       c  $6^0 = 1$
- d  $6^2 = 36$                       e  $4^{-3} = \frac{1}{4^3} = \frac{1}{64}$                       f  $2^6 = 64$

### EXERCISE 14A

1 Option D

2 a

Index Base	-3	-2	-1	0	1	2	3	4	5
2	$2^{-3} = \frac{1}{8}$	$2^{-2} = \frac{1}{4}$	$2^{-1} = \frac{1}{2}$	$2^0 = 1$	$2^1 = 2$	$2^2 = 4$	$2^3 = 8$	$2^4 = 16$	$2^5 = 32$
3	$3^{-3} = \frac{1}{27}$	$3^{-2} = \frac{1}{9}$	$3^{-1} = \frac{1}{3}$	$3^0 = 1$	$3^1 = 3$	$3^2 = 9$	$3^3 = 27$	$3^4 = 81$	$3^5 = 243$
4	$4^{-3} = \frac{1}{64}$	$4^{-2} = \frac{1}{16}$	$4^{-1} = \frac{1}{4}$	$4^0 = 1$	$4^1 = 4$	$4^2 = 16$	$4^3 = 64$	$4^4 = 256$	$4^5 = 1024$
5	$5^{-3} = \frac{1}{125}$	$5^{-2} = \frac{1}{25}$	$5^{-1} = \frac{1}{5}$	$5^0 = 1$	$5^1 = 5$	$5^2 = 25$	$5^3 = 125$	$5^4 = 625$	$5^5 = 3125$

- b The negative powers are unit fractions of the matching positive powers.
- c Every second power of two is equivalent to a power of four.
- d If it does not end in 5 it cannot be a power of 5.
- 3 a T                      b F                      c T                      d F
- e T                      f F                      g F                      h T
- i T                      j T                      k F                      l T
- 4 a 5                      b 2                      c 4                      d 5
- e 3                      f 4                      g 7                      h 50
- i 5                      j 30                      k 20                      l 25

### EXERCISE 14B

- 1 a 8                      b 36                      c 1                      d 512
- e 10 000                      f 1 000 000                      g 7                      h 50
- i 64                      j 32                      k 128                      l 729

- m 2                      n 16                      o 49 000                      p 7 000 000
- q 3200                      r 36 000 000
- 2 a 4096                      b 1728                      c 32 768                      d 2401
- e 3375                      f 10 000                      g 784                      h 15 625
- 3 a 1472                      b 159 775                      c 759 375
- d 531 522                      e 985                      f 343
- 4 a >                      b <                      c >
- d <                      e <                      f >

### EXERCISE 14C

1 Option C

- 2 a  $\frac{1}{2}$                       b  $\frac{1}{3}$                       c  $\frac{1}{4}$
- d  $\frac{1}{3^2}$                       e  $\frac{1}{4^3}$                       f  $\frac{1}{3^5}$
- g  $\frac{1}{3^4}$                       h  $\frac{1}{6^6}$                       i  $\frac{1}{34^5}$
- j  $\frac{1}{x^3}$                       k  $\frac{1}{m^2}$                       l  $\frac{3}{x^4}$
- 3 a  $3^{-1}$                       b  $5^{-1}$                       c  $7^{-1}$
- d  $3^{-2}$                       e  $4^{-5}$                       f  $2^{-6}$
- g  $7^{-2}$                       h  $10^{-5}$                       i  $2^{-2}$
- j  $12^{-3}$                       k  $10^{-4}$                       l  $\frac{2^{-2}}{3}$
- m  $x^{-2}$                       n  $x^{-3}$                       o  $4y^{-2}$
- 4 a =                      b =                      c =
- d ≠                      e =                      f =
- g =                      h =                      i ≠
- j ≠                      k ≠                      l ≠
- 5 a  $2^1$                       b  $2^4$                       c  $2^6$
- d  $2^{-3}$                       e  $2^{-2}$                       f  $2^0$
- g  $2^{-5}$                       h  $2^2$                       i  $-2^3$

### EXERCISE 14D

- 1 a  $2^7$                       b  $10^7$                       c  $4^6$                       d  $5^7$
- e  $2^{11}$                       f  $3^{-2}$                       g  $2^3$                       h  $3^2$
- i  $2^{-1}$                       j  $3^5$                       k  $10^1$                       l  $10^0$
- 2 a  $6^2$                       b  $10^3$                       c  $6^2$                       d  $6^{-2}$
- e  $10^{-2}$                       f  $3^{10}$                       g  $3^7$                       h  $10^0$
- i  $5^6$                       j  $10^{10}$                       k  $3^1$                       l  $2^{-3}$
- 3 a  $2^6$                       b  $2^9$                       c  $2^8$                       d  $10^4$
- e  $10^6$                       f  $10^8$                       g  $2^{-12}$                       h  $10^{-4}$
- i  $10^{-6}$                       j  $3^{-8}$                       k  $2^0$                       l  $2^{10}$
- 4 a T                      b F:  $3^6$                       c T
- d T                      e T                      f T
- g F:  $3^8$                       h T                      i T

### EXERCISE 14E

- 1 a  $\sqrt{3}$                       b  $\sqrt[3]{4}$                       c  $\sqrt[4]{5}$                       d  $\sqrt{6}$
- e  $\sqrt[9]{4}$                       f  $\sqrt[3]{5^{(2)}}$                       g  $\sqrt[8]{4^{(3)}}$                       h  $\sqrt[9]{6^{(2)}}$
- 2 a  $6^{\frac{1}{2}}$                       b  $4^{\frac{1}{3}}$                       c  $11^{\frac{1}{3}}$                       d  $9^{\frac{1}{4}}$
- e  $3^{\frac{4}{3}}$                       f  $7^{\frac{1}{5}}$                       g  $7^{\frac{2}{3}}$                       h  $2\left(3^{\frac{5}{3}}\right)$



### EXERCISE 15B

- 1 Display will vary depending on calculator used.  
 2 Option A  
 3 a  $1.09 \times 10^5$    b  $2.876 \times 10^{-6}$    c  $4.012 \times 10^9$    d  $1.89 \times 10^7$   
 e  $3.123 \times 10^{13}$    f  $2.876 \times 10^{-4}$    g  $9.02 \times 10^{15}$    h  $8.076 \times 10^{-12}$   
 i  $8.124 \times 10^{-11}$    j  $5.0234 \times 10^{19}$

### EXERCISE 15C

- 1 Option D  
 2 a  $5.62 \times 10^{21}$    b  $6.56 \times 10^{-17}$    c  $1.28 \times 10^{-14}$    d  $1.44 \times 10^{13}$   
 e  $1.58 \times 10^{-20}$    f  $5.04 \times 10^{18}$    g  $1.98 \times 10^{12}$    h  $1.52 \times 10^{17}$   
 i  $2.29 \times 10^8$    j  $2.09 \times 10^{-8}$   
 3 a  $1.3607 \times 10^{18}$    b  $1.0274 \times 10^{-15}$    c  $1.0458 \times 10^0$   
 d  $1.6184 \times 10^{11}$    e  $5.2132 \times 10^{19}$    f  $3.0224 \times 10^{-16}$   
 g  $2.3141 \times 10^{12}$    h  $3.8066 \times 10^{17}$    i  $3.4760 \times 10^{-3}$

### EXERCISE 15D

- 1 Option D  
 2 a  $8 \times 10^{30}$    b  $4.2 \times 10^{12}$    c  $2.25 \times 10^{26}$    d  $1.4 \times 10^{32}$   
 e  $3 \times 10^1$    f  $2 \times 10^1$    g  $3 \times 10^3$    h  $3 \times 10^{42}$   
 3 a  $8 \times 10^{-20}$    b  $6.4 \times 10^{-12}$    c  $3.15 \times 10^{-9}$    d  $3.3 \times 10^{-2}$   
 e  $2 \times 10^{33}$    f  $7 \times 10^{-37}$    g  $5 \times 10^{12}$    h  $1.65 \times 10^1$   
 4 a  $1.2 \times 10^{31}$    b  $4.5 \times 10^{11}$    c  $3.375 \times 10^{36}$   
 d  $1.32 \times 10^{-11}$    e  $2 \times 10^{26}$    f  $2.67 \times 10^5$  (3 sf)  
 g  $1.2 \times 10^2$    h  $2 \times 10^{-3}$   
 5 a  $3 \times 10^9$  m   b  $6 \times 10^9$  m   c  $3 \times 10^{10}$  m   d  $6 \times 10^{11}$  m  
 6 a  $3.125 \times 10^7$  times; 31 250 000 times  
 b  $4.5 \times 10^{16}$  blinks

### EXERCISE 15E

- 1 a  $5 \times 10^8$    b  $1.5 \times 10^{-3}$    c  $3.15 \times 10^6$   
 d  $5.6 \times 10^7$    e  $3.4 \times 10^{-3}$    f  $-2 \times 10^{-2}$   
 2 Option C  
 3 a The Pacific   b  $5.9 \times 10^7$  km<sup>2</sup>  
 c Total area =  $3.61 \times 10^8 - (2.71 \times 10^8) = 0.9 \times 10^8 = 9 \times 10^7$  km<sup>2</sup>  
 4  $9.276 \times 10^7$  miles  
 5 a Virus A   b  $2.7 \times 10^{-7}$  m   c  $3.3 \times 10^{-7}$  mm

### EXERCISE 15F

- 1 Option C  
 2 a  $1.07 \times 10^9$    b  $1.10 \times 10^{12}$   
 3 a 400 pixels   b  $1.61 \times 10^{-2}$  cm<sup>2</sup>   c 4800 pixels  
 d  $4 \times 10^{-8}$  cm<sup>2</sup>   e  $1 \times 10^{-5}$  cm<sup>2</sup>  
 4 a  $3.3 \times 10^{10}$  nm   b  $2.1 \times 10^7$  nm  
 5 a 500 seconds =  $5 \times 10^2$  seconds  
 b 19 166.67 seconds =  $1.92 \times 10^4$  seconds (3 sf)  
 6 a  $5.848 \times 10^{11}$  cells   b  $3.742 \times 10^{13}$  cells (4 sf)  
 7 Student's individual problems.

### CHAPTER REVIEW

- 1 a  $4.5 \times 10^4$    b  $8 \times 10$    c  $2.345 \times 10^6$   
 d  $3.2 \times 10^{10}$    e  $6.5 \times 10^{-3}$    f  $9 \times 10^{-3}$   
 2 a 2500   b 39 000  
 c 426 500   d 0.000 010 45  
 3 Option C  
 4 a  $9.05 \times 10^6$    b  $7.848 \times 10^{-1}$    c  $4.04 \times 10^{10}$   
 d  $3.20 \times 10^{-1}$    e  $3.429 \times 10^8$  (4 sf)   f  $3.757 \times 10^{10}$  (4 sf)  
 5 a  $5.74 \times 10^7$    b  $1.764 \times 10^{15}$   
 c  $3.4 \times 10^4$    d  $1.33 \times 10$  (3 sf)  
 6 a  $1.6125 \times 10^3$    b  $1.02 \times 10^5$   
 7 a  $8 \times 10^{-8}$  m   b  $1 \times 10^{-8}$  m   c  $4 \times 10^{-7}$  m  
 8 a The Sun   b  $6.051 \times 10^6$   
 9 a  $6.35 \times 10^3$  km   b  $3.99 \times 10^4$  km   c  $1.07 \times 10^{12}$  km<sup>2</sup>

## 16 Further algebra

### BEFORE YOU START ...

- 1 a  $-6x^2y^2 - xy$   
 b  $x^2y^2$  and  $xy$  are not like terms as they have different powers.  
 2  $12x + 5y$   
 3 Yes  
 4  $x^{3/2} + 36/y^2$   
 5 a  $-30a^2$    b  $21y^3$    c  $-2a$    d  $\frac{6}{5}$    e  $\frac{-5x}{2}$   
 6 a B   b C   c D   d A

### LAUNCHPAD

- 1 a  $x^2 + 8x + 15$    b  $x^2 + 2x - 15$    c  $x^2 - 8x + 15$   
 2 a  $(a+2)(a+3)$    b  $(x-2)(x-1)$   
 c  $(p-9)(p+5)$    d  $(y+4)(y-4)$   
 3 a  $\frac{2x}{5}$   
 b  $\frac{4x+10}{(x+1)(x+3)}$

### EXERCISE 16A

- 1 Option D  
 2 a  $x^2 + 7x + 10$    b  $x^2 - 7x + 10$    c  $x^2 - 3x - 10$   
 d  $x^2 + 3x - 10$    e  $x^2 - x - 12$    f  $x^2 + 2xy + y^2$   
 3 a  $6x^2 + 18x + 12$    b  $15x^2 + 26x + 8$    c  $6x^2 - 13x - 5$   
 d  $20y^2 - 11y - 3$    e  $6a^2 - 13a + 5$    f  $2b^2 - 11b + 15$   
 g  $6y^2 - 19y + 15$    h  $4x^2 - 4x - 24$    i  $20x^2 - 17x + 3$   
 4 a  $6x^2 + x - 2$    b  $13x^2 + 8x + 5$    c  $5x^2 + 16x + 3$   
 5 a  $\frac{4}{x^2} + 2 + \frac{x^2}{4}$    b  $4x^2 + 12xy + 9y^2$   
 c  $4x^2 - 4xy - 4xz + y^2 + 2yz + z^2$   
 6 a = -1

### EXERCISE 16B

- 1 a  $x^2 - 1$    b  $a^2 - 4$    c  $4x^2 - 1$    d  $x^2 - 4y^2$   
 2  $(x+y)(x-y) \equiv x^2 - y^2$   
 3 a Each of the two terms will itself be a perfect square and the sign between the terms will be a negative.

- b i** No. The number of  $x$ 's and  $y$ 's is not the same in each bracket.
- ii** Yes. Notice that the expression must first be divided by the common factor, 2.
- iii** Yes.

### EXERCISE 16C

- 1 a**  $x^3 + 6x^2 + 11x + 6$       **b**  $4x^3 - 16x^2 + 19x - 6$
- c**  $x^3 + x^2 - 4x - 4$       **d**  $6x^3 - 19x^2 + x + 6$
- 2 a**  $27x^3 - 108x^2 + 144x - 64$       **b**  $x^3 + 27$
- c**  $\frac{1}{125x^2} + \frac{1}{27y^3}$       **d**  $x^4y^4 - x^4$
- 3 a**  $(2x + \frac{1}{2})(x - 2)(x - 2)$       **b**  $2x^3 - 7\frac{1}{2}x^2 + 6x + 2$
- c** These are a perfect square.

### EXERCISE 16D

- 1 a**  $(x + 2)(x + 3)$       **b**  $(x + 9)(x + 2)$       **c**  $(x + 2)(x + 5)$
- d**  $(x + 5)(x + 6)$       **e**  $(x + 2)(x + 7)$       **f**  $(x + 10)(x + 9)$
- 2 a**  $(x - 2)(x - 3)$       **b**  $(x - 3)(x - 11)$       **c**  $(x - 2)(x - 15)$
- d**  $(x - 6)(x - 7)$       **e**  $(x - 4)(x - 11)$       **f**  $(x - 5)(x - 20)$
- 3 a**  $2(x + 1)(x + 2)$       **b**  $6(x - 3)(x - 1)$       **c**  $5(x - 2)(x + 1)$
- d**  $2(x + 2)(x + 5)$       **e**  $2(x - 1)(x + 3)$       **f**  $3(x + 1)(x - 11)$
- 4** Option B

### WORK IT OUT 16.1

Option B is correct.

Option A put  $13^2$  in the bracket, not 13.

Option C has a negative in both brackets.

### EXERCISE 16E

- 1 a**  $(x + 6)(x - 6)$       **b**  $(p + 9)(p - 9)$       **c**  $(w + 4)(w - 4)$
- d**  $(p + 6q)(p - 6q)$       **e**  $(12s + c)(12s - c)$       **f**  $(8h + 7g)(8h - 7g)$
- 2** Option D
- 3 a**  $2(2x + y)(2x - y)$       **b**  $3(xy + 2z)(xy - 2z)$       **c**  $2(x - 2)(2 - x)$
- d**  $3(x + 2)(x + 6)$       **e**  $7(x - 5 + y)(x - 5 - y)$
- f**  $(x + y + 8)(x - y + 2)$
- 4 a**  $(100 - 97)(100 + 97) = 3 \times 197 = 591$
- b**  $(50 - 48)(50 + 48) = 2 \times 98 = 196$
- c**  $(639 - 629)(639 + 629) = 10 \times 1268 = 12\,680$
- d**  $(98 - 45)(98 + 45) = 53 \times 143 = 7579$
- e**  $(83 - 77)(83 + 77) = 6 \times 160 = 960$
- f**  $(1234 - 999)(1234 + 999) = 235 \times 2233 = 524\,755$
- 5 a**  $(17 - 15)(17 + 15) = 2 \times 32 = 64$        $a = \sqrt{64} = 8$
- b**  $(15 - 4)(15 + 4) = 11 \times 19 = 209$        $a = \sqrt{209}$
- c**  $(20 - 14)(20 + 14) = 6 \times 34 = 204$        $a = \sqrt{204}$
- d**  $(20 - 14.5)(20 + 14.5) = 5.5 \times 34.5 = 189.75$        $a = \sqrt{189.75}$

### EXERCISE 16F

- 1 a**  $(2x + 5)(x + 1)$       **b**  $(3x + 4)(x + 2)$       **c**  $2(x - 5)(x + 9)$
- d**  $(2x + 5)(2x + 3)$       **e**  $(2x - 9)(2x - 3)$       **f**  $3(x - 7)(x + 5)$
- g**  $3(4x - 1)(x + 1)$       **h**  $(3x - 5)(x + 2)$       **i**  $2(x + 10)(x - 13)$
- j**  $(3x + 2)(x - 5)$
- 2**  $5x^2 - 13x + 6$  factorises into  $(5x - 3)(x - 2)$ , so the length is

$(5x - 3)$  cm.

**3**  $2x^2 + 11x + 12$  factorises into  $(2x + 3)(x + 4)$ .

$(x + 4)$  is half the base, so  $(2x + 3)$  is the height.

**4 a** Let  $a = (x + y)$ .

The expression becomes  $3a^2 + 13a + 12$ , which factorises into  $(3a + 4)(a + 3)$ .

Replacing  $a$  with  $(x + y)$ :  $3(x + y)^2 + 13(x + y) + 12 \equiv [3(x + y) + 4](x + y + 3)$ .

- b i**  $(3x - 1)(x - 6)$       **ii**  $(20x + 7)(10x + 1)$

### WORK IT OUT 16.2

In all cases, parts of the expressions only have been cancelled.

1: cannot be simplified

$$2: \frac{(x - 3)(x + 2)}{(x + 1)(x + 2)} = \frac{(x - 3)}{(x + 1)}$$

$$3: \frac{(x + 1)(x - 1)}{2(x + 2)} \times \frac{4(x + 2)(x - 2)}{(x + 1)} = 2(x - 1)(x - 2)$$

### EXERCISE 16G

- 1 a**  $\frac{4x}{5}$       **b** 3      **c**  $x - 3$       **d**  $2x + 9$
- e**  $\frac{x - 3}{x + 4}$       **f**  $\frac{x + 4}{4 - x}$       **g**  $\frac{x - 1}{x + 1}$
- 2** Option B
- 3 a**  $\frac{2x}{5}$       **b**  $\frac{5x}{12}$       **c**  $\frac{13}{6x}$       **d**  $\frac{3x + 5}{(x + 1)(x + 2)}$
- e**  $\frac{3x + 1}{(x - 2)(x - 1)}$       **f**  $\frac{15 - 2x}{(x - 7)^2}$
- 4 a**  $\frac{x^2 - x + 6}{(x - 2)(x + 2)}$       **b**  $\frac{2x + 5}{(x + 2)(x + 3)}$       **c**  $\frac{4x}{(2x - 3)(2x + 3)}$
- d**  $\frac{7p + 1}{p(2p + 1)}$       **e**  $\frac{2p^2 + 2p + 5}{(p - 1)(p + 2)}$       **f**  $\frac{3x - 7}{(x - 1)(x - 2)(x - 3)}$

### EXERCISE 16H

- 1 a**  $16b - 9$       **b** T      **c**  $(3x + 2)(x + 3)$
- d** T      **e** T      **f**  $\frac{(8x + 7)}{(x - 1)(x + 2)}$
- 2**  $x^2 - 25 = (x - 5)(x + 5)$  and  $(x - 5) \neq (x + 5)$
- 3**  $32x$
- 4**  $(-2x^2 - 13x - 15) = -(2x^2 + 13x + 15) = -(2x + 3)(x + 5)$ , so  $(x - 1)$  is not a factor
- 5**  $(2x - y)^2 - (x + y)(x - y) = (2x - y)^2 - (x^2 - y^2) = 3x^2 - 4xy + 2y^2$
- 6 a**  $2x^2 - x - 6$       **b**  $5x^2 + 8x + 13$       **c**  $3x^2 + 16x + 5$
- 7 a** 1.0201      **b** 0.9801      **c** 16.1604      **d** 0.9604
- 8 i**  $x^2 + 4x + 15 = (x + 2)^2 + 11 \geq 11$  as  $(x + 2)^2 \geq 0$
- ii**  $x^2 + 2x + 15 = (x + 1)^2 + 14 \geq 14$  as  $(x + 1)^2 \geq 0$
- 9** Triangle is right-angled if  $(x + 6)^2 + (x - 1)^2 = (x + 8)^2$   
Expanding and simplifying gives  $x^2 - 6x - 27 = 0$   
Factorising gives  $(x - 9)(x + 3) = 0$   
 $x > 1$ , so  $x = 9$  is the only solution
- 10 a** If perimeter = 20 cm and width is  $w$ , length is  $(20 - 2w)/2 = 10 - w$ , so area =  $w(10 - w)$
- b**  $10w - w^2 = -(w - 5)^2 + 25$   
As  $-(w - 5)^2 \leq 0$ , area  $\leq 25$

11  $(\sqrt{x} - 1/\sqrt{x})^2 \geq 0$

$(\sqrt{x} - 1/\sqrt{x})^2 = x - 2 + 1/x$

So  $x - 2 + 1/x \geq 0$  and  $x + 1/x \geq 2$

12 a  $\frac{x+2}{x-5}$       b  $\frac{3x+2}{4}$       c  $\frac{b^2(a+1)}{a^2(a-b)}$

**CHAPTER REVIEW**

1 a  $3y^2 + 12y + 14$

b  $x^3 - 7x - 6$

2 Option B

3 a  $(2x+3)(x-7)$

b  $-2(3x+4)(x+1)$

4 3997

5 Area =  $(x-5)(x+2) + 2(x-5) = 36$

$x^2 - 3x - 10 + 2x - 10 = 36$

$x^2 - x - 56 = 0$

6 a  $14x^7y^7z^7$       b  $\frac{2(x-5)}{(x+4)}$

c  $(x+1)(x+5)$       d  $2(x+5y)(x-5y)$

7 a  $\frac{7x-11}{12}$       b  $\frac{3x+4}{(x+4)(x-4)}$       c  $\frac{x^2}{6y}$

d  $\frac{3(5p-q)}{(4p+q)(p-2q)}$       e  $\frac{4-17x}{(x-2)(1-3x)}$       f  $\frac{3x-4}{(x+5)(x-5)}$

**17 Equations**

**BEFORE YOU START ...**

1 a D      b B      c A      d C

2 A:  $6x + 1 = 37$  represents the statement.

3 a  $7 + [-7] = 0$       b  $[8] - 8 = 0$

c  $-4a + [4a] = 0$       d  $5 \times [\frac{1}{5}] = 1$

e  $\frac{1}{6} \times [6] = 1$       f  $[\frac{1}{12}] \times 12x = x$

4 a C  $(x-2)(x-3)$       b A  $x(x+3)$

c B  $(x+5)(x-5)$       d D  $(+\sqrt{5})(-\sqrt{5})$

**LAUNCHPAD**

1 a D  $x = 12$       b B  $x = 17$       c A  $x = 1$

d E  $x = -3$       e C  $x = -2$

f You can check whether a solution is correct by substituting it back into the equation.

2 a  $a = 5$       b  $x = -3$       c  $a = 3$

d  $x = 1.97$  or  $x = -5.57$

3 Yes (In the second equation, taking  $x$  from both sides produces the first equation)

4  $2x + 16 = 44$        $x = 14$

5  $x = 3$  or  $x = -1$

6  $x = \pm 4$

7  $(x-3)^2 - 11 \equiv x^2 - 6x - 2$

8  $a = 1, b = -6, c = -2$

9 a  $x + y = 6$  has solutions:

$x = 1, y = 5; x = 2, y = 4; x = 3, y = 3; x = 4, y = 2; x = 5, y = 1$

b  $x = 5$  and  $y = 1$  are the only pair that satisfy the pair of equations simultaneously.

10 a £100      b  $x = 9$       c  $y = 115$

11 2 solutions; approximate solution is  $x = 4.56$

**EXERCISE 17A**

1 a  $3\frac{1}{2}$       b 2      c 4

d  $\frac{1}{2}$       e 1      f  $\frac{1}{3}$

2 a 12      b 2      c  $\frac{2}{15}$

d 1      e  $-\frac{13}{6}$       f  $\frac{15}{4}$

3 Option C

**EXERCISE 17B**

1 a  $\frac{9}{8}$       b  $-\frac{3}{5}$       c  $\frac{23}{5}$       d  $\frac{13}{2}$

e 7      f -5      g 12      h -2.8

i 2      j  $-\frac{184}{83}$

2 Option D

**EXERCISE 17C**

1 a  $3x = 348; x = 116$

b  $x - 7 = -2; x = 5$

c  $x + 6 = -4; x = -10$

d  $4x - 2 = 66; x = 17$

e  $x + x + 1 = 63; x = 31$ ; numbers are 31 and 32.

f  $2x - 3 = -2; x = \frac{1}{2}$

g  $2x + 1\frac{1}{2} = 4\frac{3}{4}; \frac{13}{8}$

2 a  $2x + 16 = 44$ ; Melissa is 14

b  $8x + 80 = 2000$ ; £2.40

c  $2x + 5 = 23 - x$ ; 6

d  $2(x+5) = (x+20) + 5$ ; Daughter is 15

e  $x + 4 + x = 22; x = 9$ ; Gina is 13

f  $3x + 4 = 2.5(x+4); x = 12$ ; woman is 36

3 a 12x cm      b  $(6x+18)$  cm

c  $6x+18 = 12x; x = 3$ ; side of square is 9 cm

4  $4x + 2 = 10x - 1; x = \frac{1}{2}$ ; length = 4 cm, width = 2.5 cm

5 A = 80°, B = 60°, C = 40°

6 37 and 46

7 70 each of apple and orange juice

8 43 articles at £5 and 58 articles at £2

9 2000 tickets at £80 and 8000 tickets at £60

**EXERCISE 17D**

1 Option C

2 a  $x = -9$  or  $x = -3$       b  $x = 6$  or  $x = -5$       c  $x = -\frac{5}{6}$  or  $x = 2$

d  $x = \frac{5}{9}$  or  $x = -1$       e  $x = 0$  or  $x = -3$       f  $x = 0$  or  $x = \frac{4}{5}$

g  $x = 10$  or  $x = -10$       h  $x = \pm\sqrt{5}$       i  $x = \pm\sqrt{6}$

3 No, it's a sum rather than a difference of two squares.

4 a  $x = 8$  or  $x = -4$       b  $x = \frac{4}{3}$  or  $x = -1$       c  $x = 8$  or  $x = 4$

d  $x = 2$  or  $x = -4$       e  $x = 5$  or  $x = -6$       f  $x = \frac{3}{2}$  or  $x = 4$

g  $x = 3$  or  $x = -\frac{1}{2}$       h  $x = -\frac{7}{6}$  or  $x = -\frac{5}{4}$

## EXERCISE 17E

- 1 a  $x^2 - 2x + 1, (x-1)^2$       b  $x^2 + 2x + 1, (x+1)^2$   
 c  $x^2 + 4x + 4, (x+2)^2$       d  $x^2 + 6x + 9, (x+3)^2$   
 e  $x^2 - \frac{2}{3}x + \frac{1}{9}, (x - \frac{1}{3})^2$       f  $x^2 - 2\sqrt{5}x + 5, (x - \sqrt{5})^2$   
 g  $x^2 + 10x + 25, (x+5)^2$       h  $x^2 - 2\sqrt{11}x + 11, (x - \sqrt{11})^2$   
 i  $x^2 + 2\sqrt{7}x + 7, (x + \sqrt{7})^2$       j  $x^2 - 10x + 25, (x-5)^2$

## 2 Option D

- 3 a  $x = 3.70$  or  $-2.70$       b  $x = 1.37$  or  $-4.37$   
 c  $x = 0.16$  or  $-6.16$       d  $x = 6.75$  or  $-7.25$   
 e  $x = 1.89$  or  $0.106$       f  $x = 5.37$  or  $-0.37$   
 g  $x = 1.30$  or  $-2.30$       h  $x = 5.45$  or  $0.55$   
 i  $x = 1.62$  or  $-1.62$

- 4 a  $x = 2.28$  or  $0.219$       b  $x = 0.631$  or  $0.227$   
 c  $x = 0.879$  or  $-0.379$       d  $x = 1.35$  or  $-2.95$   
 e  $x = -2.84$  or  $-9.16$       f  $x = 6.85$  or  $0.146$

- 5 a  $x = \frac{-5 \pm \sqrt{5}}{2}$       b  $x = -1 \pm \sqrt{5}$   
 c  $x = -6 \pm \sqrt{33}$       d  $x = \frac{-1 \pm \sqrt{22}}{3}$   
 e  $x = \frac{-3 \pm \sqrt{29}}{10}$       f  $x = \frac{3 \pm \sqrt{5}}{4}$

## EXERCISE 17F

- 1 a  $x(x+4) = 140; x = -14$  or  $x = 10$   
 b  $x(x-3) = 108; x = 12$  or  $x = -9$   
 c  $x^2 - 3x = 10; x = 5$  or  $x = -2$   
 d  $x(x+2) = 48; x = 6$ ; numbers are 6 and 8.  
 2  $x = 4$   
 3 10 cm  
 4 3 m  
 5  $x = 3$ ; dimensions of rectangles are 4 cm by 9 cm and 2 cm by 18 cm, giving an area of  $36 \text{ cm}^2$  for each rectangle  
 6 160 m and 90 m

## EXERCISE 17G

- 1 a  $x = -3, y = -5$       b  $x = -\frac{1}{2}, y = 5$       c  $x = 1, y = 2$   
 d  $x = 4, y = 2$       e  $x = 2, y = 5$       f  $x = -1, y = 3$

## 2 Option D

- 3 a  $x = 2, y = 1$       b  $x = 2, y = 1$       c  $x = \frac{21}{5}, y = \frac{13}{5}$   
 4 a  $x = 3, y = 4$       b  $x = 1, y = 2$       c  $x = 3, y = 4$   
 d  $x = 7, y = -4$       e  $x = -\frac{11}{3}, y = 17$       f  $x = -2, y = 4$

## EXERCISE 17H

- 1 a  $x = 4, y = 2$       b  $x = 3, y = 1$       c  $x = 2, y = -2$   
 d  $x = \frac{26}{7}, y = -\frac{39}{7}$       e  $x = \frac{28}{5}, y = \frac{21}{5}$       f  $x = -1, y = -2$   
 g  $x = 2, y = 3$       h  $x = 1, y = 3$       i  $x = 4, y = 1$   
 2 a  $x = 3, y = -1$       b  $x = 3, y = -1$       c  $x = 2, y = 1$   
 3 a  $x = 2, y = 3$       b  $x = 1, y = 1$       c  $x = 10, y = 5$   
 d  $x = -5, y = -2$       e  $x = -2, y = 5$       f  $x = 2, y = -1$

- 4 a  $x = 5, y = 0$       b  $x = 3, y = 1$       c  $x = -2, y = 1$   
 d  $x = \frac{45}{17}, y = \frac{38}{17}$       e  $x = 3, y = -1$       f  $x = -1, y = 1$

## EXERCISE 17I

- 1 Fizzers cost 20p; toffees cost 30p  
 2 Option B  
 3 Three 5p pieces and fifteen 10p pieces  
 4 45 and 219  
 5  $a = 70, b = 50$   
 6 62 and 14  
 7 £6.20  
 8 hard drive £25, flash £10  
 9  $12 \times 400$  and  $36 \times 450$

## EXERCISE 17J

- 1 a  $x = 1, y = 1$       b  $x = 2, y = 4$  or  $x = -1, y = 1$       c no solution  
 d The two lines will not intersect so there is no solution to the pair of equations.  
 2 a  $(-1, 1)$       b  $(1, 8)$  or  $(-5, 2)$       c  $(1, 7)$  or  $(-2, 4)$   
 d  $(0, 1)$  or  $(-\frac{1}{2}, 0)$       e  $(1, 6)$  or  $(-\frac{1}{3}, 2)$       f  $(-\frac{2}{3}, \frac{5}{3})$  or  $(-\frac{1}{2}, 2)$   
 3 Option C  
 4  $(1, 4)$  and  $(4, 1)$

## EXERCISE 17K

- 1 a 5 miles      b 48 minutes      c 10 mph  
 2 a  $-2$       b 1      c 4  
 3 a  $\sim 46$  minutes      b 30 km      c 90 km/hour  
 4 a 2000 litres      b 100 minutes  
 c Students' chosen points and explanations  
 5 a When 500 units have been sold (Costs = revenue = £6000).  
 b It tells the business owner how many units must be sold in order to make a profit.  
 6  $x = 2, y = 4$   
 7 a  $x$ -axis is distance along the ground;  $y$ -axis is height  
 b  $x = 0$  and  $x = 11.5$   
 c They are the roots  
 d  $\approx (6.25, 5.1)$   
 8 a The roots of the equation are the values where the graph crosses the  $x$ -axis.  
 b  $x = 2$  and  $x = 6$   
 c  $y = -\frac{1}{2}x^2 + 4x - 6$   
 9 a Approx  $x = 3.2, y = 4.1$   
 b  $x = 3.25, y = 4.125$   
 c The accuracy is limited by how accurate the graph is and how well the values can be read from the scale.  
 10 a  $\approx 108$   
 b  $\approx 85$   
 c 140 cm  
 11 a 9.1 kg      b 0.057 m      c 0.10 to 0.11 m

**EXERCISE 17L**

- 1  $x \approx 4.562$   
 2  $x \approx 4.243$   
 3  $x \approx 2.56$   
 4  $x \approx 0.3473$   
 5  $x \approx 2.29$

**EXERCISE 17M**

- 1  $x \approx 0.453$   
 2 a  $x \approx 2.73$     b  $x \approx -0.7$

**EXERCISE 17N**

- 1 a True  
 b False;  $x = \frac{1}{2}$  and  $x = 2$   
 c True  
 d False;  $x = -\frac{63}{19}$ ,  $y = \frac{30}{19}$   
 e False; one solution has a negative value.
- 2 a  $x = 16$     b  $x = -9$
- 3 a  $(x - 5)$  m    b length = 17.5 m, width = 12.5 m
- 4 Daughter is 8, father is 36
- 5 5
- 6 a Area =  $(2y + 1)y + 5y = 2y^2 + 6y$   
 $2y^2 + 6y = 95$  so  $2y^2 + 6y - 95 = 0$   
 b  $y = 5.55$
- 7 a  $x = 4.45$  or  $x = -0.45$     b  $n = 5.70$  or  $n = -0.70$
- 8 a students' working    b  $x = 1.22$  or  $x = -3.55$
- 9  $x = 5$
- 10 a  $a = \frac{4}{3}$     b  $a < \frac{4}{3}$     c  $a > \frac{4}{3}$
- 11  $x + y = 112$  and  $x - y = 22$   
 $x = 67$  and  $y = 45$
- 12 J = 7.5 kg, S = 5 kg
- 13  $x = 3$ ,  $y = 7$
- 14 a  $(-2, 0)$  and  $(5, 23)$   
 b  $x = 5.275$  and  $y = 22.825$  or  $x = -2.275$  and  $y = 0.175$
- 15 a 0.44    b 2.40    c -2.8

**CHAPTER REVIEW**

- 1 a  $x = 7$  or  $x = -3$     b  $x = -2$  or  $x = -6$     c  $x = -4$  or  $x = -5$   
 d  $x = -5 \pm \sqrt{30}$     e  $x = 1 \pm \frac{\sqrt{6}}{2}$     f  $x = \frac{-1 \pm \sqrt{10}}{3}$
- 2 a 2 seconds    b when  $t = 3$
- 3 a 10 and 60    b  $y = -20x^2 + 14000x - 12000$   
 c Increasing the selling price increases the profit until the price is 35, but increasing it further decreases the profit.
- 4 No; one equation is a multiple of the other.
- 5  $x = 1.16$  and  $y = 5.16$   
 $x = -2.16$  and  $y = 1.84$
- 6  $x = 1.28$  and  $y = 4.83$   
 $x = -1.88$  and  $y = -4.63$

$$\begin{aligned} 7 \quad (x+1)^2 - x^2 &= x^2 + 2x + 1 - x^2 \\ &= 2x + 1 \\ &= x + (x + 1) \end{aligned}$$

8 -2 or 8

9 a Students' working

b  $x = 0.826$  or  $x = -2.826$ 

10 a 2.41

b Students' substitutions

**18 Functions and sequences****BEFORE YOU START ...**

1 a 7, 14, 21, 28, 35

b 66, 18, 54, 36

2 a 1, 16, 25, 4, 9, 49

b 9, 15

3 a Pattern is built up using square and triangle shapes with some common sides. Each new shape has three extra matches to make the sides of the next square and two extra matches to make the sides of the triangle. Rule is  $5n + 1$ .

b 31 matchsticks

**LAUNCHPAD**

1 a 59, 71, 83    b add 12 to the previous term

2 a  $T(10) = 29$ ,  $T(20) = 59$ ,  $T(100) = 299$ b  $3n - 4$ 3 input  $\rightarrow$   $[\times 2]$   $\rightarrow$   $[-4]$   $\rightarrow$  output

4 4, 5, 6, 7, 8, ...

5  $\frac{2}{3}(x - 1)$ 6 a  $g(x) = 4x$     b  $f(x)$ 

7 It is a quadratic sequence. The differences between terms increases by 1 each time. (The difference between the first and second terms is 2, the difference between the second and third terms is 3 etc.) The second differences between terms are equal.

**EXERCISE 18A**

1 a 16, 19, 22; add 3

b 58, 63, 68; add 5

c 16, 32, 64; multiply by 2

d 8, 4, 2; divide by 2

e 108, 324, 972; multiply by 3

f 27, 9, 3; divide by 3

2 Option D

3 a 16

b add 4

4 a add 7

b subtract 4

c multiply by 4

d divide by 2

5 a add 2; 9.5, 11.5, 13.5

b multiply by 2; 9.6, 19.2, 38.4

c add  $1\frac{1}{2}$ ; 6,  $7\frac{1}{2}$ , 9

d subtract 3; -1, -4, -7

e divide by 2; 9, 4.5, 2.25

f add 3; -1, 2, 5

6 a 6 cm

b 7th bounce will be 0.75 cm

7 a  $\frac{1}{2}$ b  $1\frac{1}{3}$ 

c any negative value

**WORK IT OUT 18.1**

Option B is correct.

Option A is wrong because this sequence is defined as  $3n - 5$ .Option C is wrong because this sequence is defined as  $2n - 3$ .



### EXERCISE 18B

1 a 16th                      b 40                      c  $2n$

2 Option B

3 a 2, 5, 8, 11, 14, 17                      b 59

c no; 40th term is 119, which is not  $2 \times 59$

Position-to-term rule	1st term	2nd term	3rd term	4th term	10th term	20th term	100th term
$4n + 1$	5	9	13	17	41	81	401
$4n - 5$	-1	3	7	11	35	75	395
$8n + 2$	10	18	26	34	82	162	802
$5n - \frac{1}{2}$	$4\frac{1}{2}$	$9\frac{1}{2}$	$14\frac{1}{2}$	$19\frac{1}{2}$	$49\frac{1}{2}$	$99\frac{1}{2}$	$499\frac{1}{2}$
$\frac{n}{2} + 1$	$1\frac{1}{2}$	2	$2\frac{1}{2}$	3	6	11	51
$-2n + 1$	-1	-3	-5	-7	-19	-39	-199

5 Option D

6  $4n + 1$

7 a  $2n + 1$                       b  $4n - 1$                       c  $5n - 6$

d  $5n + 2$                       e  $3n - 6$                       f  $7n - 8$

8 Option C

9  $6n + 61$

10 a  $2.2n + 2.3$                       b 222.3 cm

c Sunflowers do not continue to grow at a constant rate. Over 2 m is unusual for a sunflower; 100 weeks is nearly two years and sunflowers are unlikely to live this long.

11 a £308                      b 75 weeks

12 a 14                      b 22

c no, because any number in the sequence  $2n + 2$  must be even

### EXERCISE 18C

1 4, 5, 6, 7, 8, 9, 10, 11, 12, 13

2 Option B

3 a -4, -3, -2, -1, 0, 1, 2, 3, 4, 5

b 3, 6, 9, 12, 15, 18, 21, 24, 27, 30

c 8, 9, 10, 11, 12, 13, 14, 15, 16, 17

d  $\frac{1}{2}, 1, 1\frac{1}{2}, 2, 2\frac{1}{2}, 3, 3\frac{1}{2}, 4, 4\frac{1}{2}, 5$

4 a 42, 44, 46, 48, 50, 52, 54, 56, 58, 60

b 13, 14, 15, 16, 17, 18, 19, 20, 21, 22

c  $7, 7\frac{1}{3}, 7\frac{2}{3}, 8, 8\frac{1}{3}, 8\frac{2}{3}, 9, 9\frac{1}{3}, 9\frac{2}{3}, 10$

d  $21\frac{1}{2}, 22\frac{1}{2}, 23\frac{1}{2}, 24\frac{1}{2}, 25\frac{1}{2}, 26\frac{1}{2}, 27\frac{1}{2}, 28\frac{1}{2}, 29\frac{1}{2}, 30\frac{1}{2}$

5 a 38, 41, 44, 47, 50, 53, 56, 59, 62, 65

b 15, 17, 19, 21, 23, 25, 27, 29, 31, 33

c  $9\frac{1}{2}, 10, 10\frac{1}{2}, 11, 11\frac{1}{2}, 12, 12\frac{1}{2}, 13, 13\frac{1}{2}, 14$

d  $44\frac{1}{2}, 48\frac{1}{2}, 52\frac{1}{2}, 56\frac{1}{2}, 60\frac{1}{2}, 64\frac{1}{2}, 68\frac{1}{2}, 72\frac{1}{2}, 76\frac{1}{2}, 80\frac{1}{2}$

e  $\frac{48}{11}, \frac{13}{3}, \frac{56}{13}, \frac{30}{7}, \frac{64}{15}, \frac{17}{4}, \frac{72}{17}, \frac{38}{9}, \frac{80}{19}, \frac{21}{5}$

6 Option A

7 a  $x \rightarrow x + 7$

b  $x \rightarrow \frac{x}{4}$

c  $x \rightarrow x - 5$

d  $x \rightarrow 3x$

e  $x \rightarrow \frac{1}{2}(x - 4)$

f  $x \rightarrow \frac{1}{4}(x + 5)$

g  $x \rightarrow 5(x - 3)$

h  $x \rightarrow \frac{4}{x+2}$

8 a  $y = 4(x - 7)$    b  $y = 2(x + 4)$    c  $y = 3(x - 2)$    d  $y = (x + 1)^2$

9 a  $x \rightarrow \frac{x}{2}$

b function 1:  $2x$ , function 2:  $\frac{x}{2}$ , composite function  $y = x$

### EXERCISE 18D

1 a 1, 4, 9, 16, 25, 36, 49, 64, 81, 100

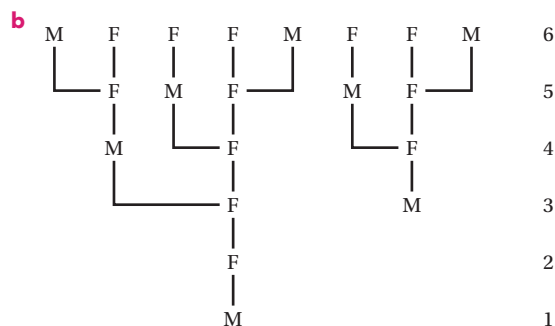
b Differences between terms are 1, 2, 5, 7, 9, 11, 13, 15, 17, 19 so add 21 onto 100 (121), then 23 onto 121 (144)

2 a Triangular   b 1, 3, 6, 10, 15, 21, 28, 36, 45, 55

c First differences: 2, 3, 4, 5, 6, 7, 8, 9, 10; second differences: 1, 1, 1, 1, 1, 1, 1, 1, 1

d Quadratic

3 a 5 (3F 2M)



i (1,) 1, 2, 3, 5, 8

ii 34

iii Add the previous two terms to get the next in the sequence.

iv This is the Fibonacci sequence.

4 Option C

5 a 3, 4, 7, 11, 18, 29, 47, 76, 123, 199

b -2, 3, 1, 4, 5, 9, 14, 23, 37, 60

c i 2, -3, -1, -4, -5, -9, -14, -23, -37, -60

ii The sequence will be the same as in part b but with each positive number being negative and vice versa

6 2, 5

7 a -27, 69                      b 36, -60                      c 1, 5

Position-to-term rule	1st term	2nd term	3rd term	5th term	10th term	20th term	50th term
$n^2 + 5$	6	9	14	30	105	405	2505
$n^2 - 3$	-2	1	6	22	97	397	2497
$2n^2 + 1$	3	9	19	51	201	801	5001
$2n^2 - 7$	-5	1	11	43	193	793	4993

- 9 2, 9, 28, 65, 126, 217  
 10 2, 6, 12, 20, 30, 42, 56, 72, 90, 110  
 11 a  $\frac{1}{2}(n^2 + n)$   
 b 10th term is 55, 25th term is 325

### EXERCISE 18E

- 1 Option D  
 2 a  $n^2 + 2n$       b  $2n^2 + n$       c  $4n^2 + 3n$   
 d  $2n^2 + 5n - 1$       e  $3n^2 - 2n$       f  $4n^2 - 3n - 4$   
 g  $-2n^2$       h  $-2n^2 + 2n$   
 3 4,  $4\sqrt{2}$ , 8  
 4 a  $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}$       b  $\frac{10}{11}$   
 5 a 0, 5, 12, 21, 32      b 252  
 6 a 5, 7, 11, 17, 25      b 35      c  $u_n = n^2 - n + 5$

### CHAPTER REVIEW

- 1 Option D  
 2 a 1 10 45 120 210 252 210 120 45 10 1  
 b i 1, 2, 4, 8, 16, 32, 64, 128, 256, 512, 1024  
 ii Powers of two (or doubles each time)  
 c i Triangular numbers  
 ii  $\frac{1}{2}(n^2 + n)$   
 3 a 28      b  $5n + 3$       c 25 days  
 4 a 339 ( $u_n = 2n^2 + 4n + 3$ )  
 b The assumption that no rabbits will die during the year is unrealistic.  
 5 4383 ( $u_n = 12n^2 - 22n + 23$ )  
 6 a 4, 6, 8, 10      b 12      c  $n \rightarrow \times 2 \rightarrow + 2$   
 d 42, 52,  $2n + 2$   
 7 a  $28 + \frac{b}{2}$       b 36 days

## 19 Surds

### BEFORE YOU START ...

- 1 a C      b B      c A  
 2 a  $a^2 = b^2 + c^2$       b  $x^2 + y^2 = z^2$   
 3 a False.  $3x$       b False. Answer is 1, because  $y^0 = 1$ , not  $y$ .  
 4 a Answer should be  $2x + 3y$ ;  $y$  terms means 1 $y$ .  
 b Answer should be  $2x^2 + 2x$ ;  $x^2$  and  $x$  are unlike terms so you cannot add them.  
 c Answer should be  $4x - y - 2$ ;  $-2y + y$  gives a negative coefficient for  $y$ .

### LAUNCHPAD

- 1 a 3.61 cm      b  $\sqrt{13}$  cm  
 c 13.0321 cm and 13 cm;  $\sqrt{13}$ , the exact length of the square, is irrational, so any decimal value of this number will be an approximation and when squared will only give an approximation of the area of the square.

- 2 a Draw a line of 5 cm.  
 b Construct a right-angled triangle with one side equal to  $\sqrt{10}$ . (An example would be the two shorter sides of 1 cm and 3 cm.)  
 3 a 3      b 4      c  $2\sqrt{3}$   
 d  $3\sqrt{3}$       e  $\sqrt{3}$   
 4 a  $\frac{2\sqrt{5}}{5}$       b  $\frac{5\sqrt{3}}{6}$

### EXERCISE 19A

- 1 Option B  
 2 a 2.646      b 3.464      c 7.141  
 d 8.660      e  $-1.732$       f  $-6.856$   
 3 a 2.828      b 6.708      c  $-10.392$   
 d 14.142      e 13.856      f  $-25.456$   
 4 a 3.146      b 1.414      c 2.236  
 d 2.449      e 10.172      f 3.244

### EXERCISE 19B

- 1 a  $\sqrt{14}$  cm      b  $\sqrt{20}$  m      c  $\sqrt{17}$  cm  
 2 Option C  
 3 Option A  
 4  $C = 2\pi\sqrt{3}$  cm  
 5 27  
 6  $\sqrt{7}$   
 7 a  $\sqrt{50}$  m      b 10 m  
 8 a  $\sqrt{8}$  cm      b  $\sqrt{8} \times \sqrt{8} = 8$  cm<sup>2</sup>  
 c i 2.83 cm      ii 2.828 cm      iii 2.8284 cm  
 d i 8.0089 cm<sup>2</sup>      ii 7.997 584 cm<sup>2</sup>      iii 7.999 846 56 cm<sup>2</sup>  
 e Exact area:  $8 \times 100 \times 1245 = \text{£}996\,000$   
 Area i:  $8.0089 \times 100 \times 1245 = \text{£}997\,108.05$   
 Area ii:  $7.997\,584 \times 100 \times 1245 = \text{£}995\,699.21$   
 Area iii:  $7.999\,846\,56 \times 100 \times 1245 = \text{£}995\,980.90$   
 f Nico would get the most profit if he used the value rounded to 2 decimal places as this gives the highest price for the metal.

### EXERCISE 19C

- 1 a C      b B      c B      d C      e E  
 2 a  $2\sqrt{2}$       b  $2\sqrt{6}$       c  $2\sqrt{7}$       d  $3\sqrt{5}$   
 e  $3\sqrt{6}$       f  $2\sqrt{17}$       g  $2\sqrt{15}$       h  $3\sqrt{14}$   
 i  $3\sqrt{10}$       j  $10\sqrt{2}$       k  $3\sqrt{13}$       l  $9\sqrt{3}$   
 3 They are all prime numbers, so they do not have factors that are squares.  
 4 a  $6\sqrt{2}$       b  $-8\sqrt{6}$       c  $10\sqrt{5}$   
 d  $-10\sqrt{15}$       e  $6\sqrt{14}$       f  $-16\sqrt{2}$   
 g  $-12\sqrt{5}$       h  $-6\sqrt{17}$       i  $42\sqrt{3}$   
 5 a i  $\sqrt{4} \times \sqrt{7} = \sqrt{4 \times 7} = \sqrt{28}$   
 ii  $-\sqrt{9} \times \sqrt{6} = -\sqrt{9 \times 6} = -\sqrt{54}$   
 b Cannot have a square root of a negative number

- 6 a  $\sqrt{18}$       b  $\sqrt{48}$       c  $\sqrt{54}$   
 d  $\sqrt{176}$       e  $-\sqrt{28}$       f  $-\sqrt{27}$   
 g  $-\sqrt{272}$       h  $-\sqrt{44}$       i  $\sqrt{432}$

7 a Students' own descriptions.

- b i  $2\sqrt{3}, 3\sqrt{3}, 4\sqrt{2}$       ii  $5\sqrt{7}, 8\sqrt{3}, 6\sqrt{7}$   
 iii  $2\sqrt{10}, 4\sqrt{3}, 3\sqrt{7}$       iv  $6\sqrt{3}, 8\sqrt{2}, 5\sqrt{6}$

### WORK IT OUT 19.1

In 2, student has incorrectly added  $\sqrt{3}$  to  $\sqrt{3}$  to get  $\sqrt{6}$ : Correct answer is option C

In 5, student has added  $\sqrt{27}$  to  $\sqrt{20}$  to get  $\sqrt{47}$ : Correct answer is option B

### EXERCISE 19D

1 Any examples using squares will show this.  $\sqrt{4} + \sqrt{9} = 2 + 3 = 5$  and  $\sqrt{13} \approx 3.605$ , so the expressions are not equal.

2 Option C

- 3 a  $6\sqrt{4} + 3\sqrt{7}$       b  $4\sqrt{2} + 2\sqrt{5}$       c  $4\sqrt{5} + 8\sqrt{3}$   
 d  $2\sqrt{2} + 5\sqrt{3}$       e  $6\sqrt{5} + 3\sqrt{2}$       f  $\sqrt{2} - 2\sqrt{3}$   
 4 a  $3\sqrt{2}$       b  $\sqrt{7}$       c  $5\sqrt{6}$   
 d  $\sqrt{5}$       e  $\sqrt{7}$       f  $8\sqrt{5}$   
 5 a  $6\sqrt{3}$       b  $6\sqrt{10} - 6\sqrt{11}$       c  $10\sqrt{3} + \sqrt{5}$   
 d  $\sqrt{3} + 4\sqrt{5}$       e  $9\sqrt{6}$       f  $46\sqrt{2} - \sqrt{6}$   
 6  $9\sqrt{3} - 3\text{ cm}$

### EXERCISE 19E

- 1 a  $\sqrt{21}$       b  $\sqrt{15}$       c 6      d  $6\sqrt{35}$   
 e  $-12\sqrt{33}$       f  $18\sqrt{5}$       g 78      h  $30\sqrt{2}$   
 i  $20\sqrt{6}$       j  $18\sqrt{6}$       k  $12\sqrt{15}$       l  $24\sqrt{30}$   
 2 a  $\sqrt{7}$       b  $\sqrt{2}$       c  $\sqrt{\frac{1}{2}}$       d  $\sqrt{\frac{1}{10}}$   
 e 3      f  $\sqrt{7}$       g  $\frac{\sqrt{6}}{2}$       h  $8\sqrt{3}$   
 i  $6\sqrt{11}$       j 6      k -6      l -3

3 Option D

4 Option B

- 5 a  $9\sqrt{2}$       b  $8\sqrt{3}$       c  $\sqrt{2}$   
 d 1      e  $-5\sqrt{2}$       f  $\frac{-3\sqrt{3}}{4}$   
 6 a  $\sqrt{15} + 2\sqrt{5}$       b  $10\sqrt{3} - 6$       c  $-\sqrt{6}$   
 d  $27 + 4\sqrt{35}$       e  $\sqrt{6} + 5\sqrt{2} + 3\sqrt{3} + 15$   
 f  $3\sqrt{5} - 7$       g 46      h  $9 + 2\sqrt{14}$   
 i  $8 - 2\sqrt{15}$   
 7 a  $\frac{5\sqrt{3}}{3}$       b  $\frac{\sqrt{5}}{5}$       c  $\frac{-2\sqrt{3}}{3}$   
 d  $\frac{\sqrt{6}}{3}$       e  $2\sqrt{6}$       f  $\frac{-3\sqrt{7}}{28}$   
 g  $\frac{2\sqrt{3} + 3}{6}$       h  $\frac{2\sqrt{5} + 5}{5}$       i  $\frac{2 - \sqrt{2}}{10}$

### EXERCISE 19F

- 1 a  $A = 18 + 72\sqrt{2}$        $P = 20\sqrt{2} + 16$   
 b  $A = 11 - 6\sqrt{2}$        $P = 12 - 4\sqrt{2}$

- c  $A = \sqrt{6} + \frac{5}{2}$        $P = 2\sqrt{2} + 2\sqrt{3} + \sqrt{10 + 4(\sqrt{6})}$   
 d  $A = \pi(19 - 6\sqrt{10})$        $P = 2\pi(\sqrt{10} - 3)$   
 e  $A = 4$        $P = 2\sqrt{11} + 2\sqrt{7}$

2 Option B

3 Option B

4  $31\text{ cm}^2$

5  $\sqrt{12}\text{ cm}$

6 a  $x = 210\text{ mm}, y = 420\text{ mm}, z = 594\text{ mm}$

b i  $10\,000\text{ cm}^2$       ii  $1457\text{ mm}$

c  $z = \sqrt{(x^2 + 2x^2)} = x\sqrt{3}$

7  $552 + 96\sqrt{15}\text{ cm}^2$

8  $\sin A = \frac{1}{\sqrt{2}}$

9  $(x + 1)\sqrt{x}$  for any non-negative value of  $x$ .

10 a  $72\text{ cm}^2$       b  $12\text{ cm}$

11  $20\sqrt{6}\text{ cm}$

### CHAPTER REVIEW

Correct answers are:

- 1 B      2 A      5 A      6 A  
 7 B      9 A

Both answers for 3, 4 and 8 are incorrect. They should be:

3  $4\sqrt{5} - \sqrt{3}$

4  $\sqrt{14} + 2\sqrt{2}$

8  $\frac{\sqrt{5} + 5}{5}$

10 a  $\sqrt{75} = \sqrt{25} \times \sqrt{3} = 5\sqrt{3}$

b  $2\sqrt{3}$

c  $4\sqrt{3}$

11 Option B

## 20 Basic probability

### BEFORE YOU START ...

1 a option B: 0.0312      b option C: 1      c option C: 0.04

2 a is correct; b and c are incorrect. The answer in b should be rounded up to 0.317, and c should be 1.0 (include the point zero to show the answer is rounded).

3  $\frac{39}{52} = 75\%$        $0.25 = \frac{13}{52}$        $0.077 = \frac{4}{52}$        $50\% = \frac{26}{52}$

### LAUNCHPAD

1 a 50      b  $\frac{113}{300}$   
 c 245 based on the experimental data, 108 based on the theoretical probability.

2 a  $\frac{1}{36}$       b  $7, \frac{1}{6}$       c  $\frac{1}{2}$       d  $\frac{1}{6}$

### EXERCISE 20A

1 a  $0.53$  or  $\frac{24}{45}$       b  $\frac{9}{50} = 0.18$       c  $\frac{9}{20} = 0.45$

2 a  $\frac{7}{40} = 0.175$       b  $\frac{33}{40} = 0.825$       c  $\frac{1}{20} = 0.05$       d  $\frac{21}{80} = 0.2625$

3 a 400

Brand	Frequency	Relative frequency
Silk-e-shine	123	$\frac{123}{400} = 0.3075$
Get knotted	105	$\frac{21}{80} = 0.2625$
Goldilocks	83	$\frac{83}{400} = 0.2075$
Bubbly stuff	89	$\frac{89}{400} = 0.2225$
Total	400	

c 0.3075

Result	Frequency	Relative frequency
spoke to customer	122	0.61
phone not answered	44	0.22
left message on answering machine	22	0.11
phone engaged or out of order	10	0.05
wrong number	2	0.01

b i Likely ii Unlikely iii Highly unlikely

5 a 479 b 0.40 c 0.64

### EXERCISE 20B

1 a Option B b Option B c Option D

2  $\frac{17}{36}$

3 0.668

4 a 0.1847 b 0.246 c 0.3337 d 0.754

5 Vijay, there are more even number products than odd.

6 a No, this is not an equally likely outcome. In English for example, few names start with X. (In China the outcomes might be different!)

b No, the chance of a boy/girl remain the same each time a woman gives birth, but they are also not equally likely outcomes.

c Technically yes, but realistically no, as skill and experience and ranking in the league will all affect the outcomes.

d This could be correct, but unlikely. People are more likely to take holidays in the summer months (which includes August) than in other months.

e No, will depend on who they are playing, which players are playing and so on.

### WORK IT OUT 20.1

Option B is correct

Option A is wrong because the fractions have not been converted to percentages correctly.

### EXERCISE 20C

1 0.74 (2 dp)

2 Option A

3 a  $\frac{1}{2}$  b 0.91 c 0.12 d 0.88

4 0.568

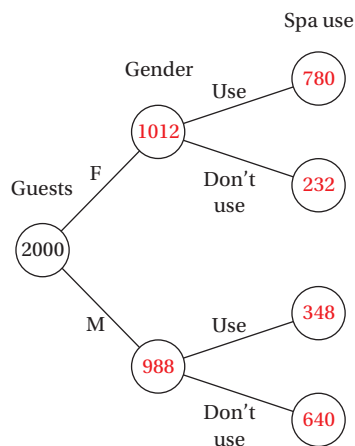
5 a 0.16 b 0.84 c 0.6

d strawberry 63, lime 66, lemon 54, blackberry 69, apple 48

6 a 0.4 b 0.97 c 11 d 114

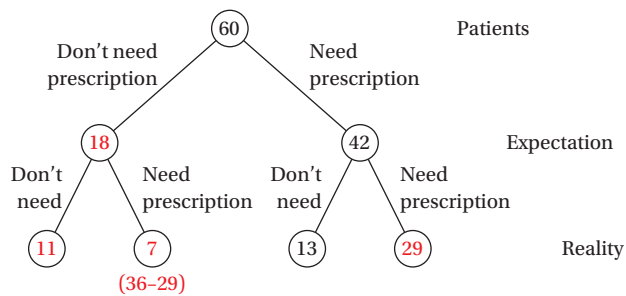
### EXERCISE 20D

1 a



b Female

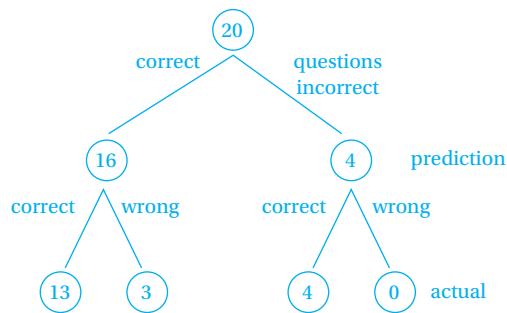
2 a



b  $\frac{29}{42}$

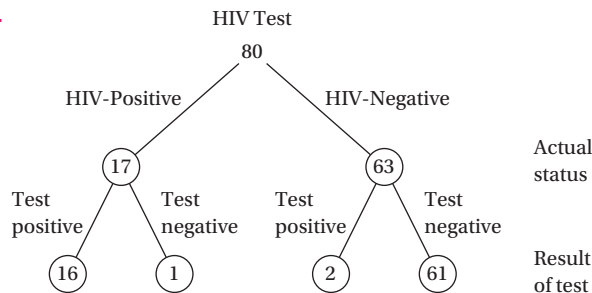
c 38.9%

3 a



b He predicted quite poorly as all the ones he thought he got wrong were actually correct.

4

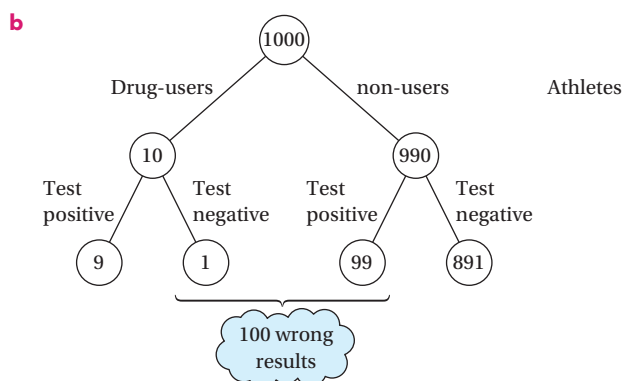


### EXERCISE 20E

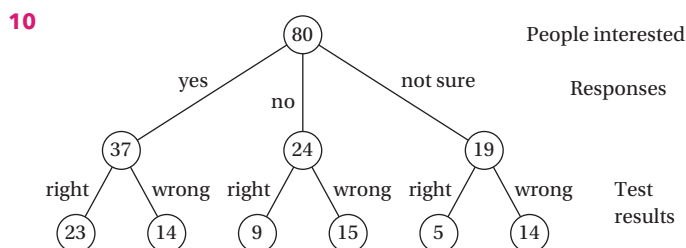
- 1** Busi is using theoretical probability to argue for an equal number of heads and tails. This is technically correct, but the reality is that each time you toss the coin there is an equal chance of getting heads or tails. 20 tosses is too small a trial to decide that the coin is biased.
- 2 a**  $\frac{13}{30}$
- b** It has been sunny for 7 tournaments and Grey College has won  $\frac{4}{7}$  (57%). When it has been cloudy Grey has won  $\frac{3}{10}$  (30%) and when it has been rainy, Grey has won  $\frac{3}{13}$  (23%).  
So it appears they have a better chance of winning when it is sunny.
- c** They are not correct. St George's has an experimental probability of  $\frac{1}{2} = 0.5$  of winning if it is cloudy, of  $\frac{6}{13} \approx 0.46$  of winning if it is rainy but only of  $\frac{2}{7} \approx 0.29$  of winning if it is sunny, so they have about 50% chance of winning unless it is sunny.
- d**  $\frac{1}{5}$ . Grey College has drawn one out of every five tournaments played.
- 3** Option C
- 4** Option C
- 5 a** 0.005
- b** Students' answers should acknowledge that  $\frac{1}{2}\%$  is a very small margin of error and most tests would be correct. However, parents are correct in claiming that 4 students could incorrectly test positive. The school could agree to retest anyone who gets a positive result for greater accuracy.
- c** 19 160

**6 a**

Status	Test positive (ie fail drug test)	Test negative (ie pass drug test)	Total
athletes who are using illegal substances	9	1	10
athletes who are not using illegal substances	99	891	990
Total	108	892	1000



- c** 91.7% (3sf)
- d** No. Diagrams show there are 100 wrong results, so the test is only 90% accurate. There is also a 1 in 1000 chance that a drug user will test negative.
- 7** Students' own opinions, but it seems to be biased in favour of red.
- 8 a** 212      **b** 89
- c** i 0.29      ii 0.63      iii 0.27
- d** Students' own ideas.
- 9 a** 42%      **b** 0.042      **c** 35      **d** 0.042 or 4.2%
- e** Outcomes are not equally likely.



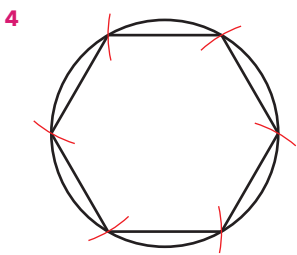
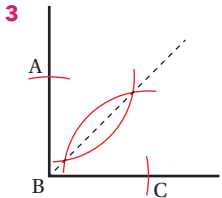
### CHAPTER REVIEW

- 1** Option B
- 2 a** 10 000 times      **b** Heads: 0.4083; Tails: 0.5917
- c** 0.41
- d** She probably is, as this is a large number of coin tosses and tails comes up 59% of the time.
- 3 a**  $\frac{1}{2}$       **b**  $\frac{9}{10}$       **c**  $\frac{9}{10}$       **d**  $\frac{1}{2}$
- 4** 5750
- 5** 97.35%
- 6**
- 
- 7 a** 0.05      **b** 138
- 8 a** Hearts 0.238, Diamonds 0.24, Spades 0.264, Clubs 0.258
- b** The relative frequencies are all quite close to 25% (theoretical frequency) so it may be fair and unbiased, but you would only be able to tell with a larger number of trials.

## 21 3D objects

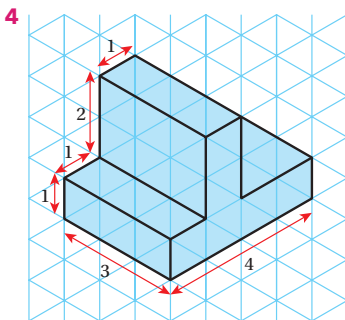
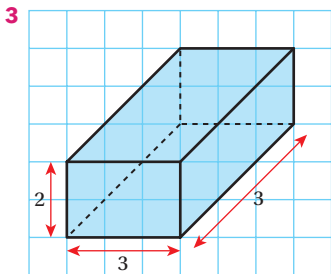
### BEFORE YOU START ...

- 1 a Square-based pyramid      b Cuboid  
 c Cube      d Triangular prism
- 2 a False; a cube has 6 faces  
 b True  
 c True  
 d False;  $F + V = E + 2$

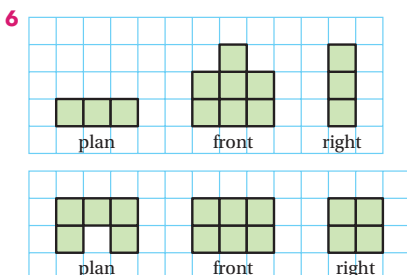


### LAUNCHPAD

- 1 a Cube      b Triangular prism  
 c Triangular-based pyramid
- 2 Net of cuboid 3 cm long, 2 cm wide and 1 cm high

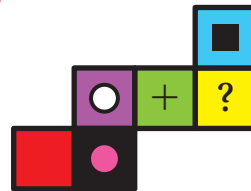


5 C

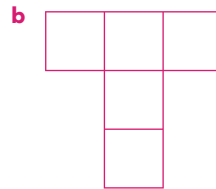


### EXERCISE 21A

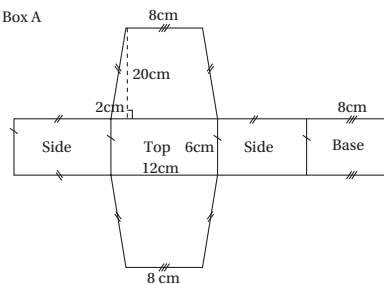
- 1 a polyhedron; triangular prism; 5 faces; 9 edges; 6 vertices  
 b not a polyhedron; cone;  
 c polyhedron; pentagonal prism; 7 faces; 15 edges; 10 vertices  
 d not a polyhedron; hemisphere
- 2 B
- 3 C
- 4 D
- 5 a Triangular-based pyramid      b Triangular prism
- 6



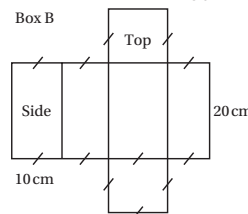
7 a B; D; F; J



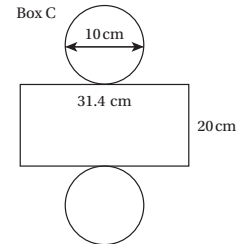
8 a Box A



Box B



Box C



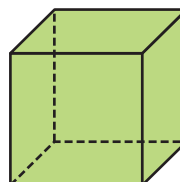
- b Box A 761.2 cm<sup>2</sup>  
 Box B 1000 cm<sup>2</sup>  
 Box C 785 cm<sup>2</sup>
- c Students' discussion

### WORK IT OUT 21.1

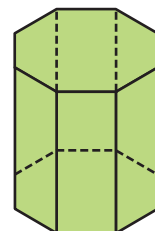
Student A is likely to end up with the correct drawing. The other two students have extended the horizontal part of the shape in wrong directions.

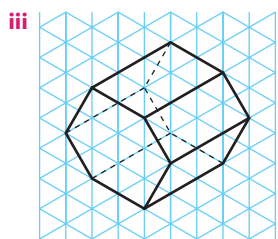
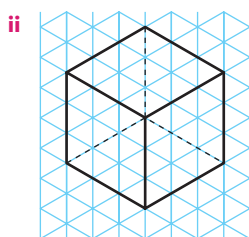
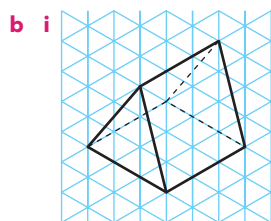
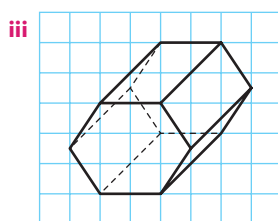
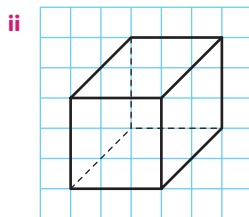
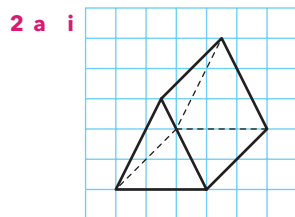
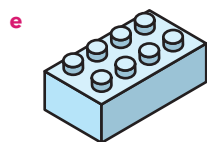
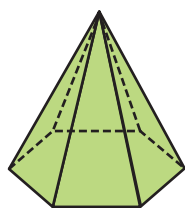
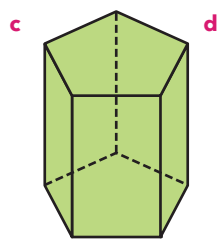
### EXERCISE 21B

1 a

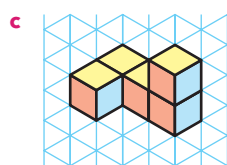
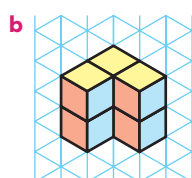
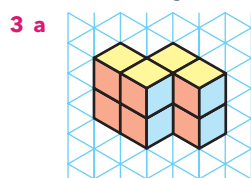


b





**c** On a square grid, the objects are drawn as if viewed 'face-on'. On an isometric grid, objects are drawn as if view from one of their edges.

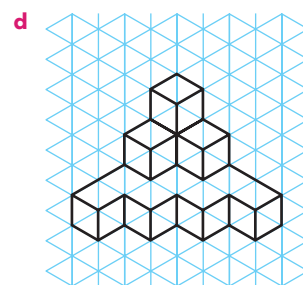
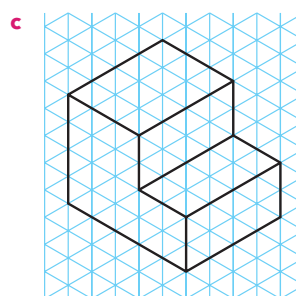
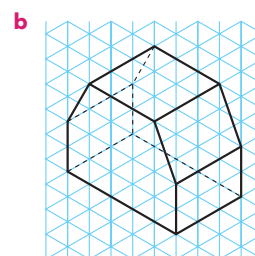
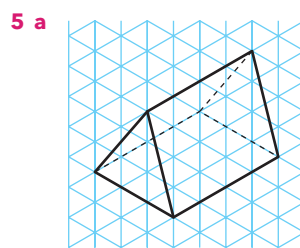
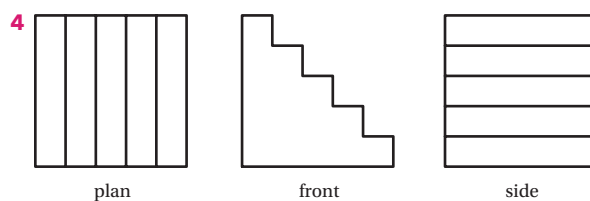
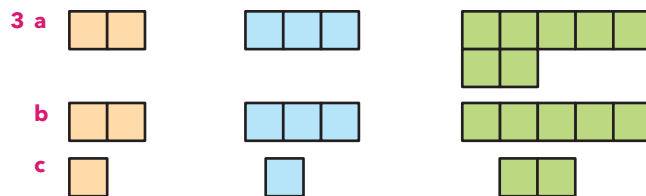


- 4 a** 5      **b** 7      **c** 11      **d** 14  
**5 a** Both      **b** Both  
**c** Student's own diagrams.

### EXERCISE 21C

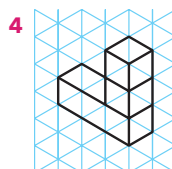
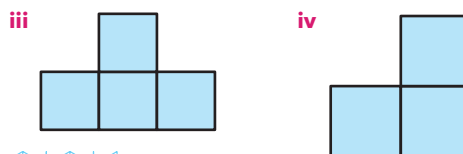
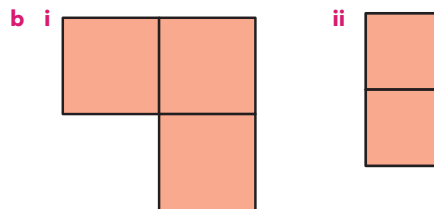
- 1 a** B      **b** B      **c** B      **d** B      **e** A

- 2 a i** B      **ii** D      **iii** C      **iv** A

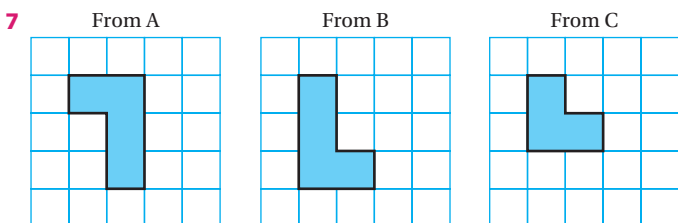
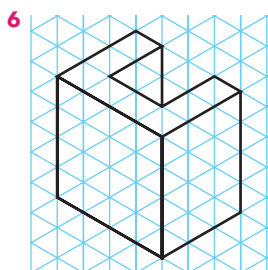


### CHAPTER REVIEW

- 1** Option D  
**2 a** Open-topped cube      **b** Cone  
**3 a i** C      **ii** D      **iii** A      **iv** B



- 5 a Students' own answer      b Students' own answer  
 c 13, 15



## 22 Units and measurement

### BEFORE YOU START ...

- 1 a 10 000      b 0.01      c 0.1  
 2 a £140      b £90      c £220  
 3 a 180p = £1.80      b 15p      c 60p

### LAUNCHPAD

- 1 a 11.569 kilograms      b 16 200 seconds      c 1230.00  
 d 0.0005 m<sup>2</sup>  
 2 a 48 kilometres per hour      b 13.33 m/s  
 3 a 24 km/h      b 90 seconds      c 167 m  
 4 800 cm or 8 metres  
 5 a Scale drawing      b 24° (23.8° (3sf))  
 c 19 km (18.6 km (3sf))

### WORK IT OUT 22.1

2.35 km so Option C

### EXERCISE 22A

- 1 Option D  
 2 10 000 mm = 10 m; 10 000 ml = 10 l; 10 kg = 10 000 g; 0.01 kg = 10 g;  
 0.1 cm = 1 mm  
 3 a 2500      b 850      c 34 000  
 d 1550      e 7      f 5400  
 g 900      h 0.102      i 0.0145  
 4 a 8500 ml      b 2.75 l      c 25.152 l  
 5 a Drawing to represent garden.  
 b 20 m  
 c 17 slabs (20 ÷ 1.2 = 16.67)  
 d £77.35  
 e £3.87  
 6 a 3.6 kg      b 776.3 cm      c 3.567 tonnes  
 d 5 000 000 mm<sup>2</sup>      e 96 350 000 cm<sup>3</sup>      f 0.345 l

- 7 a <; 50.3 cm      b >; 100 g      c <; 0.15 l  
 d >; 4 mm      e >; 20 ml      f <; 1 cm  
 g <; 1.305 kg      h >; 1976.013 t      i >; 1.8 m<sup>3</sup>  
 8 a 3000 mm      b 210 000 mm<sup>2</sup>  
 c 0.21 m<sup>2</sup>      d i 36 040 000 mm<sup>3</sup>      ii 36.04 litres

### EXERCISE 22B

- 1 1:51:52  
 2 Option C  
 3 a 832      b 5844 (16 × 365.25 to account for leap years)  
 c 140256      d 504921 600  
 4 21:05  
 5 3 tins would have to be bought  
 6 a €1 = £0.79      \$1 = £0.59      AS\$ = £0.56      1 Rs = £0.01  
 b \$212.50  
 c 4602.60 Rs  
 d £78.22  
 e You get a lot of Indian Rupees for each pound.  
 f For example, having a weaker exchange rate means that it is expensive to import goods relative to the value of any exported goods.

### WORK IT OUT 22.2

60 mph so Option A

### EXERCISE 22C

- 1 14 × £5.13 = £71.82  
 2 Sireta earns  $\frac{186.5}{22} = £8.48$  per hour. Henry earns £8.75 per hour.  
 Therefore Henry earns 27p per hour more than Sireta.  
 3 2.83 bricks per minute  
 4 21 km/hour  
 5 6 hours 2 minutes  
 6 a 10.38 m/s      b 37.38 km/hour      c 0.05 s  
 7 0.15 hour or 9 minutes  
 8 90.4 km/hour  
 9 18.2 km/hour or 5.06 m/s  
 10 93.33 km/hour

### EXERCISE 22D

- 1 a, b
- |            |                        |                         |
|------------|------------------------|-------------------------|
| Balsa wood | 0.2 g/cm <sup>3</sup>  | 200 kg/m <sup>3</sup>   |
| Ice        | 0.9 g/cm <sup>3</sup>  | 900 kg/m <sup>3</sup>   |
| Chalk      | 2.2 g/cm <sup>3</sup>  | 2200 kg/m <sup>3</sup>  |
| Tin        | 7.3 g/cm <sup>3</sup>  | 7300 kg/m <sup>3</sup>  |
| Copper     | 9.0 g/cm <sup>3</sup>  | 9000 kg/m <sup>3</sup>  |
| Gold       | 19.3 g/cm <sup>3</sup> | 19300 kg/m <sup>3</sup> |
| Petrol     | 0.7 g/cm <sup>3</sup>  | 700 kg/m <sup>3</sup>   |
| Brick      | 1.8 g/cm <sup>3</sup>  | 1800 kg/m <sup>3</sup>  |
| Aluminium  | 2.7 g/cm <sup>3</sup>  | 2700 kg/m <sup>3</sup>  |
| Iron       | 7.8 g/cm <sup>3</sup>  | 7800 kg/m <sup>3</sup>  |
| Lead       | 11.3 g/cm <sup>3</sup> | 11300 kg/m <sup>3</sup> |
- 2 option D  
 3 2.38 g/cm<sup>3</sup>



- 4  $125 \text{ cm}^3$   
 5 Block A:  $18 \text{ N/m}^2$  Block B:  $3 \text{ N/m}^2$   
 6  $60\,000 \text{ N/m}^2$

### WORK IT OUT 22.3

Student C is correct. The actual distance is  $0.85 \text{ km}$ .  
 Student A has divided rather than multiplied by  $25\,000$ .  
 Student B has incorrectly converted  $\text{mm}$  to  $\text{km}$ .

### EXERCISE 22E

- 1 a  $2 \text{ km}$  b  $25 \text{ km}$  c  $24 \text{ km}$   
 2 option B  
 3 a  $0.0054 \text{ km}$  b  $0.054 \text{ km}$  c  $0.54 \text{ km}$  d  $5.4 \text{ km}$   
 e  $54 \text{ km}$  f  $540 \text{ km}$  g  $5400 \text{ km}$   
 4 He is correct.  $15\,000 \text{ cm}$  in real life is represented by  $1 \text{ cm}$  on a  $1 : 15\,000$  map but it is represented by only  $0.1 \text{ cm}$  on a  $1 : 150\,000$  map.  
 5  $3 \text{ km}$   
 6 a  $530 \text{ km}$  b  $578.18 \text{ km/hour}$   
 7 option B  
 8 a  $2.5 \text{ m}$  b  $3 \text{ cm}$  c  $1.35 \text{ m}$   
 9  $39 \text{ cm}$   
 10 a  $180 \text{ km}$  b  $48.25 \text{ mm}$   
 11 a  $1 : 2\,000\,000$  b  $9.6 \text{ cm}$

### EXERCISE 22F

- 1 a, b, c Students' scaled diagrams.  
 2 Students' scaled diagrams.  
 3 Students' scaled diagrams.  
 4 a  $61.25 \text{ mm}$  by  $47.5 \text{ mm}$  b  $15 \text{ mm}$

### EXERCISE 22G

- 1 a  $180^\circ$  b  $045^\circ$  c  $270^\circ$   
 2 option A  
 3 a  $167^\circ$  b  $347^\circ$  c  $141^\circ$   
 d  $023^\circ$  e  $251^\circ$  f  $204^\circ$   
 4 a  $288^\circ$  b  $108^\circ$  c  $147 \text{ km}$   
 5 a  $300^\circ$  b  $225^\circ$  c  $30^\circ$   
 6 a  $9.6 \text{ km}$  b  $090^\circ$

### CHAPTER REVIEW

- 1 a  $259\,200 \text{ s}$  b  $182.5 \text{ km}$  c  $5 \text{ km}$  d  $475 \text{ litres}$   
 2 a True  
 b False. It would take  $22.5$  minutes  
 c False. It would be  $2.2 \text{ m}$   
 3  $6 \text{ m}^2$   
 4  $2\,000\,000 \text{ mm}^2$   
 5 a Scale drawing  
 b i Students' diagrams. ii  $67.5 \text{ km/hour}$   
 6  $200 \text{ kg}$   
 7 a Scale drawing b  $283^\circ$   
 c  $15.3 \text{ km}$   
 d i  $10.2 \text{ km/hour}$  ii  $2.83 \text{ m/s}$

- 8 a  $10.8 \text{ m/s}$   
 b  $13.6 \text{ m/s} = 48.96 \text{ km/h}$  so the lorry is travelling below the speed limit  
 9 Approx  $41 \text{ km}$  apart

## 23 Formulae

### BEFORE YOU START ...

- 1 a  $7.5$  b  $1.5$  c  $\frac{1}{4}$  d  $-5.5$   
 2 a  $x = 7$  b  $x = 8$  c  $x = \frac{1}{3}$  d  $x = 10$   
 3 a  $A = \text{area of a triangle}$ ,  $b = \text{base}$ ,  $h = \text{height}$   
 $A = \text{area of a circle}$ ,  $r = \text{radius}$   
 b  $\frac{1}{2}, \pi$   
 c  $A$   
 d Use of  $\pi$

### LAUNCHPAD

- 1 a C b D c A d B  
 2  $T = 45w + 20$   
 3  $-\frac{11}{6}$   
 4 a 4 variables b 3 variables c  $43.2$   
 5 a  $A$  (area);  $r$  (radius);  $\pi$  (constant)  
 b  $r = \sqrt{\frac{A}{\pi}}$  c No

### EXERCISE 23A

- 1 Option A  
 2 a  $x + y = 2000$  b  $y = 4x$  c  $y - 400 = 3x$   
 d  $y + 600 = x$  e  $\frac{1}{2}x = \frac{2}{5}y$   
 3 a  $x + 3$  b  $x - 6$  c  $10x$   
 d  $x - 8$  e  $x + x^2$  f  $3x + 1$   
 g  $\frac{2x}{(x-2)}$  h  $\frac{\pi\sqrt[3]{x}}{6}$   
 4 a  $4^\circ$  b  $1\,000 \text{ m}$   
 5 a  $Q = 0.98^n P$  b  $a = 0.95^n \times 100$

### WORK IT OUT 23.1

Option B is correct because it has the correct units for area and because  $256 = 4 \times 8^2$ .

Option A is incorrect because, although the formula is correctly used, there are no units.

Option C has the wrong value and the units are incorrect. They should be  $\text{cm}^2$ , which are the units for area, and not  $\text{cm}^3$ , which are the units for volume.

### EXERCISE 23B

- 1 a i  $68$  ii  $-18$  iii  $-\frac{7}{9}$   
 b i  $45$  ii  $-\frac{14}{9}$   
 2 a  $-3$  b  $-\frac{11}{4}$  c  $4$  d  $-\frac{3\sqrt{2}}{4}$   
 e  $0$  f  $-\frac{9\pi}{14}$   
 3  $v = 21$

4 Option A

5 a 10      b 12.5      c 20.9      d 10.4

6 a  $\frac{4}{3}$       b  $\frac{30}{17}$

7 a 29.9 m      b 69.6 m

8  $V = 190\pi$

9  $h = 44.7$

10 28.7 cm

11 99 cm

### EXERCISE 23C

1 Option B

2 a  $q = \frac{d}{a} + c$       b  $p = 8q + 2$       c  $x = \frac{4y + 15}{3y + 5}$

d  $h = \frac{2a^2}{3}$       e  $t = \frac{py}{2p + y}$       f  $b = \frac{5a + 2}{a + 7}$

g  $x = \frac{pc}{q - p}$

3  $n = \frac{S}{180} + 2$

a 8      b 12      c 20

4  $m = \frac{2E}{v^2}$

a 8 kg      b 3.5 kg      c 20 kg

5 Option C

6  $C = \frac{5(F - 32)}{9}$

a 20°C      b -5°C      c 100°C

7 3 cm

8 2.5 cm

9 30.5 m/s

10  $n = 5$

11  $L = 2.45$

### EXERCISE 23D

1 a T      b F      c T      d T      e F

2 16.97 cm<sup>2</sup>

3 a Total accrued = £6749      b Interest = £749

4 This is the formula for solving a quadratic equation.

In order to evaluate the two possible solutions both the positive and negative square root values must be considered.

5  $ax^2 + bx + c = 0 \Rightarrow x^2 + \frac{b}{ax} = -\frac{c}{a} \Rightarrow \left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a^2} = -\frac{c}{a}$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2}{4a^2} - \frac{c}{a}$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$$

Taking the square root of both sides:

$$x + \frac{b}{2a} = \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$\text{So } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

6 a  $\sqrt{n}, \frac{n}{2} + 1, n, \frac{36}{n}, n^2, 8n$       b  $n^2, n, \sqrt{n}, \frac{n}{2} + 1, 8n, \frac{36}{n}$

7 a Yes, gallium melts above 29.8°C.

b 2199°F

8 a  $Q = -1.077$       b  $m = -6$

9  $l = 13.5$  cm

10 a 30 m      b 34.125 m      c 38.5 m

11 a  $a = \frac{v - u}{t}$

12  $s = \frac{[(b\sqrt{n} - a)^2]}{3}$

13 30 m/s

14  $x = -5$  or  $-4$

### CHAPTER REVIEW

1 Option A

2 a 5 cm      b 7.4 cm      c  $h = \frac{S}{2\pi r} - r$       d  $4\pi\sqrt{3}$

e The total surface area is the sum of the area of the curved surface and the area of the flat surface; the area of flat surface is the area of a circle with radius  $r$ , which is  $\pi r^2$ , so the total surface area is  $2\pi r^2 + \pi r^2 = 3\pi r^2$

3 a  $A = \pi r(2C - r)$       b  $A = \pi(C^2 - h^2)$

4  $C = 55^\circ$

5  $s = 7.5$  m/s and 5.5 m/s

6  $(w - 3)^2$

## 24 Volume and surface area

### BEFORE YOU START ...

1 a cube      b cylinder      c square-based pyramid  
d cone      e polyhedron      f triangular prism

2  $A = \pi r^2$

3 6 cm<sup>2</sup>

4 a cuboid      b The shape of the faces.

### LAUNCHPAD

1 125 cm<sup>3</sup>

2 120 cm<sup>3</sup>

3 volume =  $1.087 \times 10^{12}$  km<sup>3</sup> surface area =  $5.112 \times 10^8$  km<sup>2</sup>

4 volume = 8820 m<sup>3</sup>

### WORK IT OUT 24.1

Calculation A is correct. The volume of the skip is 5.44 m<sup>3</sup>.

### EXERCISE 24A

1 Option D

2 a volume = 169.6 cm<sup>3</sup> surface area = 213.4 cm<sup>2</sup>

b volume = 80 cm<sup>3</sup> surface area = 138.16 cm<sup>2</sup>

c volume = 168 m<sup>3</sup> surface area = 244 m<sup>2</sup>

d volume = 141.37 cm<sup>3</sup> surface area = 150.8 cm<sup>2</sup>

e volume = 126 cm<sup>3</sup> surface area = 190 cm<sup>2</sup>

f volume = 42 cm<sup>3</sup> surface area = 96 cm<sup>2</sup>

3 180 litres

- 4 Option A  
 5  $2500 \text{ m}^3$   
 6  $33.03 \text{ cm}^3$   
 7 volume =  $w \times b \times h$  the width and breadth of the base remain constant so is proportional to  $h$  (height). Volume of oil = 4.05 l  
 8  $6.03 \text{ m}^2$   
 9 They are the same.  
 10 a  $2764.6 \text{ cm}^3$  b  $6283.19 \text{ cm}^3$   
 11 5 cm  
 12  $60 \text{ m}^2$   
 13 178.5 m  
 14 volume =  $x(x+2)(x+3) = x^3 + 5x^2 + 6x$   
 surface area =  $6x^2 + 20x + 12$   
 15 volume  $(a+b)^3$

### EXERCISE 24B

- 1 a  $3\sqrt{135\pi}$ ;  $45\pi$  or  $9\pi\sqrt{15}$  b  $\frac{175\pi}{12}$ ;  $24.83\pi$  c  $\frac{125\pi}{6}$ ;  $25\pi$   
 d  $18\pi$ ;  $9(1+\sqrt{5})\pi$  e  $\frac{128\pi}{3}$ ;  $48\pi$   
 2  $2.2 \times 10^{10} \text{ km}^3$   
 3 Option C  
 4 a  $8659.01 \text{ mm}^3$  b  $126.68 \text{ cm}^3$   
 c  $706.86 \text{ cm}^3$  d  $5728.03 \text{ mm}^3$   
 e  $1465.74 \text{ cm}^3$  f  $2026.83 \text{ cm}^3$   
 g  $28.27 \text{ cm}^3$  h  $17\,203.36 \text{ mm}^3$   
 i  $153.94 \text{ cm}^3$   
 5 a volume 226.72 surface area 235.62  
 b volume 9786.68 surface area 2940.53  
 c volume 1015.94 surface area 615.75  
 d volume 4047.96 surface area 1910.09  
 e volume 83 959.91 surface area 23 373.45  
 f volume 2069.06 surface area 989.6  
 g volume 40 030.35 surface area 7728.32  
 6  $6.12 \text{ m}^3$

### EXERCISE 24C

- 1 a 60.82 b 11 713.3  
 2 a  $\pi(90 \times 219.3 - 50 \times 148.6) + \pi(50^2) = 46\,500$  (3sf)  
 b  $5 \times 46\,500 = 232\,500$   
 3  $2.49 \text{ m}^3$   
 4 volume =  $298\,174.77 \text{ cm}^3$ , surface area =  $26\,817.48 \text{ cm}^2$   
 5 a i 8250 ii 11 281.9  $\text{cm}^2$   
 b i 3200 ii  $3602.8 \text{ mm}^2$   
 6  $495 \text{ m}^3$   
 7  $7794.23 \text{ cm}^3$   
 8 volume =  $130 \text{ m}^3$   
 9 Assuming a height of 15 cm the radius of a cylinder must be between 4.12 and 4.6 cm.  
 Assuming a height of 15 cm the radius of a cone must be between 7.14 and 7.98 cm.

### EXERCISE 24D

- 1 a 865.3; 600  
 b 480; 423.9  
 c You can't have a slant height less than the perpendicular height.  
 d 48; 96  
 e 23.09; 67.83  
 f 28.22; 61.25  
 2 Option B  
 3 2 460 000 (3sf)  
 4 Pyramid A = 96; Pyramid B =  $24\sqrt{3} = 41.57$ .  
 Difference =  $96 - 41.57 = 54.43$  (3sf)  
 5  $V = 48x^3$ ;  $SA = 96x^2$   
 6  $V = \frac{\sqrt{3}}{6} = 0.289$   
 7 a volume =  $3.24 \text{ cm}^3$  surface area =  $15.57 \text{ cm}^2$   
 b volume =  $0.12x^3$  surface area =  $1.73x^2$   
 8  $V = 145$ ;  $SA = 268$  (3sf)

### CHAPTER REVIEW

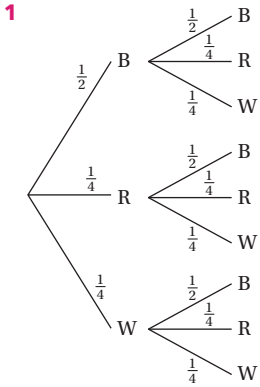
- 1  $2 \times 0.6 \times 1.58 + 2 \times 2 \times 1.7 = 8.7 \text{ m}^2$  (2 sf)  
 2  $227.5 \text{ cm}^3$   
 3  $24.64 \text{ m}^3$   
 4  $4186 \text{ m}^3$   
 5 1.1 m  
 6 A  
 7 a 552.9  
 b  $276.5 \text{ cm}^2$

## 25 Further probability

### BEFORE YOU START ...

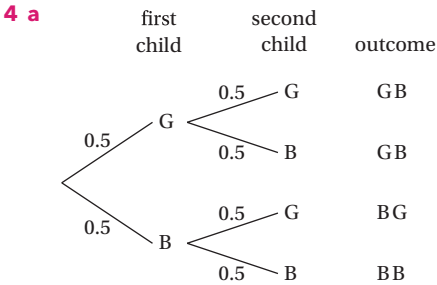
- 1 a  $\frac{3}{8}$  b  $\frac{13}{15}$  c  $\frac{2}{5}$   
 d  $\frac{4}{15}$  e 0.18  
 2 a Relative frequency  
 b outcomes  
 c event  
 d random  
 e sample space  
 3 a FF, FM, MM, MF  
 b HH, HT, TH, TT  
 c AB, AC, BA, BC, CA, CB

**LAUNCHPAD**



2 a  $\frac{1}{52}$       b  $\frac{4}{13}$       c  $\frac{9}{13}$

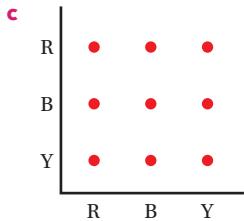
3 a  $\frac{5}{9}$       b  $\frac{41}{81}$       c  $\frac{20}{81}$



b  $\frac{1}{2}$

**EXERCISE 25A**

1 a Option B      b Option D



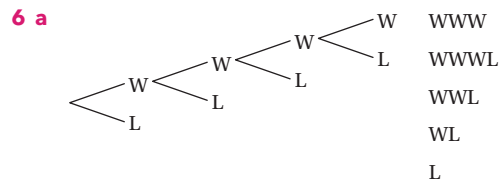
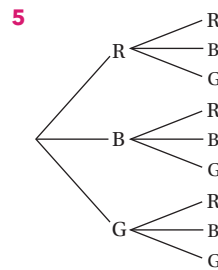
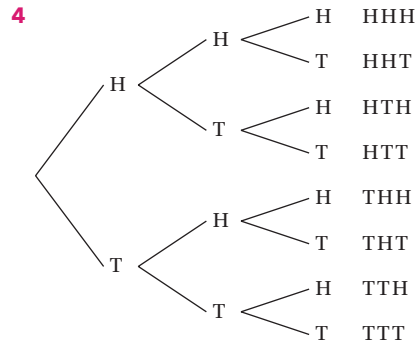
3 a i

Dice 1 \ Dice 2	1	2	3	4	5	6
1	1, 1	2, 1	3, 1	4, 1	5, 1	6, 1
2	1, 2	2, 3	3, 2	4, 2	5, 2	6, 2
3	1, 3	2, 3	3, 3	4, 3	5, 3	6, 3
4	1, 4	2, 4	3, 4	5, 4	5, 5	6, 6
5	1, 5	2, 5	3, 5	4, 5	5, 5	6, 5
6	1, 6	2, 6	3, 6	4, 6	5, 6	7, 6

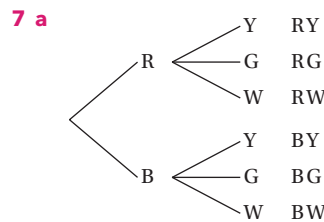
ii

Coin \ Spinner	H	T
A	HA	TA
B	HB	TB
C	HC	TC
D	HD	TD

b Students' own answers.



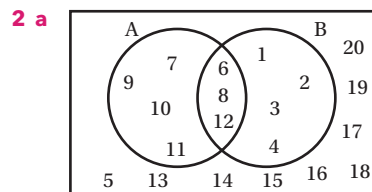
b 5      c  $\frac{1}{2}$



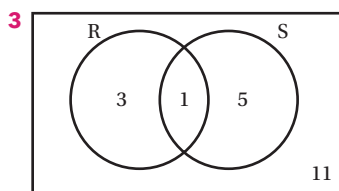
b Not really, in reality people have colour preferences, so relative frequency of choices would probably not be equal.

**EXERCISE 25B**

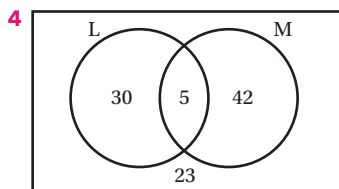
1 a Option D      b Option B      c Option B



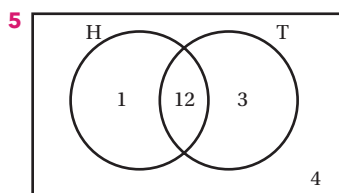
- b i {6, 8, 12}      ii {1, 2, 3, 4, 6, 7, 8, 10, 11, 12}  
 iii 7      iv 13  
 v {5, 7, 9, 10, 11, 13, 14, 15, 16, 17, 18, 19, 20}



$$p(\text{not red, not sports shoes}) = \frac{11}{20}$$



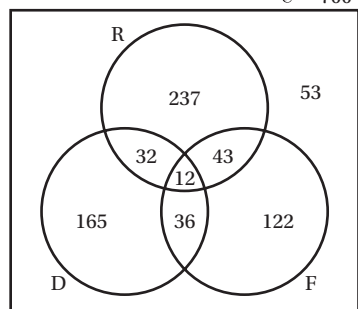
$$p(\text{tie hair back}) = \frac{1}{20}$$



12 students wearing headphones and sending texts

- 6 a 0.3      b 0.25      c 0.375

7      £ = 700



- a 12      b 36      c 165

### WORK IT OUT 25.1

Option B is correct.

If entrances numbered 1 and 2, and exits labelled A, B and C, the possible combinations are:

1A, 1B, 1C, 2A, 2B, 2C

### EXERCISE 25C

- 1 3 276 000 options
- 2  $3 \times 3 \times 5 = 45$
- 3  $9 \times 8 \times 7 \times 6 = 3024$
- 4  $4^5 = 1024$
- 5  $8 \times 7 \times 6 = 336$

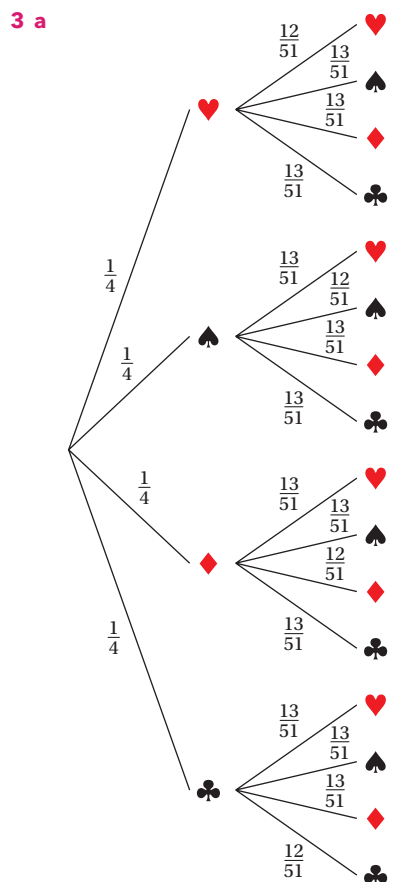
### EXERCISE 25D

1 a possible sample space:

	U	E	E
D	DU	DE	DE
N	NU	NE	NE
D	DU	DE	DE

- b  $\frac{4}{9}$       c  $\frac{2}{3}$       d  $\frac{8}{9}$

- 2 a  $\frac{1}{15}$       b  $\frac{2}{15}$       c  $\frac{1}{45}$       d  $\frac{1}{30}$   
 e  $\frac{2}{45}$       f  $\frac{1}{15}$       g  $\frac{7}{30}$



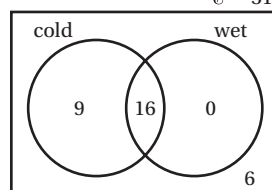
- b i  $\frac{3}{51} = \frac{1}{17}$       ii  $\frac{3}{4} \times \frac{38}{51} = \frac{19}{34}$       iii  $\frac{13}{51}$

- 4 a i  $\frac{1}{24}$       ii  $\frac{1}{24}$       iii 0

- b  $\frac{1}{4}$       c  $\frac{1}{24}$

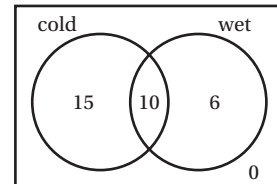
- 5 a  $\frac{1}{4}$       b  $\frac{1}{13}$       c  $\frac{1}{52}$       d  $\frac{4}{13}$

6      £ = 31



maximum

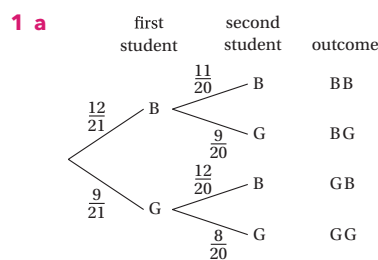
£ = 31



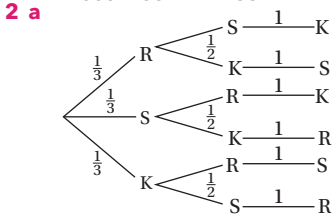
minimum

Maximum = 16 days  
 Minimum = 10 days

### EXERCISE 25E

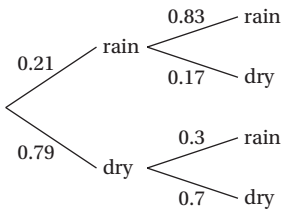


**b** i  $\frac{11}{35}$     ii  $\frac{6}{35}$     iii  $\frac{9}{35}$   
 c i  $\frac{42}{665} = \frac{6}{95}$     ii  $\frac{111}{133}$     iii  $\frac{99}{665}$



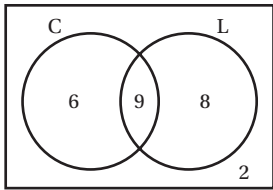
**b** dependent, one affects the other  
**c** 1    **d** 6    **e**  $\frac{1}{6}$     **f**  $\frac{1}{2}$

**3 a**    Friday    Saturday



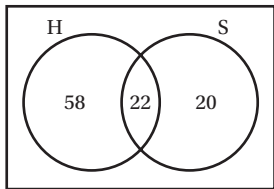
**b** i 0.1743    ii 0.3

**4**    £ = 25



**a**  $\frac{3}{5}$     **b**  $\frac{9}{17}$

**5 a**    £ = 100



**b** i 0.58    ii  $\frac{22}{80} = 0.275$

**6**  $\frac{4}{15}$  or 0.16

**7 a**  $\frac{5}{14}$     **b**  $\frac{9}{14}$     **c**  $\frac{15}{28}$     **d**  $\frac{3}{14}$

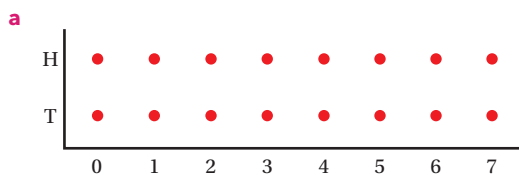
**8 a**  $\frac{26}{30}$     **b** 0.12 (2dp)

**9 a** If these two probabilities are the same then the events are independent.

**b** The probability of red is changed when it is preceded by blue, so the events are independent, in other words, the counter must have been replaced.

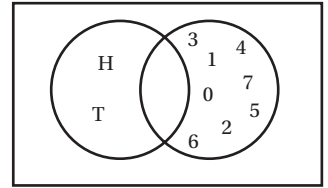
**CHAPTER REVIEW**

**1** Student's own diagrams. Possible answers could be:

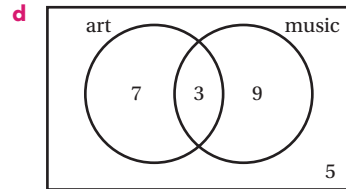


**b**

	Pink	Yellow
A	AP	AY
B	BP	BY
C	CP	CY



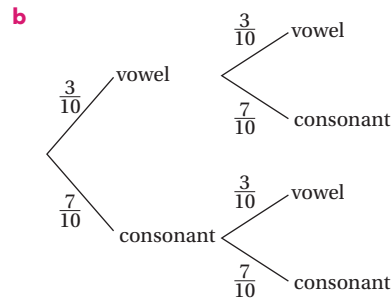
**c** PAN    ANP    NAP  
 PNA    APN    NPA  
 $3 \times 2 \times 1 = 6$



**2** Student's own diagrams.

**a**  $\frac{1}{36}$     **b**  $\frac{11}{36}$     **c**  $\frac{1}{6}$     **d**  $\frac{11}{36}$

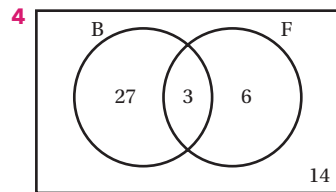
**3 a**  $\frac{3}{10}$



**c** i  $\frac{9}{100}$     ii  $\frac{49}{100}$     iii  $\frac{21}{50}$     iv 0.91

**d** Letter are replaced, so independent.

**e** Remove a letter and don't replace it.



**a**  $\frac{7}{25}$     **b**  $\frac{27}{50}$     **c**  $\frac{3}{30} = 0.1$     **d**  $\frac{14}{20} = 0.7$

**5** Option D

**6**  $\frac{11!}{2! \times 2!} = 9979200$

**7 a**  $\frac{3}{4} \times \frac{4}{5} = \frac{3}{5}$

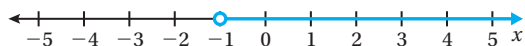
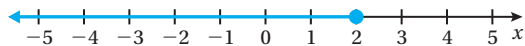

**b**  $P(\text{DOESN'T HIT}) = 1 - \frac{(x+2)}{(x+3)} = \frac{1}{(x+3)}$   
 $\left(\frac{3}{4} \times \frac{1}{5}\right) + \left(\frac{1}{4} \times \frac{4}{5}\right) = \frac{7}{20}$

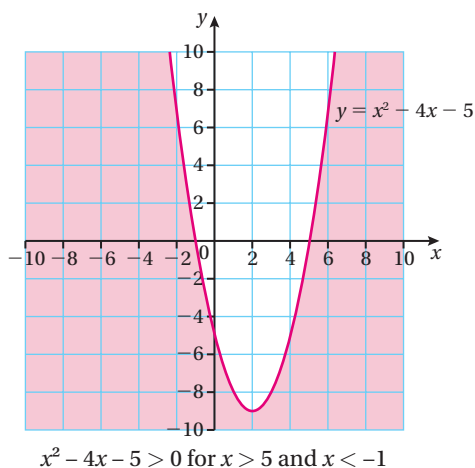
**26 Inequalities**

**BEFORE YOU START ...**

**1 a** 3    **b** 9    **c**  $\frac{55}{28}$     **d** -8  
**2**  $x = -1$   
**3 a** 3    **b**  $y = 3x - 9$

### LAUNCHPAD

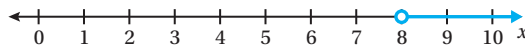

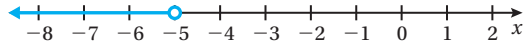
- 1 a  $p < 0.45$     b  $x \geq -4$     c  $11 < y < 18$   
 2 a 3, 4    b -1, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12  
 c 3, 4, 5  
 3 a   
 b   
 c   
 4 a True    b False  
 5 a  $x < 2$     b  $x \geq -2$   
 6  $\{-1, 5\}$   
 7

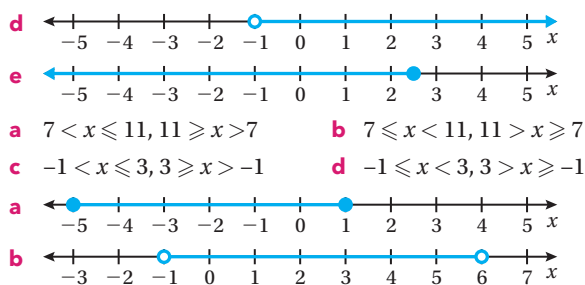


### EXERCISE 26A

- 1 a  $4 + 7 > 4 + 3$     b  $8 - 5 < 13 - 5$     c  $-5 + 3 < -1 + 3$   
 d  $-4 - 6 > -11 - 6$   
 2 a  $2 \times 7 > 2 \times 3$     b  $2 \times 8 < 2 \times 13$     c  $7 \div 2 > 3 \div 2$   
 d  $8 \div 2 < 13 \div 2$   
 3 a  $(-2) \times 7 < (-2) \times 3$     b  $(-2) \times 8 > (-2) \times 13$     c  $7 \div (-2) < 3 \div (-2)$   
 d  $8 \div (-2) > 13 \div (-2)$   
 4 option D  
 5 a any four integers greater than 14  
 b any four integers greater than or equal to 6  
 c any four integers less than or equal to -2  
 d any four integers greater than or equal to 4  
 e any four integers less than or equal to 9  
 6 infinitely many numbers all greater than 6  
 7 four values,  $x = 4, 5, 6, 7$ ; including decimals and fractions there are an infinite number of values for  $x$  that satisfy  $3 < x < 8$   
 8 5, 4, 3

### EXERCISE 26B

- 1 Option C  
 2 a  $x < -2$     b  $x \leq -2$     c  $x > -2$     d  $x \geq -2$   
 3 a   
 b   
 c 



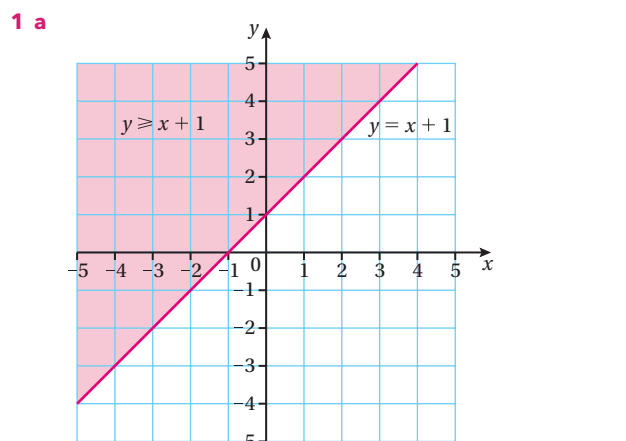
### EXERCISE 26C

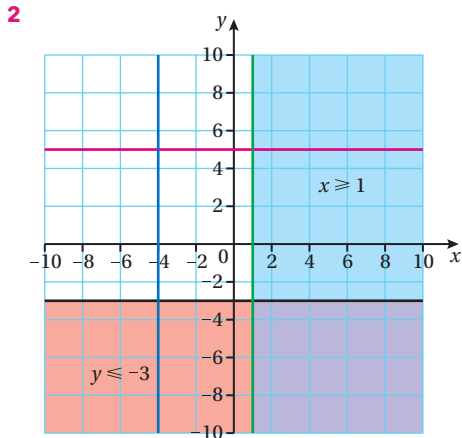
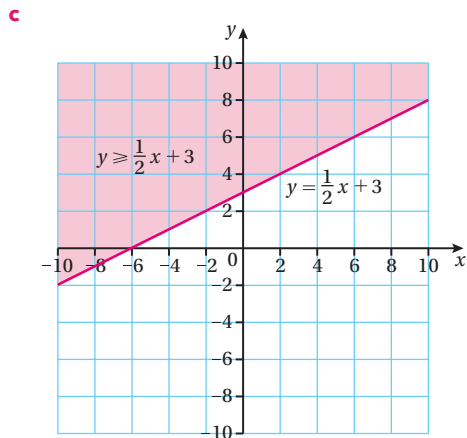
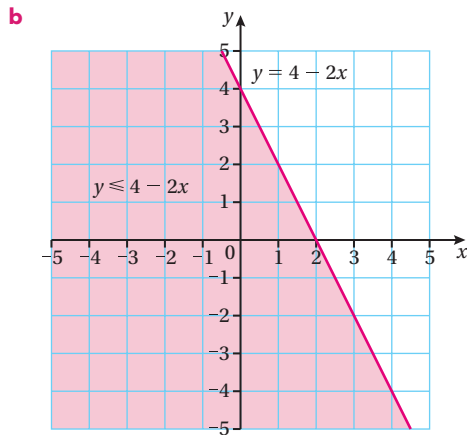
- 1 a  $x \leq 5$     b  $x \leq -13$     c  $x < 3\frac{1}{2}$     d  $x \geq -10$   
 e  $x < -20$     f  $x < -1$   
 2 Option D  
 3 a  $h < 19$     b  $y \leq 30$     c  $x < -46$     d  $h \geq -\frac{3}{2}$   
 e  $y \geq -\frac{44}{3}$     f  $n < 264$     g  $v \leq -\frac{13}{6}$     h  $z > 62$   
 i  $k > 33$     j  $e > \frac{31}{28}$   
 4  $p > 6$   
 5  $q < 68$   
 6  $p \leq 3$   
 7  $d > 6$   
 8  $a > -6$   
 9 a  $y \geq \frac{150 - x}{2}$   
 b i  $y \geq 57.5$     ii  $y \geq 50.5$

### EXERCISE 26D

- 1 a Estimates from the graph:  $x^2 - 3x - 3 \geq 0$  when  $x \geq 3.8$  and  $x \leq -0.8$   
 b  $x^2 - 3x - 3 < 0$  when  $-0.8 < x < 3.8$   
 2 a  $-2x^2 + 16x - 24 \geq 0$  when  $2 \leq x \leq 6$   
 b  $-2x^2 + 16x - 24 \leq 0$  when  $x \geq 6$  or  $x \leq 2$   
 3 Students' own sketches  
 a  $x > 3$  or  $x < -2$     b  $-4 \leq x \leq -1$     c  $x \geq 5$  or  $x \leq 2$   
 d  $-3 < x < 0$   
 4 a  $-4 < x < \frac{3}{2}$     b  $0 \leq x \leq 5$   
 c  $x \leq -2; x \geq 4$     d  $x < -4$  or  $x > \frac{3}{2}$   
 5 a  $x^2 - x - 6 > 0$     b  $x^2 - x - 6 < 0$     c  $x^2 - x - 6 \leq 0$

### EXERCISE 26E





Any two points with  $x$ -coordinate greater than 1 and a  $y$ -coordinate less than  $-5$ .

**3**  $y \leq \frac{1}{3}x - 2$

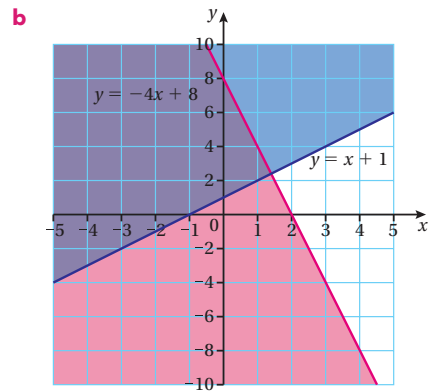
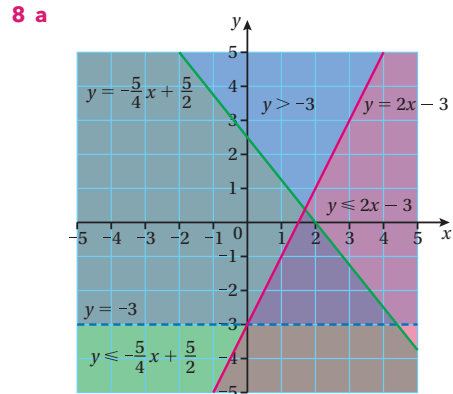
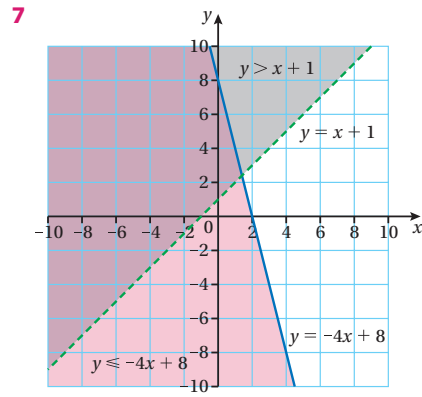
Test a point either side of the line.

**4 a**  $y \geq -1.2x + 3$     **b**  $y \leq 2x - 4$

**5** UNSHADED area is  $x + 2y < -4$  and  $3x + y > 3$

**6**  $y = -\frac{3x}{2}; y = \frac{1}{2}x - 4$

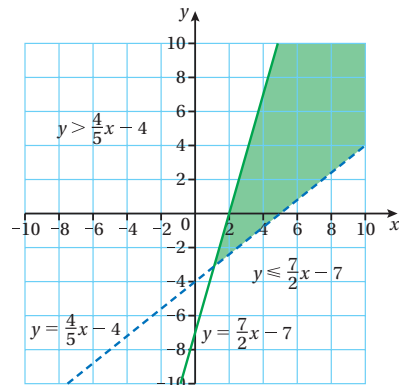
$y \geq -\frac{3x}{2}$  and  $y > \frac{1}{2}x - 4$



**9**  $y \geq 5x + 1$      $y \geq -3x - 1$      $y < \frac{x}{3} + 4$

**10** Region defined by  $y \geq -\frac{4}{3}x$ ,  $y \geq 2x - 12$  and  $y \leq x - 6$

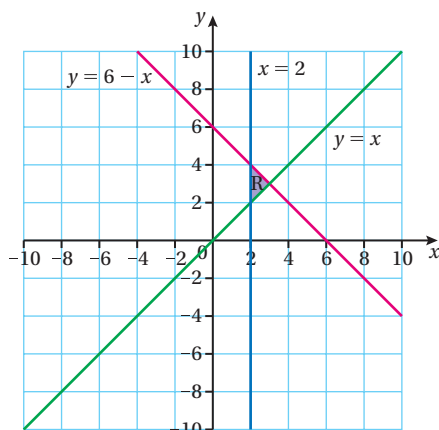
**11**  $x = \frac{10}{9}, y = -\frac{28}{9}$





### CHAPTER REVIEW

- 1 Option B  
 2 (1, 1), (1, 2) and (2, 1)  
 3



- 4  $x \leq 150, y \leq 120, y \leq -x + 200$   
 5  $y \leq 5; x \geq 2; y \geq x$

### 27 Ratio

#### BEFORE YOU START ...

- 1 a  $\frac{2}{5}$       b  $\frac{3}{4}$   
 2 28  
 3 68

#### LAUNCHPAD

- 1 4:7  
 2 8:7  
 3  $15:65 = 3:13$   
 4 10:25  
 5 345 grams

#### EXERCISE 27A

- 1 Option B  
 2 Option A  
 3 a 45 : 36, simplifies to 5 : 4      b 81 : 9, simplifies to 9 : 1  
    c 81 : 90, simplifies to 9 : 10      d Yes 9 pupils per teachers  
 4 a 3 : 5                      b 1 : 2                      c 1 : 2  
 5 a 1 : 4                      b 1 : 2                      c 2 : 5  
 6 3  
 7 1 : 400  
 8 1 : 950  
 9 1 : 40  
 10 a i 2 : 3                      ii 2 : 3                      iii 2 : 3  
      b They are similar.                      c They are parallel.  
      d 1 : 1  
 11 11 : 9  
 12 3 : 2

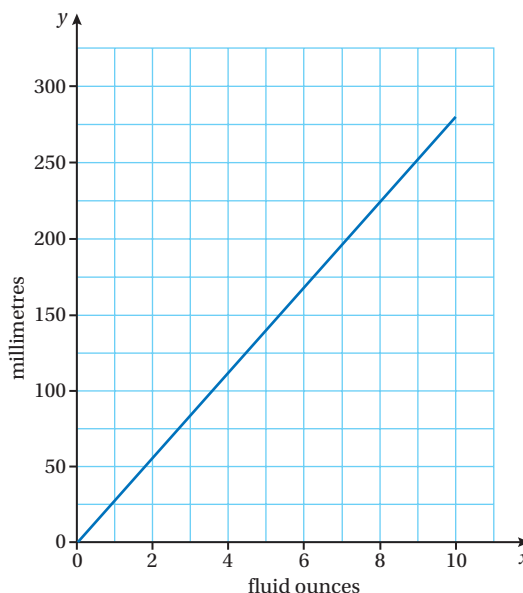
- 13 3 : 2  
 14 225  
 15 2 : 5 : 3

#### EXERCISE 27B

- 1 a 36 : 108                      b 64 : 80                      c 132 : 12  
    d 48 : 72 : 24                      e 18 : 36 : 90                      f 16 : 56 : 40 : 32  
 2 Option C  
 3 Option C  
 4 a 20 kg                      b  $\frac{1}{5}$   
 5 45 g  
 6 a 200 g flour, 75 g margarine, 75 g lard                      b  $\frac{3}{7}$   
 7 a 16 cm by 40 cm                      b area is 640 cm<sup>2</sup>  
 8 4.5 litres  
 9 0.5 litres  
 10 200 g biscuit, 240 g dried fruit, 80 g butter, 80 g cocoa powder  
 11  $2 \times 175 \div 25 = 14$  oboe players  
 12 34 pairs  
 13 18 kg

#### EXERCISE 27C

- 1 Option C  
 2 Latte, flat white, cappuccino, espresso, double espresso  
 3 a 5 : 6, 1 : 1.2                      b 6 : 5, 1 : 0.83  
 4 a 8:200  $\rightarrow$  1:25  
    b 200:8  $\rightarrow$  1:0.04  
 5 a



- b 1 : 0.0357  
 6 a 75 sweets      b  $\frac{8}{25}$   
 7 24 km  
 8 8 sausages; 2 tins of tomatoes; 300 g of potatoes; 6 tsp mixed herbs; 400 ml vegetable stock  
 9 42 chocolates

10 9 more milk chocolates than dark chocolates

- 11 a 3 : 4                      b 3 : 2                      c 1 : 2  
     d 48 turns                      e 18 turns

12 a 5 : 2                      b 150 cm and 60 cm

13 1 : 2.744

14 1 kg for £1.99

15 a 6.7%                      b 145 cm

### EXERCISE 27D

This is an investigative exercise where students will produce their own individual answers.

### EXERCISE 27E

- 1 1 :  $\pi$   
 2 It is an isosceles triangle with angles  $54^\circ$ ,  $54^\circ$  and  $72^\circ$ .  
 3 It is a regular pentagon, with each interior angle  $108^\circ$ . The ratio 1 : 1 : 1 : 1 : 1 means all the angles are the same size which means the pentagon must be regular.  
 4 8 sides (so it is an octagon)  
 5 It is a right-angled triangle with angles  $45^\circ$ ,  $90^\circ$  and  $45^\circ$ ; its other sides are  $\sqrt{50}$  cm.  
 6 3 : 4  
 7 6 white chocolates; 8 milk chocolates; 6 dark chocolates  
 8 9 : 16  
 9 a 25 : 4                      b 125 : 8

### CHAPTER REVIEW

- 1 5 : 21  
 2 2 : 1  
 3 120, 200, 40  
 4 1 : 1.6  
 5 Option C  
 6 12 silver; 18 blue; 6 red; 9 black; 3 yellow  
 7 In one day he earns  $8 \times £6.50 = £52$ . He shares this in the ratio of 4 : 1 so his share is  $£52 \times \frac{4}{5} = £41.60$ . Number of days taken to save £1040 is  $1040 \div £41.60 = 25$  days

## 28 Proportion

### BEFORE YOU START ...

- 1 a 30 min                      b 15 min                      c 20 min                      d 12 min  
 2 a  $\frac{1}{12}$                       b  $\frac{2}{5}$                       c  $\frac{9}{10}$   
 3 a 21                      b 24                      c 0.4

### LAUNCHPAD

- 1 450 g  
 2 100 km  
 3 €17.14  
 4 a  $c = 6.5a$                       b £208                      c  $50.6 \text{ m}^2$   
 5 a  $c = 15.25s^2$                       b £857.81                      c 6.50 m  
 6 a 12 days                      b 5 days

### EXERCISE 28A

- 1 students' own answers, such as 9 km in an hour, 1 km in 400 seconds, ...  
 2 option A  
 3 option D  
 4 £5.50 per hour  
 5 a 60p                      b £3.60                      c £12                      d 350 g  
 6 a £6.30  
     b Ben: 35p per minute, Danny: 32p per minute. Danny's phone is better value.  
 7 36 people  
 8 a i 640 km                      ii 160 km                      iii 80 km                      iv 5.33 km  
     v 0.89 km  
     b 125 hours (5.2 days)  
 9 500 m

### EXERCISE 28B

- 1 option B  
 2 option D  
 3 AU\$316.75  
 4 £26.19  
 5 a 600 ml = £7.20                      b 1 litre = £12                      c 4.5 litres = £54  
     d 48.75 litres = £585  
 6 a £1 = €1.21                      b €1 = £0.826  
 7

	Accommodation	Food	Ski rental	Flights
Bun di Scuol	£340	£65	£300	£69
Flims-Laax-Falera	£300	£100	£111	£144

Bun di Scuol = £774, Flims-Laax-Falera = £655; cheaper by £119

- 8 a £1 = JIB2.4                      b JIB1 = £0.42  
 9 a €423.50                      b KSh26 825                      c ₹153 000  
     d Price of hostels (cheapest first): Mongolia, India, Brazil, New Zealand, Kenya, France  
 10 UK 100 g = £7.69, Spain 100 g = £7.80: cheaper in the UK  
 11 Area of each playing field. number of people using each playing field. Calculate number of people per unit of area, such as  $10 \text{ m}^2$ .

### WORK IT OUT 28.1

Graph 1 is correct.

Graph 2 is incorrect because it shows a non-linear relationship; the equation for this line would have an  $x^2$ .

Graph 3 does not show a pair of variables that are directly proportional to each other because it has a negative gradient.

Graph 4 does not show a pair of variables that are directly proportional to each other because it does not pass through the origin. A graph of a directly proportional relationship passes through the origin.

### WORK IT OUT 28.2

Formula 1 does not represent directly proportional because it has a constant ( $c$ ). This means that its graph would not pass through the origin. The constant shows that it has an  $y$ -intercept of 5.

Formula 2 does represent directly proportional because it shows a relationship of the form  $y = kx$  where  $k$  is the constant of proportionality.

Formula 3 does represent directly proportional because it can be rearranged to be of the form  $y = kx$  where  $k$  is the constant of proportionality.

Formula 4 does not represent directly proportional because it has  $d^2$ . It is not a linear relationship and is therefore not directly proportional.

### EXERCISE 28C

- option B
- 3 km
  - 2.25 h (accept answers between 2 h 12 min and 2 h 16 min)
  - runner B
  - runner A = 6 km/h, runner B = 8 km/h
  - The athletes run at a constant speed.
- $c = 2l$
- The triangles produced by the object, its shadow and the line joining the height of the object to the end of its shadow are similar. Therefore the length of shadow is directly proportional to the object's height.
  - $s = 0.75h$
    - 15 m
    - 6.7 m
- $q = 8p$
  - $q = 30.4$
  - $p = 1.85$
- $d = \frac{20h}{3}$

### EXERCISE 28D

- Option A
- $w = 4m^3$
  - 500
  - 2.5
- $r = 2\sqrt{s}$
  - 8.94
  - 36
- $j = 70I^2$
  - 2.39 amps
- $t = 0.32\sqrt{l}$
  - 39.06 cm
- $m = 19.3s^3$
  - 8.63 cm

### WORK IT OUT 28.3

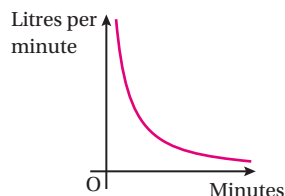
Graph 3 is the correct answer because the greater  $x$  is, the less  $y$  is. None of the others is correct because each line intersects one or both axes. This means that the length or width could be zero for a non-zero width or length. This would result in a zero value for the area of the rectangle, which would be incorrect.

### EXERCISE 28E

- option B
- 12 person-days
  - 6 days
  - 2 days
  - 12 people
  - Everybody works at the same rate; several people can paint the hall at the same time.
- \$12
  - 15
  - \$1.50
- $h = \frac{180}{s}$
  - 45 mph
  - 2 hours 24 minutes
  - 80 mph
- £300
  - £3000
  - $w = \frac{3000}{n}$

<b><math>m</math> (minutes)</b>	10	20	30	40	50	60	70	80	90	100
<b><math>r</math> (litres per minute)</b>	60	30	20	15	12	10	8.57	7.5	6.67	6

$$r = \frac{600}{m}$$



- $a = \frac{50}{b^2}$
  - 12.5
  - 10
- $r = \frac{6.63}{\sqrt{M}}$  (2 dp) or  $r = 2\sqrt{\frac{11}{M}}$
- 0.069 minutes (4.11 seconds)

### CHAPTER REVIEW

- £3.60
  - 611 g
- 10 people
- option B
- $c = 2.2m$
- $F = 3\sqrt[3]{g}$
  - 15
  - 343
- 20 minutes
- $t = \frac{100}{w}$
- $d = \frac{24}{e^3}$
  - 0.89 (2 dp)
  - 3.63 (2 dp)
- $R = \frac{18.15}{A}$
  - $R = 4.5$

## 29 Graphs of linear functions

### BEFORE YOU START ...

<b>1</b>	<b>Term number</b>	1	3	5	10
	<b>Term</b>	1	7	13	28

- A(-3, 4), D(1, -4), E(4, 0)
  - B
    - F
  - The origin
- $x = -3$
  - $x = 42$
  - $x = -0.4$
- 1
  - 1.5
- $y = 1 - 2x$
  - $y = \frac{6 - 2x}{3}$
  - $y = \frac{x + 2}{2}$

### LAUNCHPAD

- $x - y = 2$

<b><math>x</math></b>	-2	-1	0	1
<b><math>y</math></b>	-4	-3	-2	-1

- $x + y = 4$

<b><math>x</math></b>	-2	0	1	2
<b><math>y</math></b>	6	4	3	2

c  $2x + y + 2 = 0$

<b>x</b>	-3	-2	0	1
<b>y</b>	4	2	-2	-4

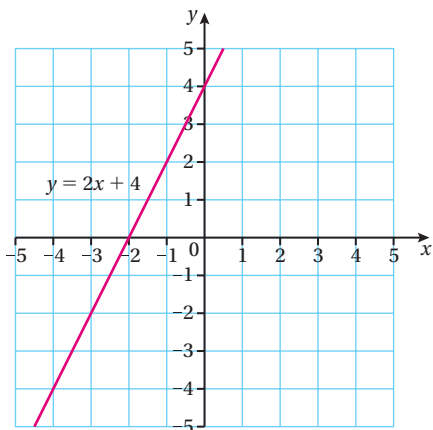
d  $x - 2y + 2 = 0$

<b>x</b>	-2	0	2	4
<b>y</b>	0	1	2	3

2 a is (1d)  $x - 2y + 2 = 0$

b is (1c)  $2x + y + 2 = 0$

3 a



b Gradient = 2    y-intercept (0, 4)

4  $y = \frac{3}{2}x + \frac{5}{2}$      $2y = 3x + 5$

5  $-\frac{5}{4}$

6  $y = -3x + 3$  and  $y = 7 - 3x$

7  $y = \frac{1}{2}x + 3$      $2y = x + 6$

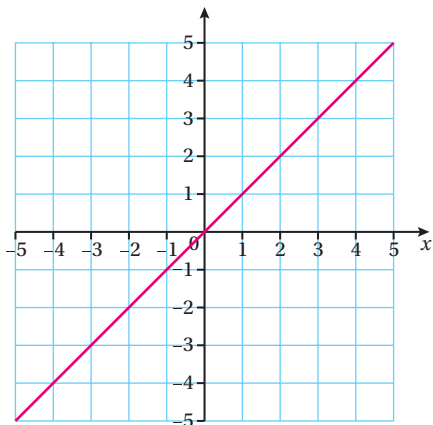
8 The product of the gradients  $(\frac{1}{2} \times -2) = -1$

9  $y = \frac{3x}{4} + \frac{25}{4}$      $4y = 3x + 25$

**EXERCISE 29A**

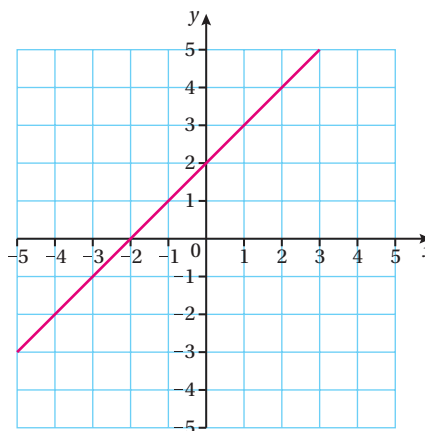
1 a  $y = x$

<b>x</b>	-2	-1	0	1
<b>y</b>	-2	-1	0	1



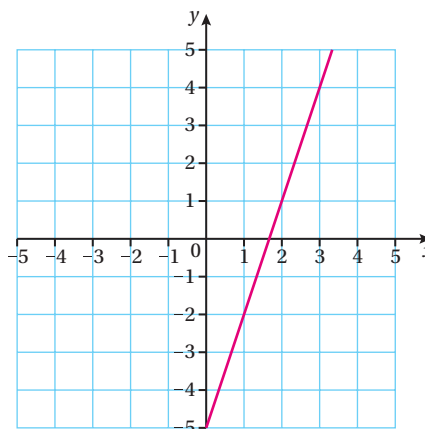
b  $y = x + 2$

<b>x</b>	-2	-1	0	1
<b>y</b>	0	1	2	3



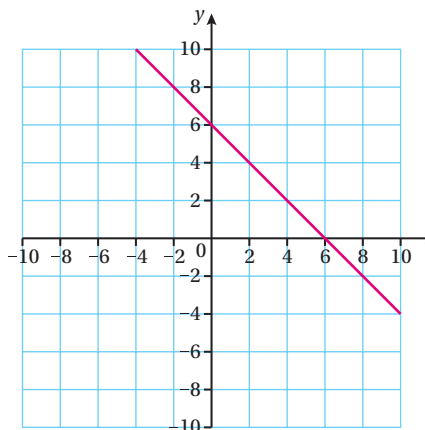
c  $y = 3x - 5$

<b>x</b>	0	1	2	3
<b>y</b>	-5	-2	1	4



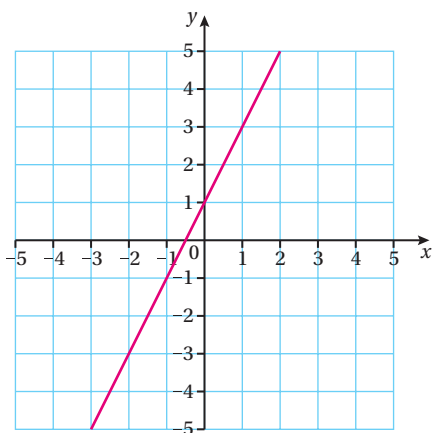
d  $y = 6 - x$

<b>x</b>	0	2	6	8
<b>y</b>	6	4	0	-2



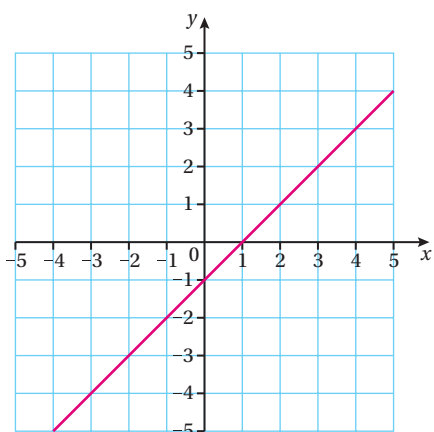
e  $y = 2x + 1$

$x$	-2	-1	0	1
$y$	-3	-1	1	3



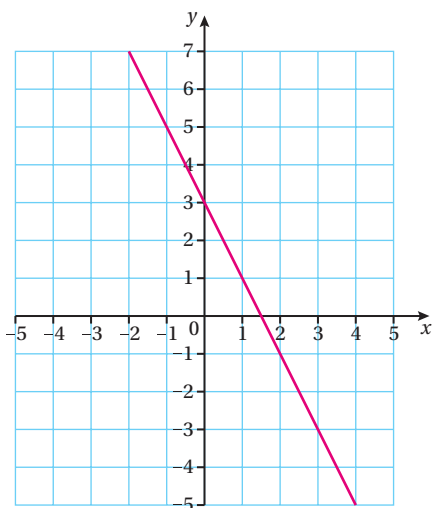
f  $y = x - 1$

$x$	-2	-1	0	1
$y$	-3	-2	-1	0



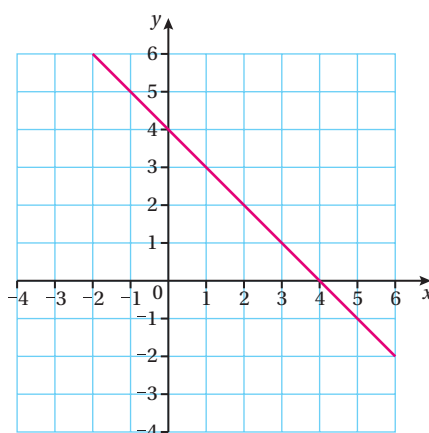
g  $y = -2x + 3$

$x$	-2	-1	0	1
$y$	7	5	3	1



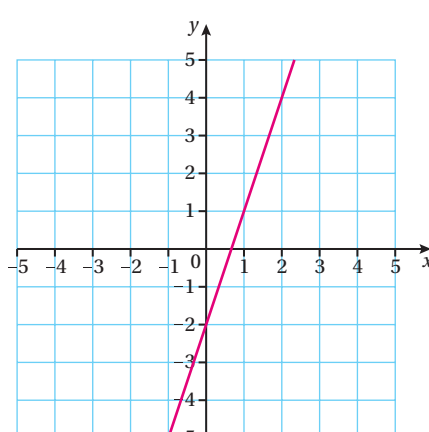
h  $y = 4 - x$

$x$	0	1	3	6
$y$	4	3	1	-2



i  $y = 3x - 2$

$x$	-1	0	1	2
$y$	-5	-2	1	4



2 Two points are sufficient to define a line.

3 option B

### EXERCISE 29B

1 option C

2 a 3                      b 1                      c -2                      d  $-\frac{1}{2}$

e  $\frac{2}{3}$                       f  $-\frac{5}{4}$

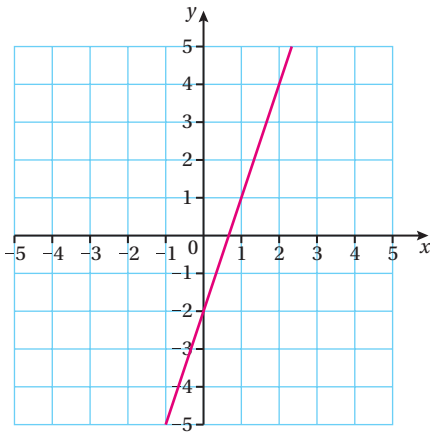
3 a 3                      b 1                      c -3                      d  $\frac{7}{4}$

### WORK IT OUT 29.1

Option B is correct. The coefficient of  $x$  is  $-2$  and not 2.

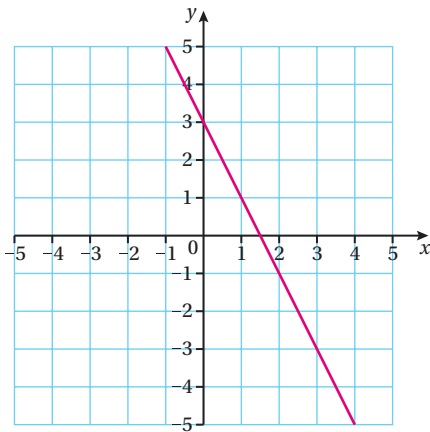
**EXERCISE 29C**

**1 a**



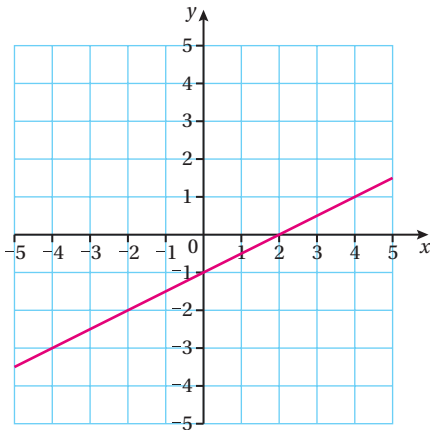
$y = 3x - 2$ ; this line has a positive gradient of 3 and the  $y$ -intercept is  $(0, -2)$ .

**b**



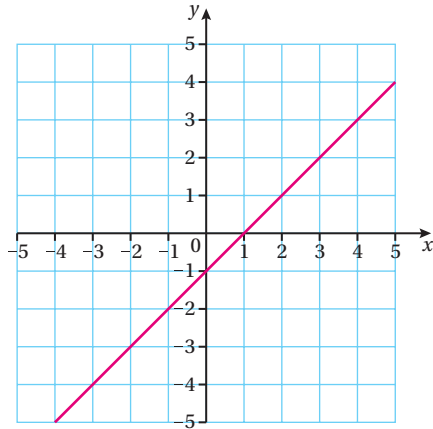
$y = -2x + 3$ ; this line has a negative gradient of  $-2$  and the  $y$ -intercept is  $(0, 3)$ .

**c**



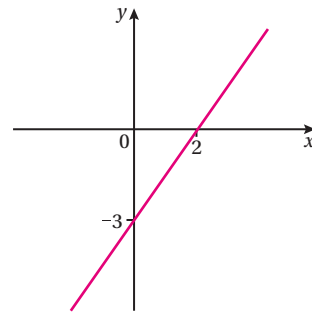
$y = \frac{1}{2}x - 1$  has a positive gradient of  $\frac{1}{2}$  and the  $y$ -intercept is  $(0, -1)$ .

**d**

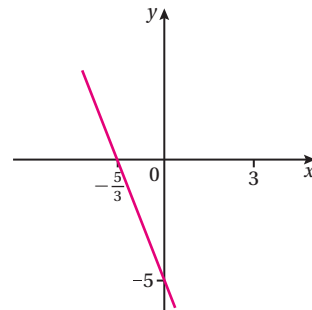


$y = x - 1$ ; gradient = 1, intercept  $(0, -1)$ .

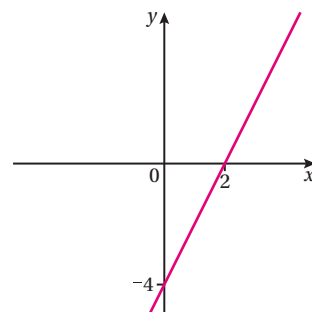
**2 a**  $y = \frac{3}{2}x - 3$



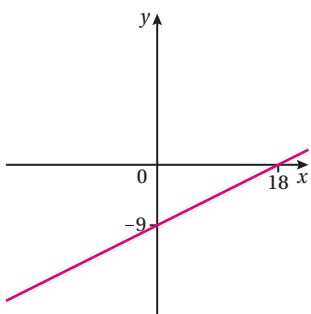
**b**  $y = -3x - 5$



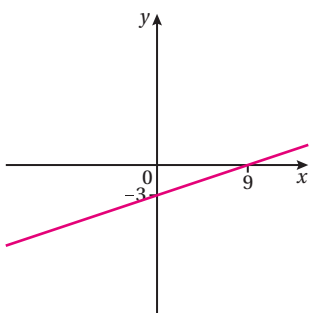
**c**  $y = 2x - 4$



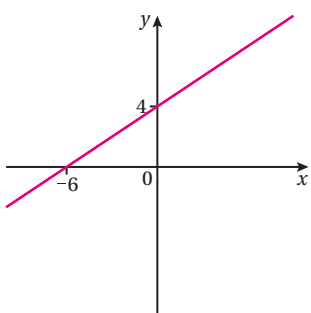
**d**  $y = \frac{1}{2}x - 9$



**e**  $y = \frac{1}{3}x - 3$

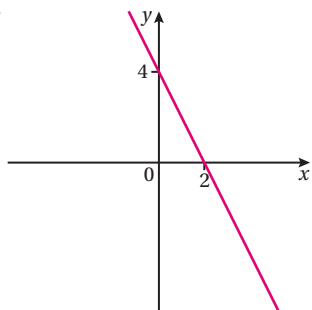


**f**  $y = \frac{2}{3}x + 4$

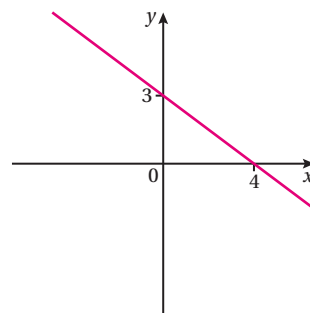


**3 a** i      **b** iii      **c** iv      **d** ii      **e** v

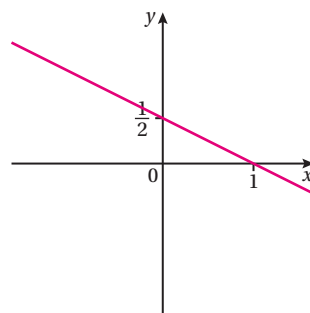
**4 a**  $y = -2x + 4$ ; gradient = -2



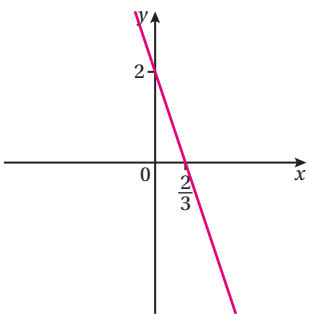
**b**  $y = -\frac{3}{4}x + 3$ ; gradient =  $-\frac{3}{4}$



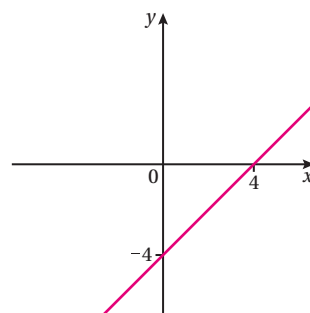
**c**  $y = -\frac{1}{2}x + \frac{1}{2}$ ; gradient =  $-\frac{1}{2}$



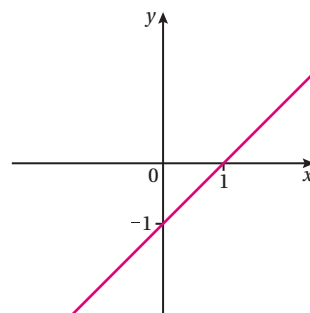
**d**  $y = -3x + 2$ ; gradient = -3



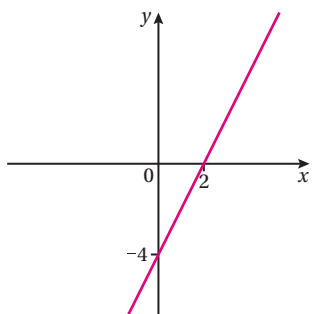
**e**  $y = x - 4$ ; gradient = 1



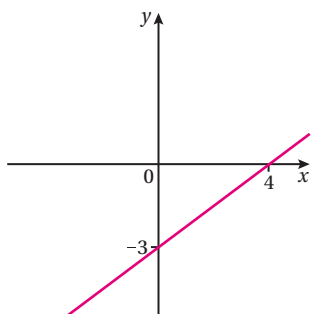
**f**  $y = x - 1$ ; gradient = 1



g  $y = 2x - 4$ ; gradient = 2



h  $y = \frac{3}{4}x - 3$ ; gradient =  $\frac{3}{4}$



**EXERCISE 29D**

- 1 a 2      b -4      c -9      d -8

2 Option A

3 a  $y = -\frac{x}{3}$       b  $y = \frac{3x}{2}$       c  $y = -2x - 9$       d  $y = -2x + 5$

4 a  $y = 2x$ , when  $x = 3$ ,  $y = 6$       b  $y = \frac{1}{2}x + 6$ , when  $x = 8$ ,  $y = 10$   
 c  $y = -x + 5$ , when  $x = 4$ ,  $y = 1$

**WORK IT OUT 29.2**

Option A is correct. If the equations are written in the form  $y = mx + c$ , then the coefficients of  $x$  for each group of three lines can be seen to be the same.

**EXERCISE 29E**

1 a  $y = 2x + 3$       b  $y = 3x + 5$

2 a  $a = 3$       b  $b = -1$

3  $y = -2x - 2$

4 a Gradient of AB = 4      b Equation of AB  $y = 4x + 2$

c Prove ABCD is a parallelogram

If ABCD is a parallelogram then  $AB \parallel CD$  and  $BC \parallel AD$ .

Gradient of CD =  $\frac{8 - 16}{13 - 15} = \frac{-8}{-2} = 4$ ,  $CD \parallel AB$  same gradient;

gradient of BC =  $\frac{16 - 14}{15 - 3} = \frac{2}{12} = \frac{1}{6}$ ; gradient of AD =  $\frac{8 - 6}{13 - 1} = \frac{2}{12} = \frac{1}{6}$ ,  $BC \parallel AD$  same gradient. ABCD is a parallelogram.

5 All the equations are  $x = a$  constant and  $y = a$  constant.

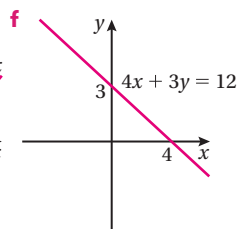
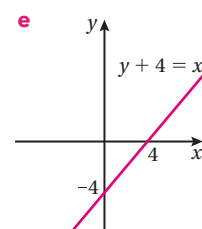
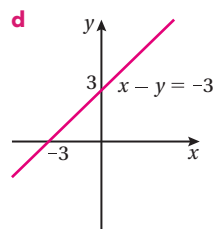
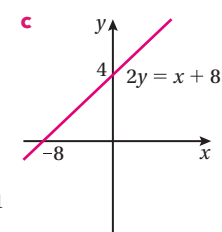
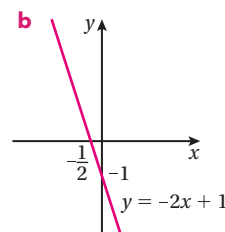
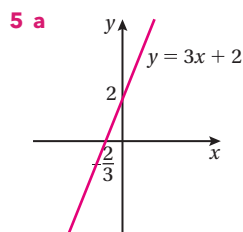
**EXERCISE 29F**

1 Option A

2 a  $m = \frac{1}{2}$       b  $m = \frac{1}{2}$

3 a  $y = -\frac{2x}{5} - 2$       b  $11y + 4x + 20 = 0$

4  $y = 3x + 1$        $y = -\frac{x}{3} + 1$        $3 \times -\frac{1}{3} = -1$

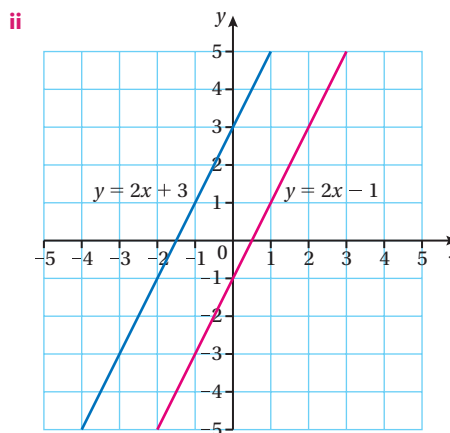
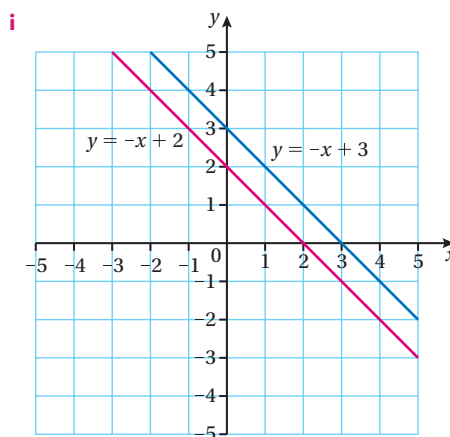


6  $m_{PR} = 3$   $m_{PQ} = -\frac{1}{3}$   $3 \times -\frac{1}{3} = -1$  so yes the triangle is right angled

**EXERCISE 29G**

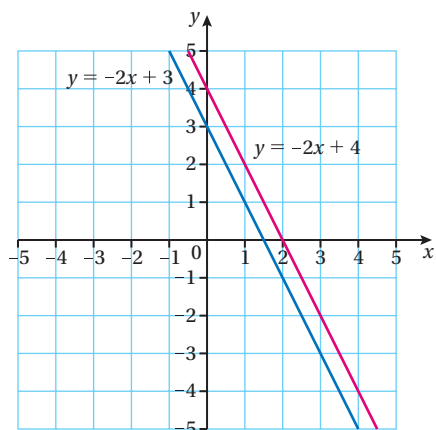
1 a i  $y = x - 2$       ii  $y = -x + 2$

b, c

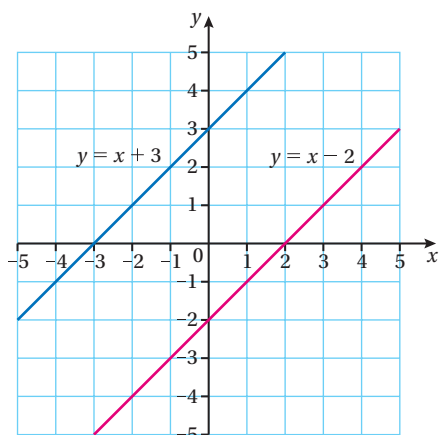




iii



iv



d  $1, -\frac{1}{2}, \frac{1}{2}, -1$

2 a  $y = 3x + 5$    b  $y = -x + 4$    c  $y = \frac{3}{4}x - 2$    d  $y = -\frac{1}{7}x$

3 a Any equation with a negative gradient and a negative y intercept, e.g.  $y = -x - 3$

b Examples:  $y = 2x$  and  $y = 4x + 3$

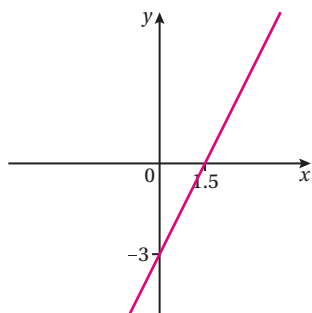
c  $y = 3x - 3$

d Examples:  $y = 7$  and  $y = 3$

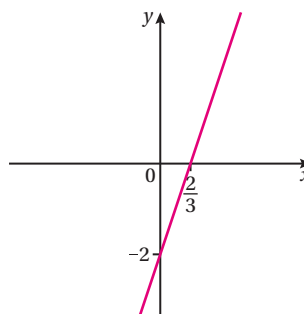
4 a option C   b Values of  $m \neq 1$    c  $m = 2$

5 Gradient =  $\frac{1}{2}$ ,  $y = \frac{1}{2}x + 3$

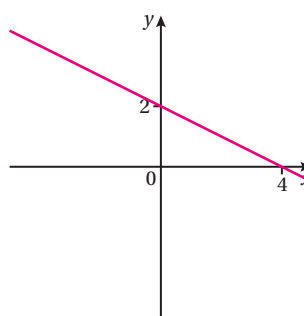
6 a



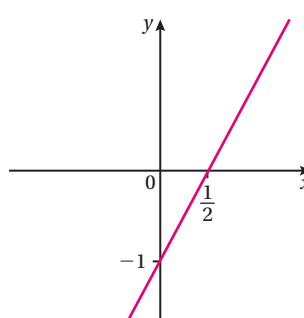
b



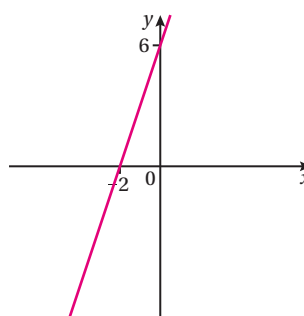
c



d



e



7 a  $9y + 4x = 74$    b  $4x + 5y = 32$    c  $3y = 4x + 26$

8 a  $y = -4x - 6$    b  $y + 4x = 20$    c  $y = 5x + 28$

9  $b = -4$

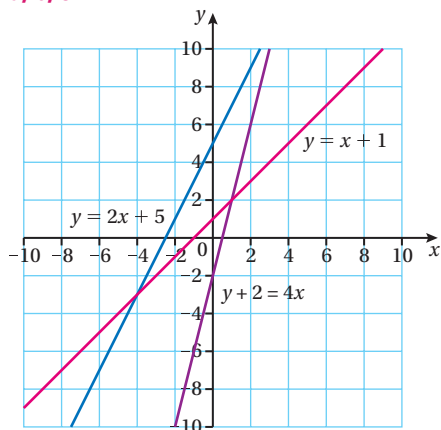
10 a  $2, y = 2x + 1$    b  $y = -x + 1$

11  $y + x = 3$     $y + x = -3$     $y - x = -3$     $y - x = 3$

- 12** The equation of line AB is  $y = 5x - 4$   
 $C(-2, -14) - 14 = 5 \times -2 - 4 \rightarrow -14 = -14$ , hence C lies on the line  $y = 5x - 4$
- 13**  $m_{\text{radius}} = 2\sqrt{6}$  so  $m_{\text{tangent}} = -\frac{1}{2\sqrt{6}}$ ;  $y + 2\sqrt{6} = -\frac{1}{2\sqrt{6}}(x + 1)$ ;  
 $2\sqrt{6}y + 24 = -x - 1$ ;  $2\sqrt{6}y + x + 25 = 0$

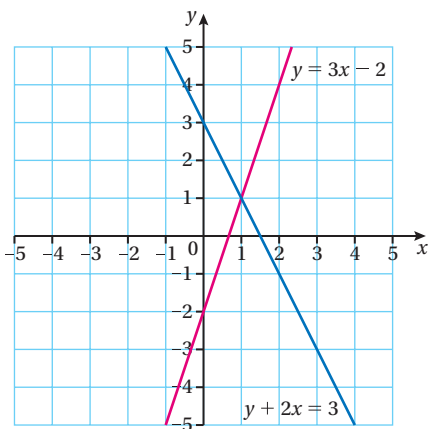
**CHAPTER REVIEW**

**1 a, b, c**



- 2 a** **i**  $y = 6x - 1$       **b**  $y = 1 - x$   
**c**  $y = 3x + 1$       **d**  $y = 2 - x$   
**ii**  $y = -1 - x$   
**iii** yes
- 3**  $m = -4$ ;  $y - 4 = -4(x - 2)$ ;  $y = -4x + 12$  so Option A

**4 a**



- b**  $3x - 2 = -2x + 3$   
 $5x = 5$   
 $x = 1$   
 $3 - 2 = 1$   
 $(1, 1)$
- 5 a**  $y = 3 - \frac{3x}{2}$       **b**  $y = 3x - 9$
- 6 a** Line A  $y = -\frac{1}{2}x$       Line B  $y = x + 4$       Line C  $y = 2x - 6$
- b** Gradient of A =  $-\frac{1}{2}$  gradient of C = 2       $-\frac{1}{2} \times 2 = -1 \therefore A \perp C$
- c** Gradient of B = 1 gradient of C = 2 the gradients are not identical  $\therefore B$  is not  $\parallel C$
- d** x-intercept of B is when  $y = 0$        $0 = x + 4$        $x = -4$
- e** y-intercept of C is when  $x = 0$        $y = 0 - 6$        $y = -6$
- f**  $y = -\frac{1}{2}x + 4$

**30 Interpreting graphs**

**BEFORE YOU START ...**

- 1** A: inverse proportion means that  $y = \frac{k}{x}$ , where  $k$  is a constant.  
A is the graph of  $y = \frac{6}{x}$  therefore represents an inverse proportion.
- 2** 1.5
- 3** 47.25 cm<sup>2</sup>

**LAUNCHPAD**

- 1 a** Something traveling at constant speed  
**b** Something at rest/stationary  
**c** Something traveling at a constant speed then very rapidly returning to its starting point  
**d** Something traveling very rapidly, stopping for a time, then traveling very rapidly again, then stopping for a time  
**e** Something traveling for a time at a slow speed, then very rapidly, then at a slower speed again but faster than the initial speed  
**f** Something traveling back towards a fixed point (It started at the far left hand side of the sketch!) and then moving back away from it again (or passing it and continuing)

**2** Check that students' justifications are sensible.

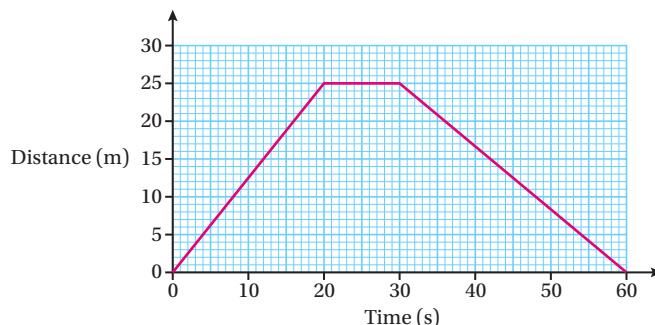
- a** A  
**b** D  
**c** B  
**d** B  
**e** A  
**f** C

**3** B: area under the graph is the rate of flow  $\times$  time

**EXERCISE 30A**

- 1** Option C
- 2 a** Driving at constant speed; stationary; driving back to base at constant speed; stationary; driving at constant speed.
- b**  $50 + 40 + 40 = 130$  minutes (or 2 hours 10 minutes) as we presume this is why the taxi is stationary for these periods where the graph is horizontal and no distance is traveled.
- c** 25 miles
- d** **i** 12 mph      **ii** 10 mph      **iii** 6 mph  
**iv** 6.25 mph
- 3 a** 720 m      **b** 7 mins      **c** 0907 and 0921
- d** going to the supermarket

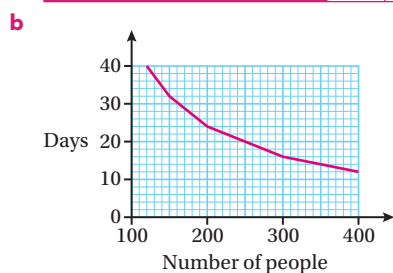
**4 a**



- b** 15m      **b** 5m

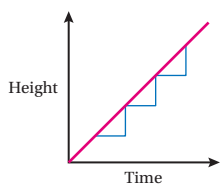
**5 a**

<b>No. of people</b>	120	150	200	300	400
<b>Days the water will last</b>	40	32	24	16	12

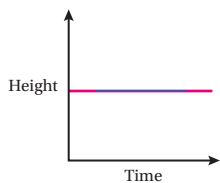


**EXERCISE 30B**

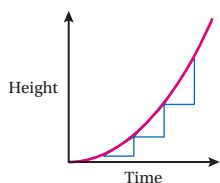
**1 a** Tank being filled at a constant rate



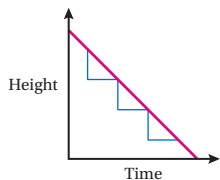
**b** No water going in



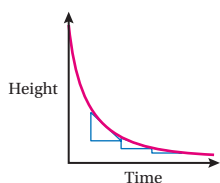
**c** Water flow is increasing



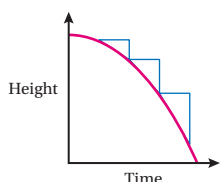
**d** Water draining out of tank at a constant rate



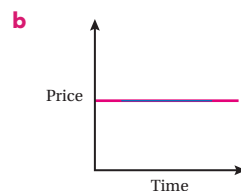
**e** Water draining out of tank, flow is slowing



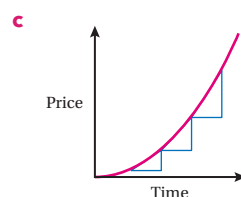
**f** Water draining out of tank, flow is increasing



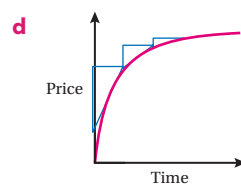
The price is falling at a decreasing rate – the gradient triangles all have negative gradients getting less steep as time goes on



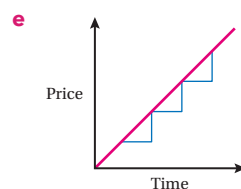
The price is constant – gradient triangle has a gradient of 0



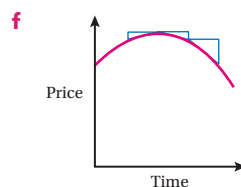
The price is rising at an increasing rate – the gradient triangles all have positive gradients getting more steep as time goes on



The price is rising at a decreasing rate – the gradient triangles all have positive gradients getting less steep as time goes on



The price is rising at a fixed rate – each gradient triangle has the same gradient



The price rises (gradients positive) then decreases (gradients negative)

**3** accelerating at decreasing rate; constant velocity; decelerating at decreasing rate; constant velocity

### EXERCISE 30C

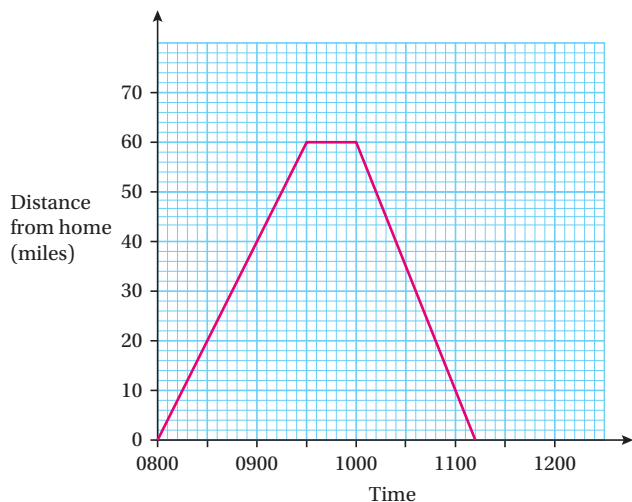
- 1 a 135 m (allow 5 m either way)  
 b 1.2 seconds  
 c 2.5 to 4 seconds  
 d i Approx. 32 m/s                      ii Approx. 140 m/s  
 e 99 m/s
- 2 a 7.15am      b 10.30am  
 c i Approx. 0.35 m/h approx.      ii 1 m per hour  
 d To holiday makers, the coast guard and anyone using the beach
- 3 a A: The car accelerates from 0 km/h to 60 km/h in 5 s. B: It travels at a constant speed of 60 km/h for 5 s. C: It accelerates to a top speed of 90 km/h in 10 s. D: It then decelerates at a constant deceleration from 90 km/h to 0 km/h in 40 s.  
 b 75 km/h  
 c The car reaches its top speed of 90 km/h.  
 d 12 km/h/s or 3.33 m/s<sup>2</sup>  
 e 2.25 km/h/s or 0.625 m/s<sup>2</sup>
- 4 a i 4              ii -2  
 b (-1.5, 2.25)

### EXERCISE 30D

- 1 a 80 km                                      b 120 km  
 c 93.75 km                                  d approx 64 km
- 2 a area under graph would be the amount of water that had flowed down the river.  
 b can't tell as the graph doesn't start at zero  
 c 210 000 cubic feet  
 d 180 000 cubic feet  
 e When it rains there will be more water running into the river upstream

### CHAPTER REVIEW

1 a



- b Yes, the line representing his journey crosses the x axis before 1130
- 2 a 2.5 min                                      b 0.3 km/min  
 c 15 km                                        d 17.5 km  
 e 0.3 km/min<sup>2</sup>                                f 90 km/h

- 3 a -4.75 (±2.5)  
 b The gradient represents the rate at which the population is decreasing.

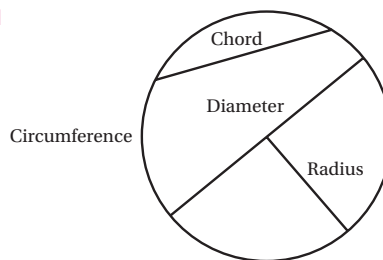
## 31 Circles

### BEFORE YOU START ...

- 1 a False      b True      c False      d True
- 2 a Arc length =  $\frac{6}{5}\pi \approx 3.8$  cm; area =  $\frac{9}{5}\pi \approx 5.7$  cm<sup>2</sup>  
 b Arc length =  $\frac{175}{4}\pi \approx 137.4$  mm; area =  $\frac{4375}{8}\pi \approx 1718.1$  mm<sup>2</sup>  
 c Arc length =  $10\pi \approx 31.4$  cm; area =  $60\pi \approx 188.5$  cm<sup>2</sup>
- 3 a Correct      b InCorrect      c Correct

### LAUNCHPAD

1



- 2 12756 km
- 3 a 65 degrees  
 b Angle APB = 90 because the angle at the circumference subtended from the diameter is a right angle. Angle AQB = 90 because opposite angles in a cyclic quadrilateral sum to 180. Angle PBA = Angle BAQ and Angle PAB = Angle ABQ because alternate angles are equal.  
 Hence Angle PAQ = Angle AQB = 90 and hence as all four angles are right angles the shape APBQ must be a rectangle.
- 4 Angle ADB = 90° because the angle on the circumference subtended from the diameter is 90°. From first principles: angles in a triangle sum to 180°, so  $x + y + y + x = 180$  and hence  $2x + 2y = 180$  so therefore  $x + y = 90$  and hence Angle ADB =  $x + y = 90$ .

### EXERCISE 31A

- 1 a E radius      b F major segment      c A sector  
 d B tangent      e D circle                      f C minor arc
- 2 a diameter      b radius                                      c larger  
 d minor arc      e major arc                              f chord  
 g centre, EF      h AB
- 3 a They are equal because base angles in an isosceles triangle are equal.  
 b As OE = OF, because both are radii of the smaller triangle, triangle OEF is an isosceles triangle and the base angles OFE = OFE. Similarly, OB = OC as they are both radii of the larger circle and hence angles OBC = OCB. Triangles OBC and OEF both share angle EOF and hence they are similar triangles because they share the same angles and angle OEF = OFE = OCB = OBC.
- 4 a The radius is half the diameter, so 12.5 cm  
 b The diameter is twice the radius, so 50 cm

### EXERCISE 31B

- 1 Option A  
 2 Let Angle OAC =  $x$  and angle OAB =  $y$ . Angle OAC = OCA as base angles in an isosceles triangle are equal. Similarly, Angle

$OAB = OBA$ . Therefore angle  $CAB = x + y$ . As angles in a triangle sum to 180 angle  $AOC = 180 = 2x$  and angle  $AOB = 180 = 2y$ . Angles around a point sum to 360 and hence  $COB = 360 - (180 - 2x) - (180 - 2y) = 2x + 2y = 2(x + y) = 2CAB$  as required.

**3** Let  $OXY = x$  and  $OZY = y$ . Angle  $OXY = OYX$  as base angles in an isosceles triangle are equal. Similarly, angle  $OZY = OYZ$ . As angles in a triangle sum to 180 degrees  $x + x + y + y = 180$  and hence  $2x + 2y = 180$  and hence  $x + y = 90$ . Therefore angle  $XYO + OYZ = x + y = 90$  so angle  $y$  is a right angle as required.

**4** By the theorem proved in question 1 angle  $DOC = 2DAC = 50^\circ$ . By the same theorem angle  $2DBC = DOC$  and hence angle  $DBC = \frac{1}{2}DOC = \frac{1}{2} \times 50 = 25$  as required.

**5**  $136^\circ$

**6**  $35^\circ$

**7** Angle  $BCD = 180 - 42 = 138$

**8**  $CAB = 60$  and  $CBA = 30$

### EXERCISE 31C

**1** 18 units

**2** 10 units

**3**  $42^\circ$  (Assuming CE is a diameter)

**4**  $x = 20$

**5**  $a$  66,  $b$  24

### EXERCISE 31D

**1** Option B

**2 a** Angle  $BAN = ACB = x$  and angle  $TAC = CBA = x$  by the alternate segment theorem and hence angle  $TAC = ACB$  and angle  $NAB = ABC$ . Therefore the alternative segment theorem holds and hence CB is parallel to TN.

**b** Angle  $ACB = CBA$  and hence triangle ACB is an isosceles triangle because base angles in an isosceles triangle are equal. Therefore as triangle ACB is an isosceles triangle  $AC = AB$ .

**3 a**  $x$                       **b**  $180 - 2x$                       **c**  $2x$

**d**  $90 - x$                       **e**  $90 - x$

**4 a**  $a = 78$  because opposite angles in a cyclic quadrilateral sum to 180,  $b = 102$  because co-interior angles sum to 180 degrees,  $c = 78$  because opposite angles in a cyclic quadrilateral sum to 180.

**b**  $x = 36$  because  $2x + 3x = 180$  because opposite angles in a cyclic quadrilateral sum to 180,  $3x + y = 180$  because angles on a straight line sum to 180 so  $y = 72$ ,  $z = 96$  because opposite angles in a cyclic quadrilateral sum to 180.

**c**  $a = 62$  because opposite angles in a cyclic quadrilateral sum to 180,  $b = 90$  because angles subtended from the diameter at the circumference are right angles,  $c = 28$  because angles in a triangle sum to 180.

**d**  $p = 120$  because  $LM = LP$  and hence angle  $LMP = LPM$  because base angles in an isosceles triangle are equal and angles on a straight line sum to 180.

### WORK IT OUT 31.1

Option A is incorrect because it states angle  $OTQ = PQT$  which is not true.

Option B is correct.

Option C is incorrect because angle BA is not equal to OCB.

### EXERCISE 31E

**1 a** 14                      **b** 36                      **c** 64

**d** 80                      **e** 60

**2** Example reasons given, there may be more than one way of finding the missing angle.

**a** Angle  $EHO = 90^\circ$ , radii bisect chords at  $90^\circ$  only.

**b** Angle  $GFB = 62^\circ$ , because of the alternate segment theorem.

**c** Angle  $GBF = 28^\circ$ , angles in a triangle sum to  $180^\circ$  and the angle on the circumference subtended from the diameter is  $90^\circ$ .

**d** Angle  $FEG = 28^\circ$ , the angle on the circumference subtended from the diameter is  $90^\circ$  (angle  $BEF$ ) and  $BEG$  is  $62^\circ$  because of the alternate segment theorem.

**e** Angle  $DBF = 62^\circ$ , since GF and BD are parallel angle  $DBG = 180 - \text{angle } BGF = 90^\circ$  (complementary angles). Thus, angle  $DBF = 90 - \text{angle } GBF = 90 - 28 = 62^\circ$ .

**f** Angle  $GEH = 90^\circ$ , since opposite angles in a cyclic quadrilateral sum to  $180^\circ$ . Cyclic quadrilateral here is BGED, so angle  $GEH = 180 - \text{angle } GBD = 180 - 90 = 90^\circ$ .

**3**  $2x$

**4 a**  $90 - x$                       **b**  $180 - 2x$                       **c**  $2x - 90$

### CHAPTER REVIEW

**1** Option B

**2 a**  $x = 140$  because the angle subtended from the chord at the centre is twice the angle subtended from the same chord at the circumference in the same arc.

**b**  $y = 110$  because opposite angles in a cyclic quadrilateral sum to  $180^\circ$ .

**3 a**  $a = 54$

**4** angle  $x = 50^\circ$ ; angle  $y = 28^\circ$ ; angle  $z = 124^\circ$

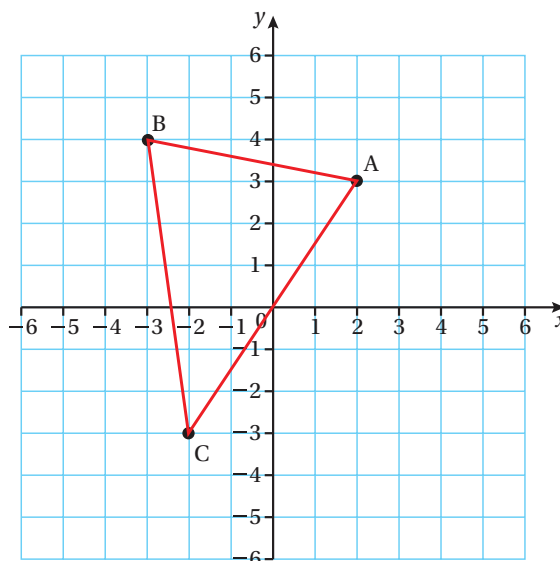
**5 a**  $x = 150^\circ$

**b**  $y = 57^\circ$

## 32 Vector geometry

### BEFORE YOU START ...

**1 a**

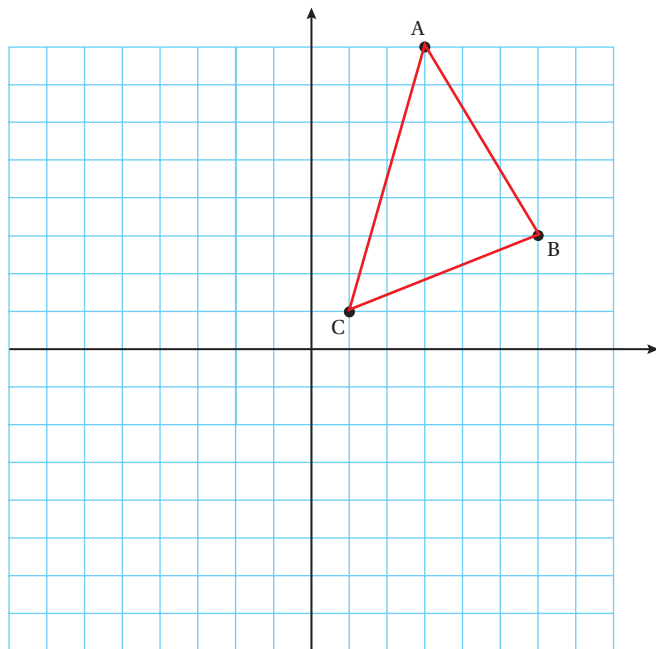


- 2 a -4    b 7    c -23    d -28    e 27  
 3 a  $m = 12$     b  $k = -4$     c  $d = -7$   
 4  $x = 2, y = 1$

**LAUNCHPAD**

1  $\vec{HG} = \begin{pmatrix} -3 \\ -4 \end{pmatrix}$

2



3 a  $\mathbf{j} + \mathbf{k} = \begin{pmatrix} 1 \\ 4 \end{pmatrix}$     b  $2\mathbf{k} - \mathbf{l} = \begin{pmatrix} 8 \\ 4 \end{pmatrix}$

4  $f = 3, g = 6$

5  $\vec{CA} = \begin{pmatrix} -14 \\ -2 \end{pmatrix}$      $\vec{CA} + \vec{AB} = \begin{pmatrix} -5 \\ 10 \end{pmatrix}$

6  $\begin{pmatrix} -3 \\ 4 \end{pmatrix}$  and  $\begin{pmatrix} 15 \\ -20 \end{pmatrix}$

7 a  $\vec{BE} = \mathbf{j}$     b  $\vec{AF} = 2\mathbf{j} + \mathbf{k}$     c  $\vec{AM} = 2\mathbf{j} + \frac{\mathbf{k}}{2}$

**EXERCISE 32A**

- 1 1 C iv    2 D ii    3 B i    4 E i    5 A iii

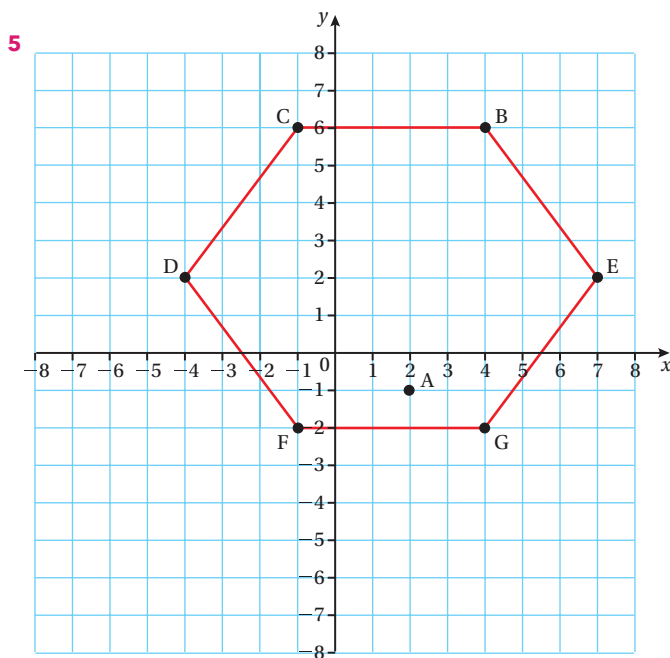
2 Option D

3 Option C

4 a i  $\begin{pmatrix} 2 \\ -1 \end{pmatrix}$     ii  $\begin{pmatrix} -2 \\ 1 \end{pmatrix}$     iii  $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$

iv  $\begin{pmatrix} -3 \\ -2 \end{pmatrix}$     v  $\begin{pmatrix} -5 \\ 1 \end{pmatrix}$     vi  $\begin{pmatrix} 6 \\ -3 \end{pmatrix}$

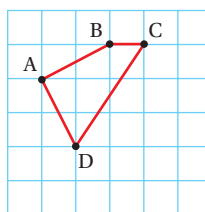
- b Vectors  $\vec{AB}$  and  $\vec{DC}$  are the same length and go in opposite directions.  
 c Vectors  $\vec{AB}$  and  $\vec{BH}$  are parallel,  $\vec{BH}$  is 3 times as long as  $\vec{AB}$ .



6 a  $\begin{pmatrix} -4 \\ 6 \end{pmatrix}$     b an example is E(0, 0) and F(-4, 6)

7 a  $\begin{pmatrix} -6 \\ 10 \end{pmatrix}$     b (-5, 4)

8



9 a  $k \begin{pmatrix} 1 \\ 1 \end{pmatrix}$  where  $-7 \leq k \leq 7$

b  $\begin{pmatrix} 1 \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \begin{pmatrix} -1 \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ -1 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix} \begin{pmatrix} 1 \\ -1 \end{pmatrix} \begin{pmatrix} -1 \\ 1 \end{pmatrix} \begin{pmatrix} -1 \\ -1 \end{pmatrix}$

c  $\begin{pmatrix} 2 \\ 1 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \end{pmatrix} \begin{pmatrix} -2 \\ -1 \end{pmatrix} \begin{pmatrix} -1 \\ -2 \end{pmatrix} \begin{pmatrix} 2 \\ -1 \end{pmatrix} \begin{pmatrix} 1 \\ -2 \end{pmatrix} \begin{pmatrix} -2 \\ 1 \end{pmatrix} \begin{pmatrix} -1 \\ 2 \end{pmatrix}$

- 10 By using Pythagoras' theorem; Magnitude doesn't have a direction; it is a scalar quantity;  $\sqrt{((-4)^2 + 2^2)} = \sqrt{20} = 4.47$  (3 dp);  $\sqrt{((-3)^2 + 5^2)} = \sqrt{34} = 5.83$  (3 dp)

**WORK IT OUT 32.1**

Option C

**EXERCISE 32B**

1 Option D

2 a i  $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$     ii  $\begin{pmatrix} 7 \\ -5 \end{pmatrix}$     iii  $\begin{pmatrix} -12 \\ 8 \end{pmatrix}$     iv  $\begin{pmatrix} -12 \\ 21 \end{pmatrix}$

v  $\begin{pmatrix} -1 \\ -1 \end{pmatrix}$     vi  $\begin{pmatrix} -9 \\ 17 \end{pmatrix}$

b result iii (vector  $4\mathbf{p}$ ) is parallel to  $\begin{pmatrix} 3 \\ -2 \end{pmatrix}$

3 three vectors of the form  $\begin{pmatrix} 2k \\ -3k \end{pmatrix}$  for any three values of  $k$

4 a  $x = 4, y = 0$     b  $x = 12, y = 5$     c  $x = 17, y = -5$

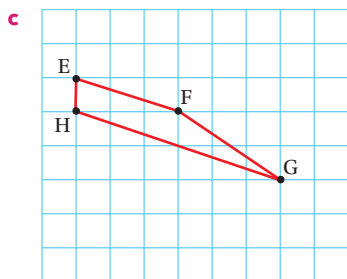
d  $z = -2, x = -3.5$     e  $z = -\frac{1}{4}, y = 32$     f  $z = 3, y = 6$

g  $z = 3, x = 5$     h  $z = 4, t = 3$

5 a  $\begin{pmatrix} 5 \\ 4 \end{pmatrix}$       b  $\begin{pmatrix} -15 \\ -12 \end{pmatrix}$

6 a sides EF and HG are parallel

b trapezium



c  $\vec{GF} = \begin{pmatrix} -3 \\ 2 \end{pmatrix}$

7 It is a parallelogram, because opposite sides, AB and DC, and DA and CB, are parallel. (It could also be a square or a rhombus or a rectangle.)

### EXERCISE 32C

1 Option D

2 a  $\begin{pmatrix} -10 \\ -2 \end{pmatrix}$       b  $\begin{pmatrix} -2 \\ 12 \end{pmatrix}$       c  $\begin{pmatrix} -1 \\ 6 \end{pmatrix}$

3 a i  $\vec{DE} = 2\vec{AB}$     ii  $\vec{DF} = 2\vec{AC}$

b The two triangles are similar. DEF is an enlargement of ABC, scale factor 2

4 a p; ABCD is a square, so BC is parallel to AD and of equal length (magnitude)      b -m      c m + p

d m - p

5 Option D

6 a i  $\vec{AC} = \mathbf{p} + 2\mathbf{q}$ ;  $\vec{BC} = 2\vec{BM} = 2\mathbf{q}$

ii  $\vec{DB} = \mathbf{p} - 2\mathbf{q}$ ;  $\vec{DB} = \vec{CB} = 2\vec{BC} = -2\mathbf{q}$

iii  $\vec{MD} = \mathbf{q} - \mathbf{p}$ ;  $\vec{MC} = \vec{BM} = \mathbf{q}$  and  $\vec{CD} = \vec{BA} = -\vec{AB} = -\mathbf{p}$

b  $\vec{NM} = \frac{1}{2}\mathbf{p} - \mathbf{q}$ ;  $\vec{DB} = \mathbf{p} - 2\mathbf{q}$ ;  $2\left(\frac{1}{2}\mathbf{p} - \mathbf{q}\right) = \mathbf{p} - 2\mathbf{q}$ , so  $\vec{NM}$  is parallel to  $\vec{DB}$

7 a -n      b -m      c n + m      d 2n - m

8 a i  $\frac{1}{2}\mathbf{e}$       ii  $-\frac{1}{2}\mathbf{g}$       iii g - e

iv  $\frac{1}{2}\mathbf{g}$       v  $-\frac{1}{2}\mathbf{e}$

b Triangle HIJ is equilateral and similar to triangle EFG and an enlargement, sf  $\frac{1}{2}$ .

9 a  $-\frac{1}{2}\mathbf{e}$       b  $\frac{1}{2}(\mathbf{g} - \mathbf{e})$       c  $\frac{1}{2}\mathbf{g}$

d g      d  $\frac{1}{2}(\mathbf{g} - \mathbf{e})$

10 a 5T      b 4T + R

c  $\frac{1}{2}(5T + R)$       d  $-2\frac{1}{2}T$

11 a q - p      b  $\frac{3}{4}(\mathbf{q} - \mathbf{p})$

c  $\vec{OM} = \vec{OP} + \vec{PM} = \mathbf{p} + \frac{3}{4}(\mathbf{q} - \mathbf{p}) = \frac{3}{4}\mathbf{q} + \frac{1}{4}\mathbf{p} = \frac{1}{4}(3\mathbf{q} + \mathbf{p})$

12 24m; 26.8m

### CHAPTER REVIEW

1 The coordinate (-2, 3) is a point in two-dimensional space. The vector  $\begin{pmatrix} -2 \\ 3 \end{pmatrix}$  describes movement from one point to another. It has magnitude and direction.

2 a and c; b and d; e, f and g

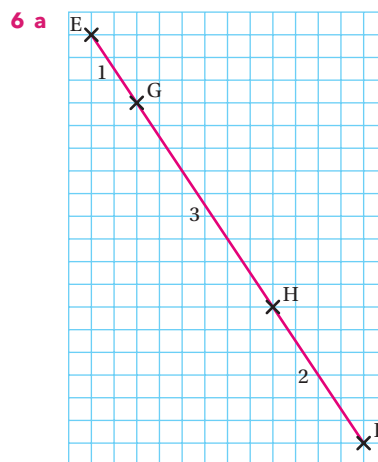
3 a  $\begin{pmatrix} -1 \\ -3 \end{pmatrix}$       b  $\begin{pmatrix} 2 \\ -7 \end{pmatrix}$       c  $\begin{pmatrix} -6 \\ 3 \end{pmatrix}$

4 option A

5 a  $\vec{EG} = 5(2\mathbf{e} + \mathbf{f}) = 10\mathbf{e} + 5\mathbf{f}$

b  $\vec{FG} = 3(2\mathbf{e} + \mathbf{f}) = 6\mathbf{e} + 3\mathbf{f}$

c  $\vec{OF} = 7\mathbf{e} + 4\mathbf{f}$



b i  $6\mathbf{e} - 9\mathbf{f}$       ii  $12\mathbf{f} - 8\mathbf{e}$

### 33 Transformations in a plane

#### BEFORE YOU START ...

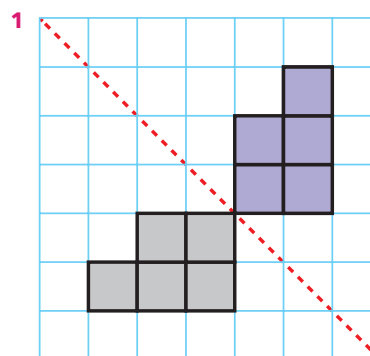
1 a 90° anti-clockwise      b 180°      c 270° clockwise

2 a y = 2      b y = x      c x = -1      d y = -x

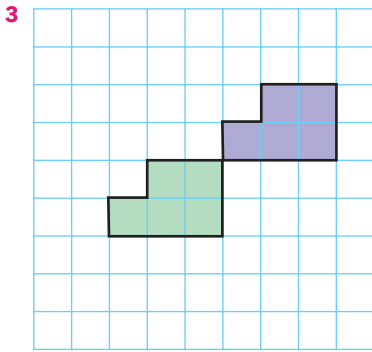
3 a The coordinate (3, 2) is a single point in x,y space. The vector  $\begin{pmatrix} 3 \\ 2 \end{pmatrix}$  has magnitude and direction, and determines the relative position of one point in x,y space to another.

b The vectors are perpendicular to each other.

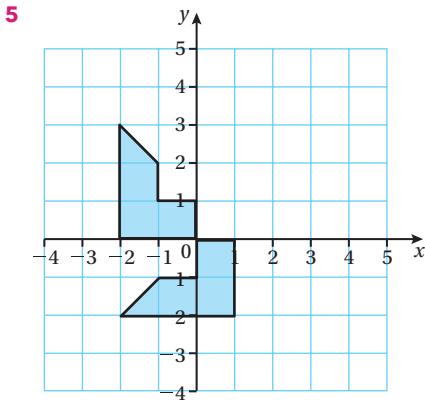
#### LAUNCHPAD



2  $y = -x$

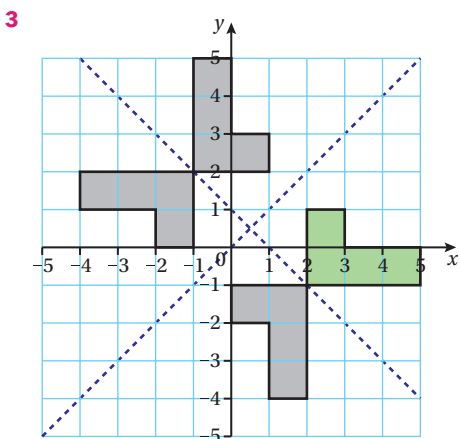
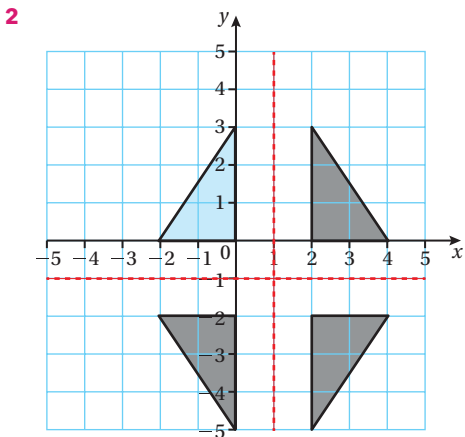


4  $\begin{pmatrix} 2 \\ -4 \end{pmatrix}$

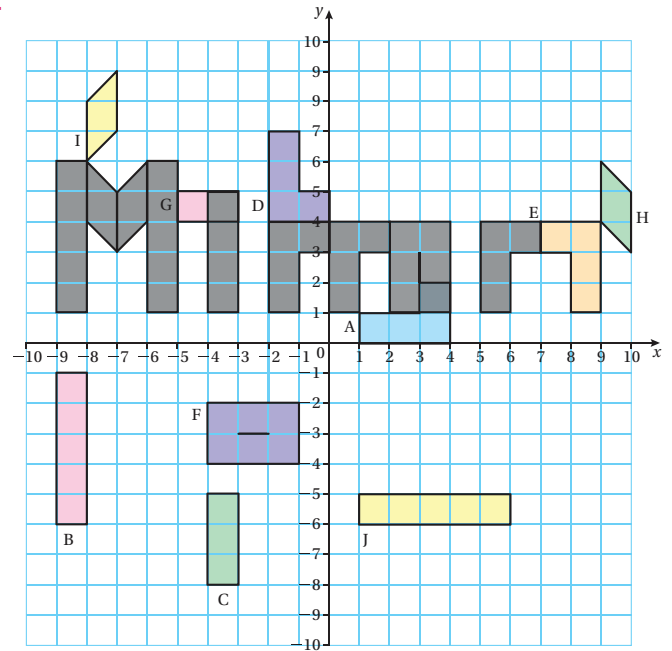


**EXERCISE 33A**

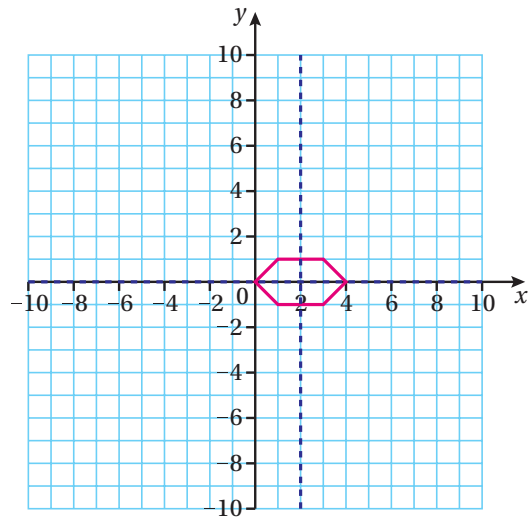
1 Option B



4



5



Reflect in  $y = 0$ , then reflect in  $x = 2$  (or reverse order).

6 Students' own puzzles

**WORK IT OUT 33.1**

Option C is correct.

Option A is a reflection in  $x = -1$ .

Option B is not a reflection.

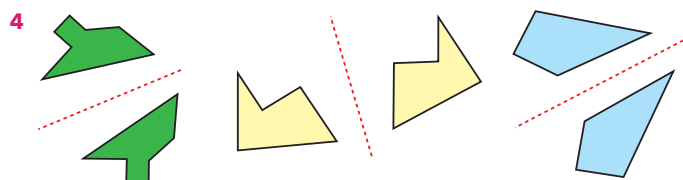
**EXERCISE 33B**

1 Option C

2 a  $y = x - 1$     b  $y = 2 - x$     c  $y = 0.5$     d  $y = 1.5 - x$

3 a i  $y = 2.5$     ii  $x = 6.5$     iii  $y = x - 4$     iv  $y = 9 - x$





- 5 a i** One possibility is reflection in  $x = 1$ , followed by reflection in  $y = 1$ .
- ii** One possibility is reflection in  $y = -x$ , followed by reflection in  $y = -0.5$  and then in  $x = -0.5$ .
- b** There is more than one answer.
- c** In some cases the order does matter.
- d** If the shapes have been 'flipped' as in **i** you need an odd number of reflections, if they haven't been 'flipped' you need an even number of reflections.

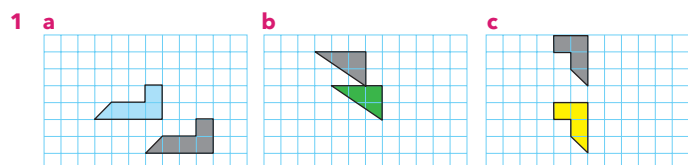
### WORK IT OUT 33.2

Option B is correct.

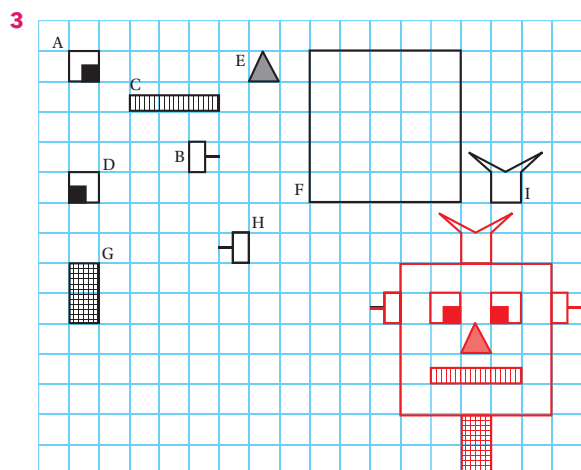
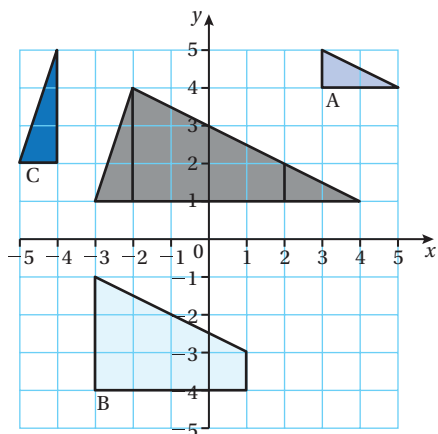
Option A is translation  $\begin{pmatrix} 5 \\ -2 \end{pmatrix}$

Option C is a translation  $\begin{pmatrix} -1 \\ 3 \end{pmatrix}$

### EXERCISE 33C



- 2** Shape that is created is a triangle.



### WORK IT OUT 33.3

Transformation B is a reflection because if the line  $x = y$  is drawn it acts as a mirror line. Each vertex of the object triangle is the same distance from the mirror line as its image is on the other side of the mirror line.

Transformations A and E are translations because each object and its image are oriented the same way and the translation can be described by a column vector in each example.

### EXERCISE 33D

- 1** Option D
- 2**  $S \begin{pmatrix} 3 \\ -7 \end{pmatrix}$      $L \begin{pmatrix} 0 \\ 6 \end{pmatrix}$      $I \begin{pmatrix} -8 \\ -10 \end{pmatrix}$      $D \begin{pmatrix} 4 \\ 5 \end{pmatrix}$      $E \begin{pmatrix} -13 \\ 5 \end{pmatrix}$
- 3** Students' own answers.

### WORK IT OUT 33.4

Option B is correct.

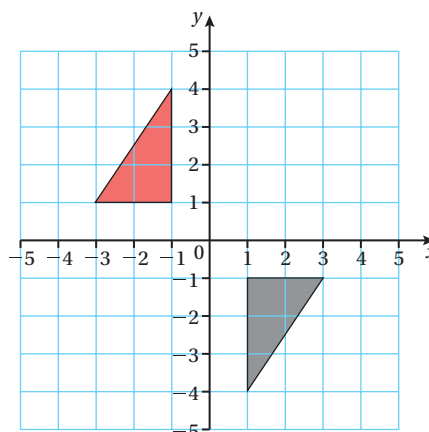
Option A is a rotation about the origin.

Option C is a rotation in clockwise direction.

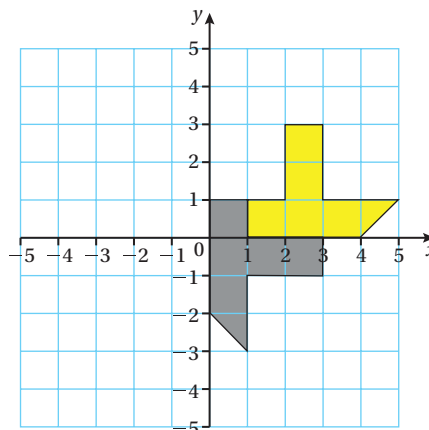
### EXERCISE 33E

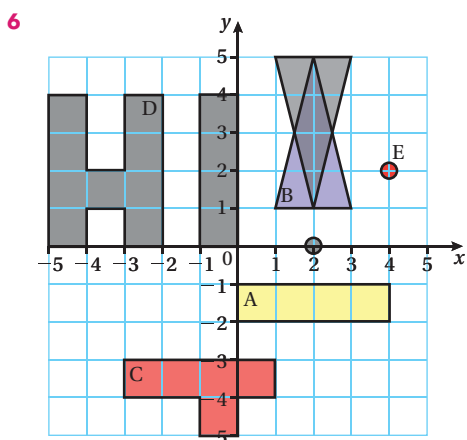
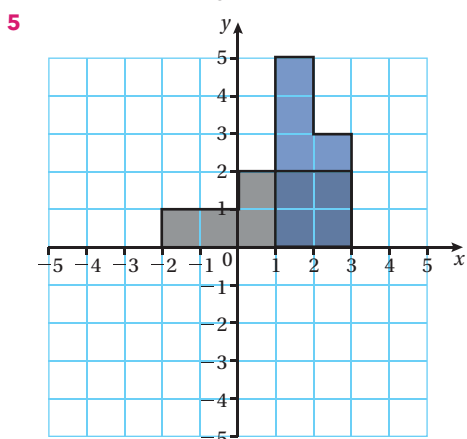
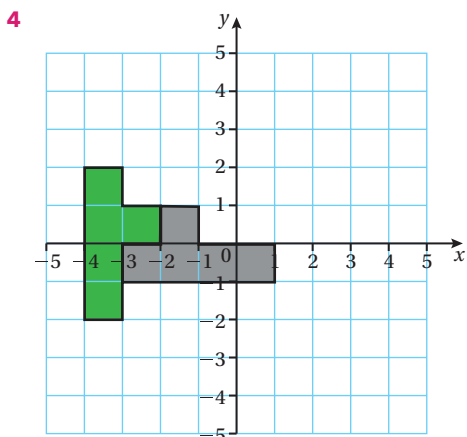
- 1** Option B

**2**



**3**





7 Each coordinate is a combination of the same two numerals; for example  $(-1, 5)$ ,  $(5, 1)$ ,  $(1, -5)$ ,  $(-5, -1)$ . This result is only obtained if the centre of rotation is the origin.

### WORK IT OUT 33.5

Transformation A is a rotation because each of the vertices of the object can be rotated around a centre of rotation  $(-1.5, \frac{1}{2})$ . Each vertex is rotated by the same angle, clockwise around the centre of rotation.

Transformations B and F are reflections because the mirror lines can be drawn and the respective vertices of the objects and images are equidistant from the mirror lines. The mirror lines are  $y = 0$  and  $y = x$  respectively.

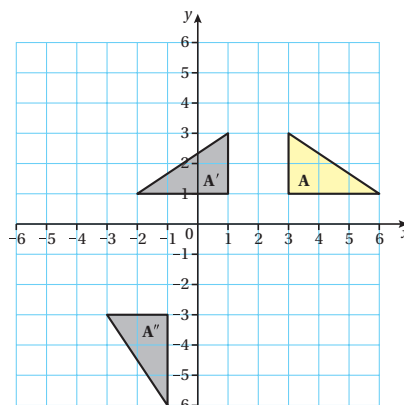
Transformation D is a translation because the object and its image have the same orientation and the translation can be described by a column vector.

### EXERCISE 33F

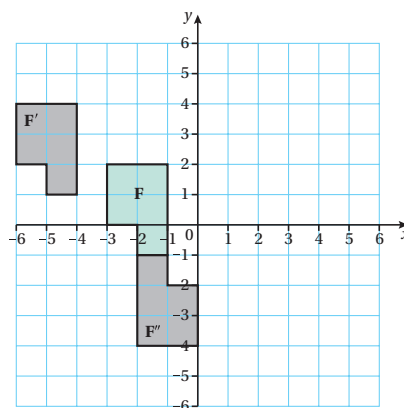
- 1 Option D
- 2 a Rotate  $180^\circ$  centre  $(-1, 0)$
- b Rotate  $90^\circ$  anticlockwise centre  $(2, -3)$
- c Rotate  $90^\circ$  clockwise centre  $(-4, 2)$
- d Rotate  $90^\circ$  anticlockwise centre  $(-2, 3)$
- 3 There are many possible answers for this question which should encourage discussion and comparison of suggestions.
- 4 Check students' diagrams.

### EXERCISE 33G

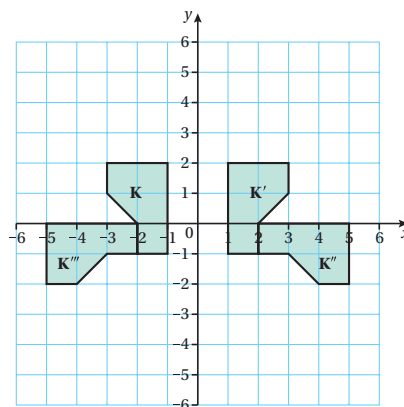
- 1 Option C
- 2 The single transformation from A to A'' is a reflection in the line  $y = -x$ .



- 3 The single transformation from F to F'' is a rotation  $180^\circ$  around the point  $(-1.5, -1)$ .



- 4 Rotation  $90^\circ$  anti-clockwise about the point  $(-2, -1)$ .



- 5 a One possibility - rotate  $90^\circ$  clockwise around  $(1,1)$  and then reflect in the line  $y = 2$ .
- b Yes
- c Students' own work
- 6 Rotation  $180^\circ$  around the origin.
- 7 Many possible answers. Check students' pairs of transformations.

### CHAPTER REVIEW

- 1 a T      b T      c F      d T

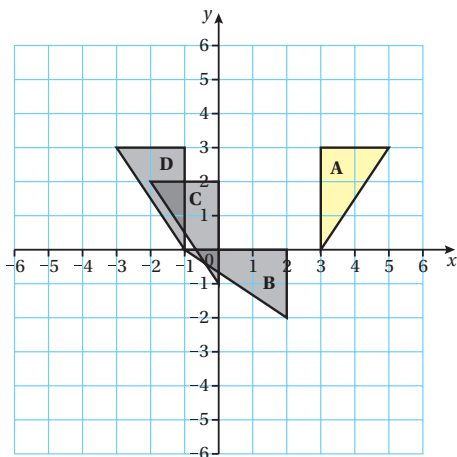
- 2 a Rotation  $180^\circ$  around  $(4, 0)$ .
- b Rotation  $180^\circ$  around  $(3, -2)$ .
- c Translation through vector  $\begin{pmatrix} 2 \\ 4 \end{pmatrix}$ .

3 For example:

Translate the original shape through vectors  $\begin{pmatrix} 2n \\ 0 \end{pmatrix}$  and  $\begin{pmatrix} 0 \\ 6m \end{pmatrix}$  where  $n$  and  $m$  are integers.

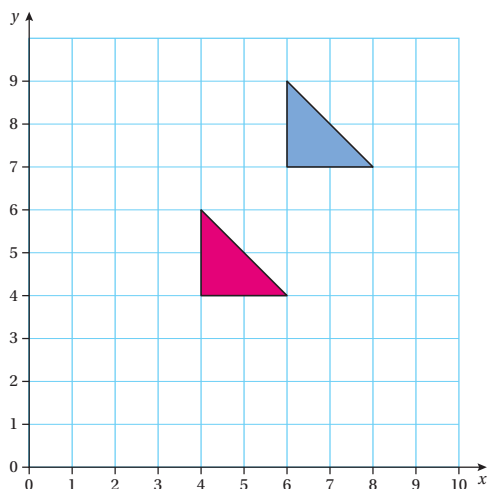
Rotate the original triangle  $180^\circ$  around  $(2, 3)$  and translate this new triangle using the same vectors.

4



The single transformation that maps shape A onto shape D is a reflection in the line  $x = 1$ .

5 a



b Clockwise rotation of  $90^\circ$  about the point  $(4, 3)$

6 For example:

Reflect A in  $y$ -axis to give  $A'$ .

Rotate A,  $90^\circ$  anticlockwise around  $(-2, 3)$  and then translate through vector  $\begin{pmatrix} 0 \\ -5 \end{pmatrix}$  to give  $A'''$ .

Reflect  $A'''$  in  $x = 0.5$  to get  $A''$ .

Rotate B,  $90^\circ$  anticlockwise around  $(0, 4)$  to give  $B'$ .

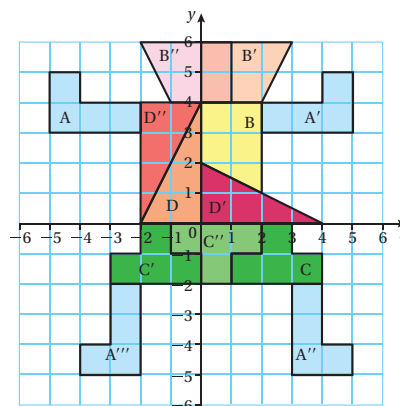
Reflect  $B'$  in the line  $x = 0.5$  to give  $B''$ .

Reflect C in the line  $x = 0.5$  to give  $C'$ .

Rotate C,  $180^\circ$  around  $(1.5, -1)$  to give  $C''$ .

Rotate D,  $90^\circ$  clockwise around the origin to give  $D'$ .

Rotate D,  $180^\circ$  around  $(-1, 2)$ .



## 34 Construction and loci

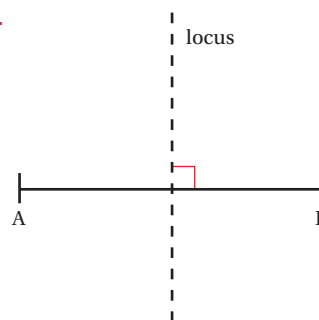
### BEFORE YOU START ...

- 1 a  $120^\circ$       b  $40^\circ$
- 2 Check students' drawings.
- 3 a C      b C
- 4 a side      b vertex      c centre
- d radius      e diameter

### LAUNCHPAD

- 1 A False      B True      C False      D False
- 2 A False      B True      C False      D True
- 3 a Angle bisector
- b Placed a compass at B to draw arcs on BA and BC, then drew arcs from points P and Q and then drew a line BR from point B through the intersection of these arcs
- c  $24^\circ$

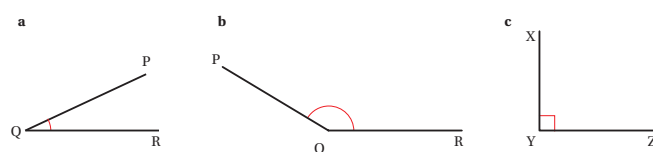
4



### EXERCISE 34A

1 Option A

2 a



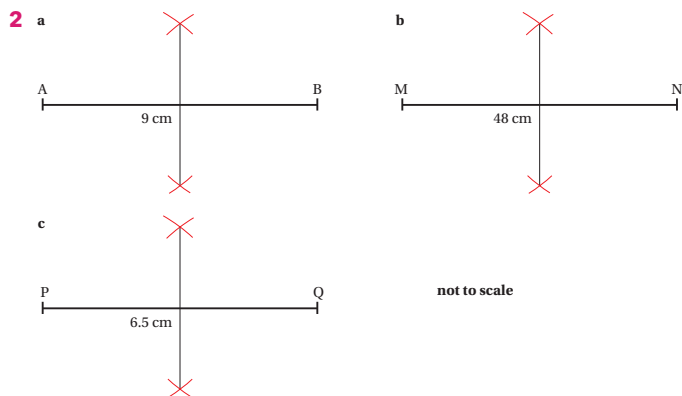
3 Use the protractor to measure the smaller (non reflex) angle at the point, which in this case will measure  $122^\circ$  and subtract this from  $360^\circ$ .

- 4 a Student drawings as specified in question
- b The lengths of the lines are irrelevant because the length of the line does not change the size of the angle.
- 5 Student drawings as specified in question.
- 6 Student drawings as specified in question.
- 7 Student drawings as specified in question. Angles measured according to their diagram.
- 8 Student drawings as specified in question.
- 9 a Draw a line longer than 6.4 cm and mark a point A on this line.  
Set the compass to 6.4 cm and placing the compass at A draw an arc that crosses this line.  
Mark this point B.  
Placing the compass at A draw an arc above the line the crosses above the midpoint of AB.  
Repeat this step from point B.  
Mark the point where the arcs intersect as C.  
Use the ruler to join A to C and B to C.  
ABC is an equilateral triangle with sides of 6.4 cm.
- b Draw a line longer than 60 mm and mark a point O towards the middle of this line  
Set the compass to 30 mm.  
Place the compass at O draw a semicircle by drawing the arc above the line.

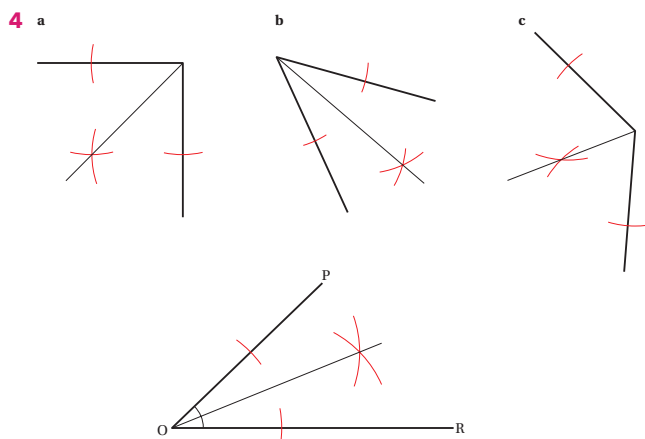
10 Students' own answers

**EXERCISE 34B**

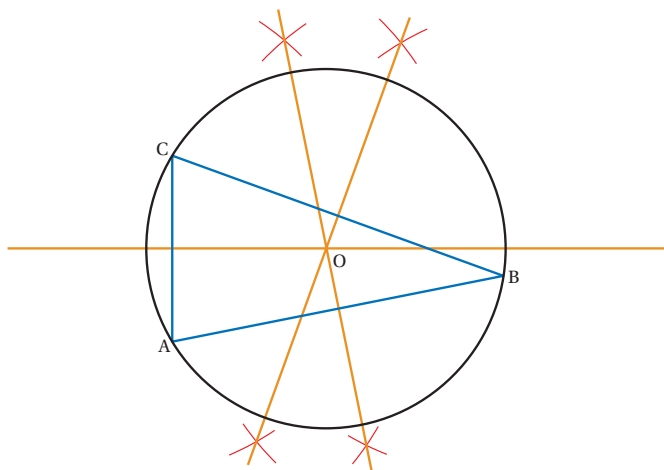
1 Option D



- 3 a Student drawings of angle bisectors.
- b Constructions can be checked for accuracy with a protractor.

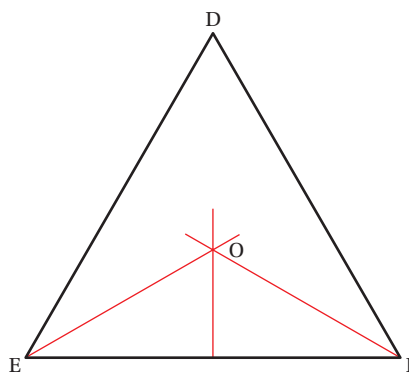


5 a b



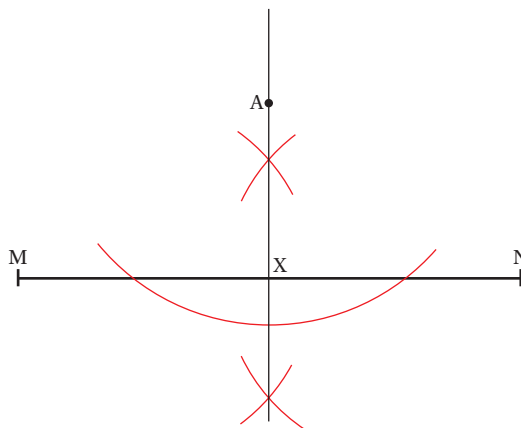
- c For any triangle ABC, a circle with a centre at the intersection of the perpendicular bisectors and passing through point A will also pass through points B and C. This is called the 'circumcircle' of the triangle.

6 a

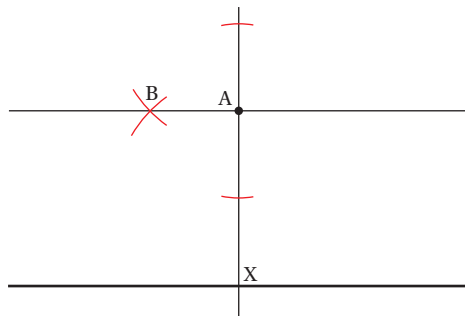


- b  $DO = EO = FO \approx 4$  cm

7 a



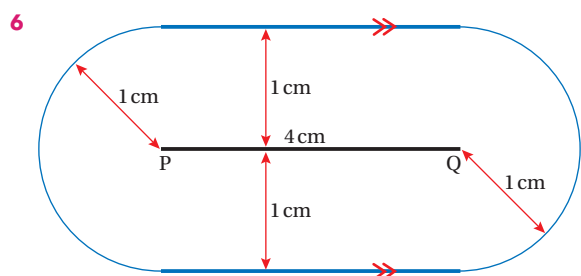
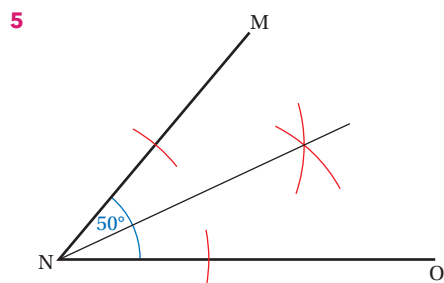
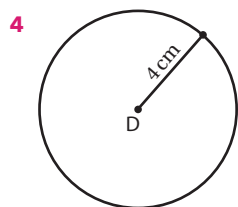
b



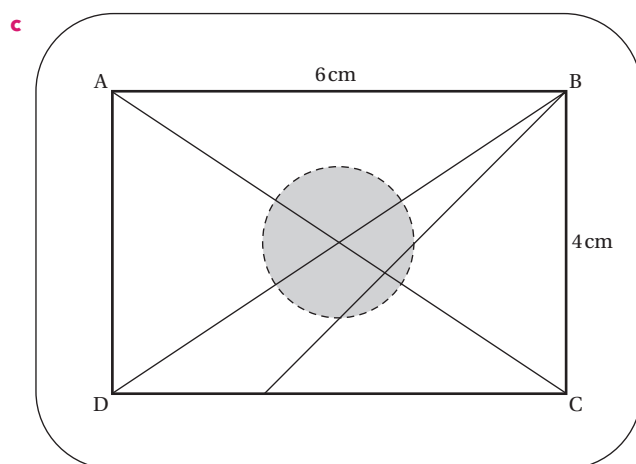
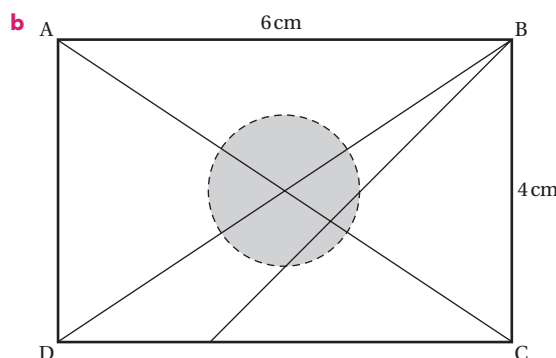
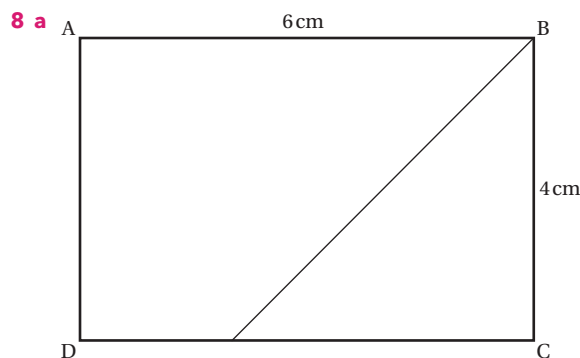
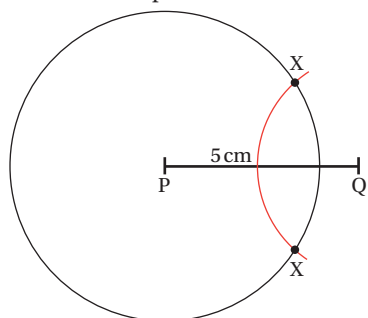
- 8 a Student drawings as specified in question.
- b Yes.  $SOQ = POT$  (vertically opposite)
- 9 a Student drawings as specified in question.
- b Centre of circle
- c Centre lies on perpendicular bisector of any chord; OR Perpendicular from centre to any chord bisects the chord.

### EXERCISE 34C

- 1 Discussion question
- 2 Option C
- 3 a Points on a circle centre at X, radius 200 km.
- b Points within the area defined by lines A and B: line A is an oval defined by parallel lines 1 km long and at 2 km either side of the straight fence joined by semicircles of radius 2 km at each end; line B is an oval defined by parallel lines 1 km long and at 3 km either side of the straight fence joined by semicircles of radius 3 km at each end.
- c Points on the straight line across the centre of the court.
- d The centre spot.
- e Points within the area defined by two parallel lines that are 1 km either side of the railway line.



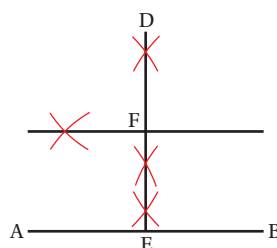
- 7 There are two points that are 4 cm from P and 2.5 cm from Q.



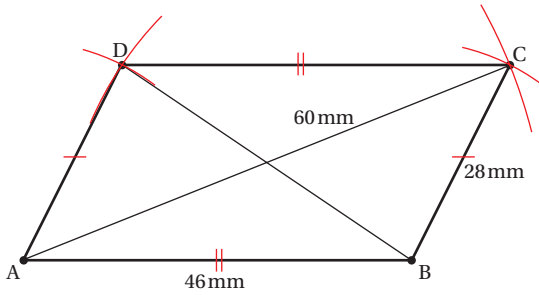
- 9 Draw a square 1 cm bigger than the object but with rounded corners. Draw another square 1 cm smaller than the object with rounded corners. Shade in the area between the outer and inner loci.
- 10 a a succession of vertical semicircles (called a cycloid)
- b a straight line parallel to the road surface

### EXERCISE 34D

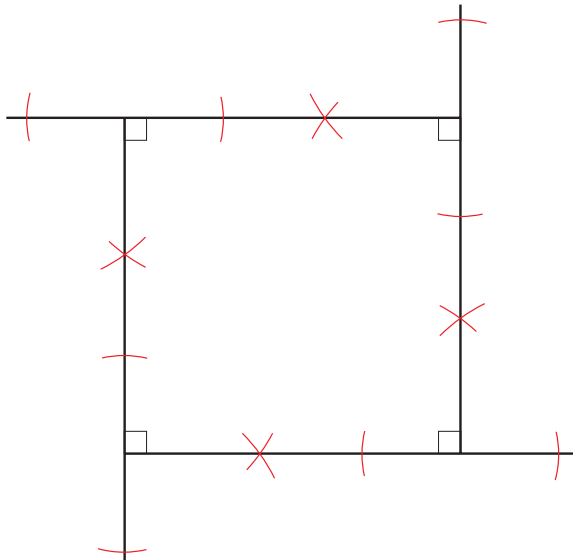
- 1 Option B
- 2



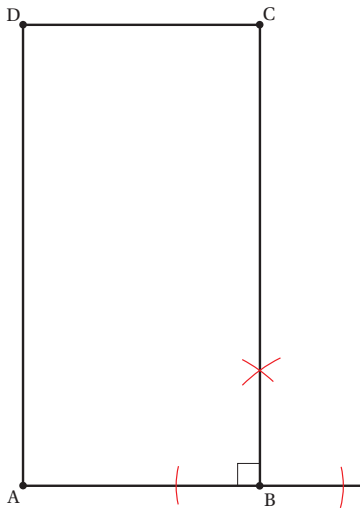
3 Length of diagonal DB is 47 mm.



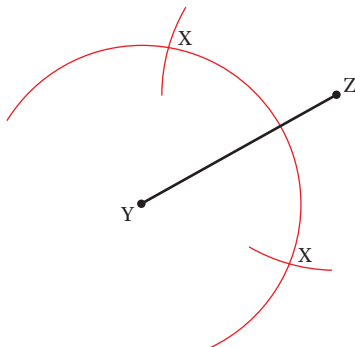
4



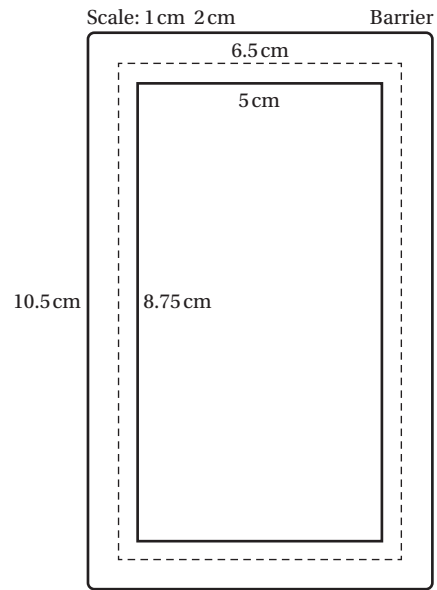
5 A rectangle.



6 Drawings should be to scale 1 cm to 5 m, so line YZ measures 3 cm.



7 a

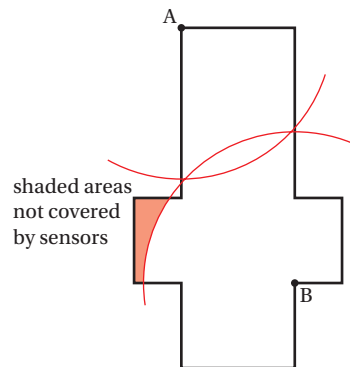


b The barrier should be 1.5 m away from enclosure because people can reach out about 1 m. This means that visitors will not be able to touch the monkeys when both are stretching out (leaving a small margin for error).

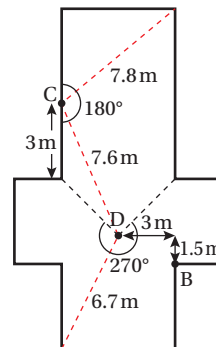
8 Semicircle with diameter 10m with similar semicircle with diameter 11m enclosing the first. This outer shape should have rounded corners at the ends of the diameter.

9 Rectangle at suitable scale with one diagonal perpendicularly bisected within the diagram only.

10 a

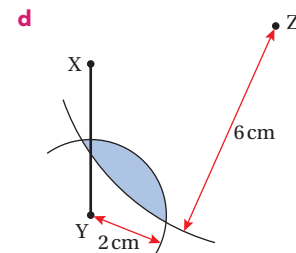
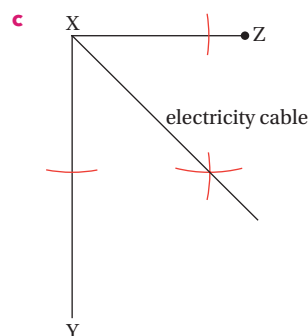
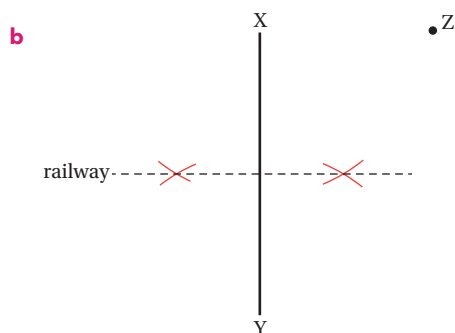
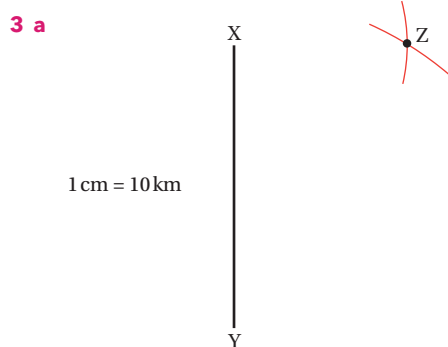
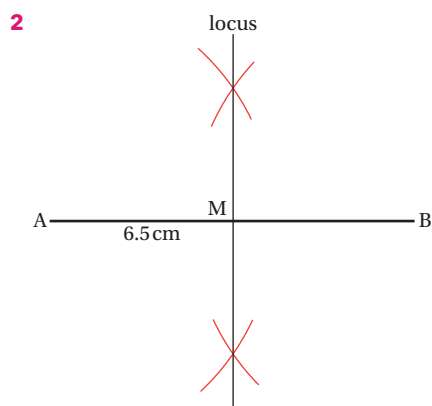
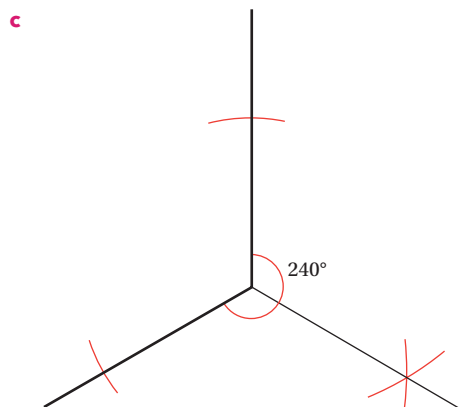
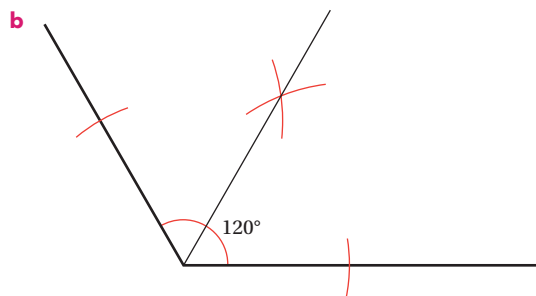


b Sensors should be better placed at C and D to provide maximum coverage.

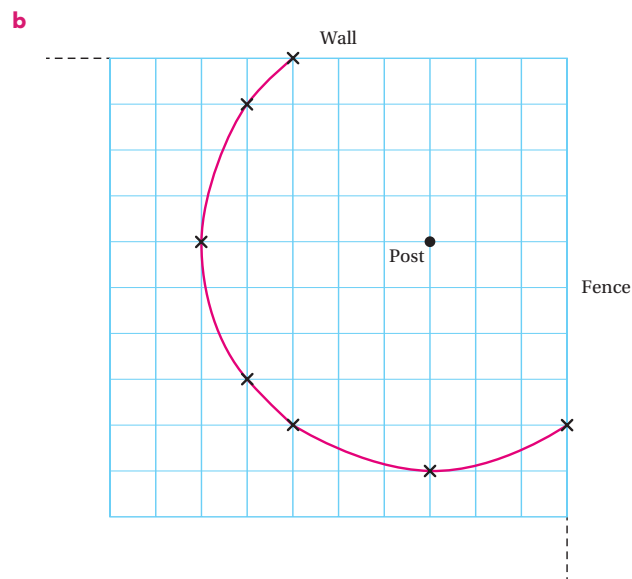


### CHAPTER REVIEW

1 a  $120^\circ$  and  $240^\circ$



4 a 2 m



c 1:50

5 3 concentric circles, radii 1.5, 2.5, 3.5 cm. All shaded except for the smallest circle

6 Students' own answers

## 35 Similarity

### BEFORE YOU START ...

1 a BOC      b  $80^\circ$       c  $150^\circ$

2 Side DF = side AC, angle EDF = angle BAC, side DE = side AB, SAS congruency

3 a  $x = 8$       b  $h = 2.5$       c  $k = 0.25$

4 C

5  $x = 0.67$

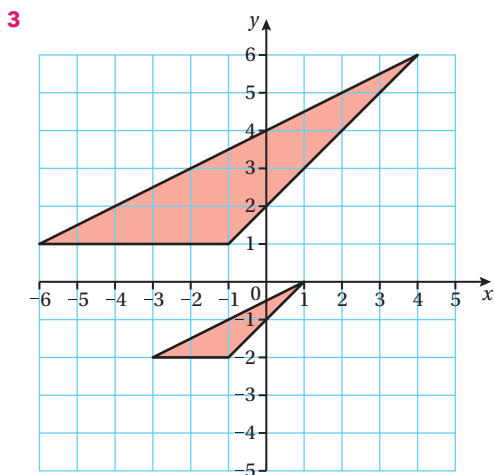
6 a  $\frac{1}{8}$       b 20.25      c 3375

### LAUNCHPAD

1 a Not necessarily, for example, a square is a special case of a rectangle but is not similar to a rectangle that has different lengths and widths.

b These are similar; corresponding angles are equal in size.

2 AC = 14.1 cm, BC = 4.7 cm, AE = 4 cm



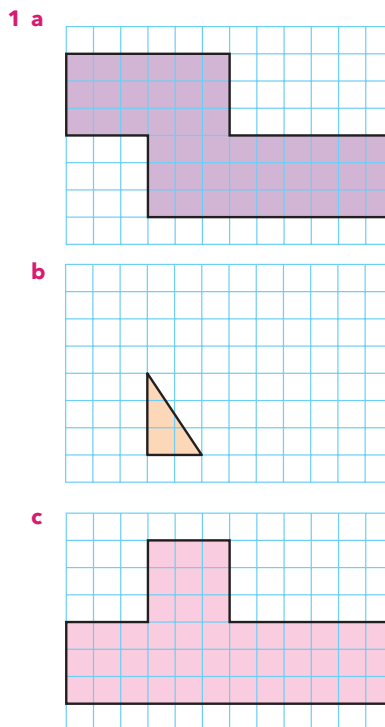
- 4** Enlargement centre (4, 2) scale factor 0.5  
**5** No. Although angles are the same, the side ratios are not. AB ratio is 2 : 3, but BC ratio is 9 : 14.  
**6** 12 cm

**EXERCISE 35A**

- 1 a** CDE and BDA.  $\angle EDC = \angle ADB$  (common),  $\angle DEC = \angle DAB$  (corresponding),  $\angle DCE = \angle DBA$  (corresponding)  
**b** YZV and YWX.  $\angle FGH = \angle LMN$  (alternate), extending lines FH to X and LN to Y:  $\angle GHF = \angle XHN$  (vertically opposite),  $\angle XHN = \angle MNL$  (corresponding) so  $\angle GHF = \angle MNL$ ,  $\angle GFH = \angle MLN$  (angles in a triangle)  
**c** PQS and RSQ.  $\angle PQS = \angle RSQ$  (alternate),  $\angle PSQ = \angle RQS$  (alternate),  $\angle QPS = \angle SRQ$  (angles in a triangle)  
**d** HFG and NLM.  $\angle FGH = \angle LMN$  (alternate), extending lines FH to X and LN to Y:  $\angle GHF = \angle XHN$  (vertically opposite),  $\angle XHN = \angle MNL$  (corresponding) so  $\angle GHF = \angle MNL$ ,  $\angle GFH = \angle MLN$  (angles in a triangle)
- 2** Option B
- 3 a** Yes (angles the same)  
**b** No (angles not the same)  
**c** Yes (ratio of corresponding sides is the same)
- 4 a** False (ABC with base angles  $30^\circ$ , WXY with base angles  $45^\circ$ )  
**b** True (all angles are  $60^\circ$ )  
**c** False (two other angles could be  $30^\circ$  and  $60^\circ$  or  $45^\circ$  and  $45^\circ$ , so not the same)  
**d** True (third angle will be  $60^\circ$  for each triangle)  
**e** True (base angles are always both  $45^\circ$ )  
**f** False (ratio of corresponding sides could be in same proportion so corresponding angles could be equal)
- 5 a** Triangles NMO, KJO and LJM ( $\angle JKO = \angle JLO$  (alternate),  $\angle JOK = \angle JML$  (alternate)  $\angle OJK = \angle MJK$  (common) and  $\angle JOK = \angle MON$  (vertically opposite),  $\angle MNO = \angle JKO$  (alternate)  $\angle KJO = \angle NMO$  (alternate))  
**b** Triangle ABC, BDC and ADB ( $\angle CBA = 90^\circ$  (straight line), if  $\angle CAD = x$  then  $\angle DBA = 90 - x$  and  $\angle CBD = x$  and  $\angle BCD = 90 - x$ , so angles are the same for all three triangles))
- 6**  $c = 8$  cm,  $d = 15$  cm  
**7**  $e = 16$  cm,  $f = 13.5$  cm  
**8**  $AE = 1.5$  cm,  $CE = 10$  cm,  $AD = 7.5$  cm  
**9**  $YZ = 3$  cm,  $XY = 9$  cm

- 10** 2.975 m (3dp)  
**11** 192 m

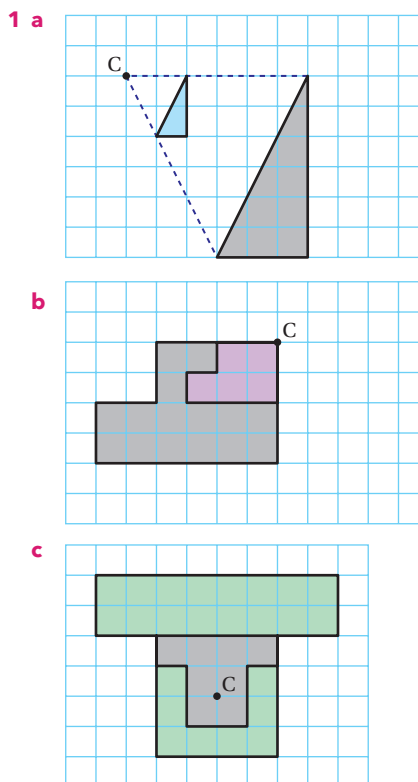
**EXERCISE 35B**



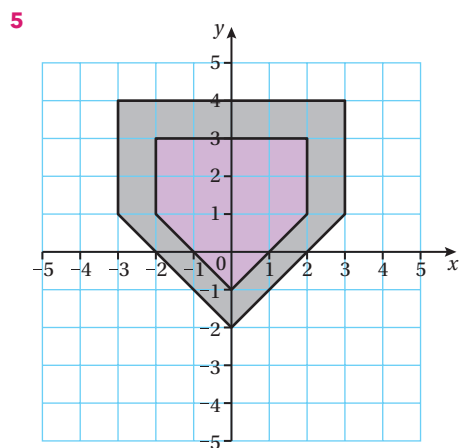
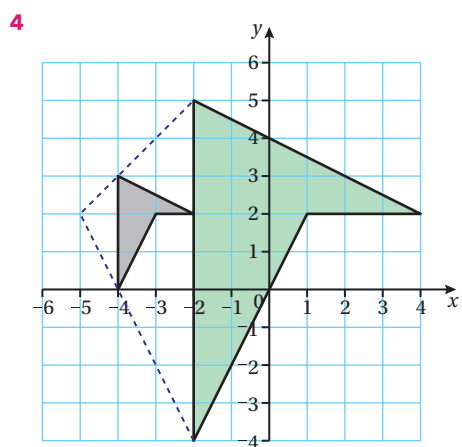
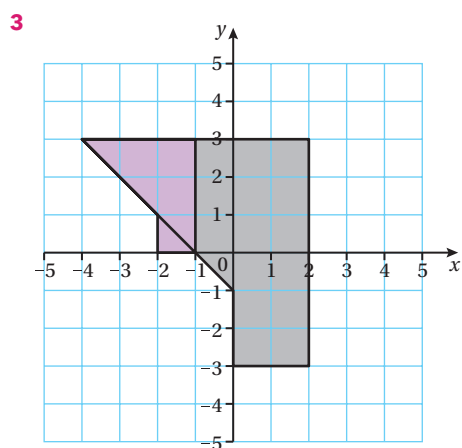
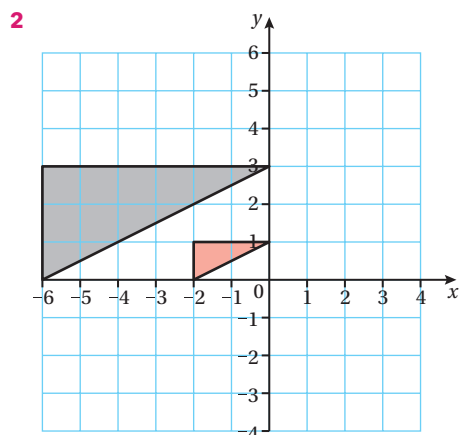
**WORK IT OUT 35.1**

- Option A is correct.  
 Option B is wrong because the base length has not doubled in the image.  
 Option C is wrong because a scale factor of 3 has been used.

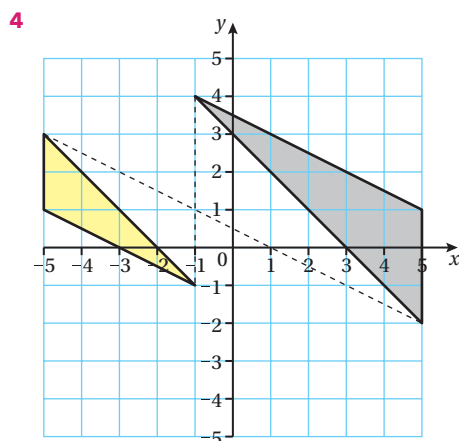
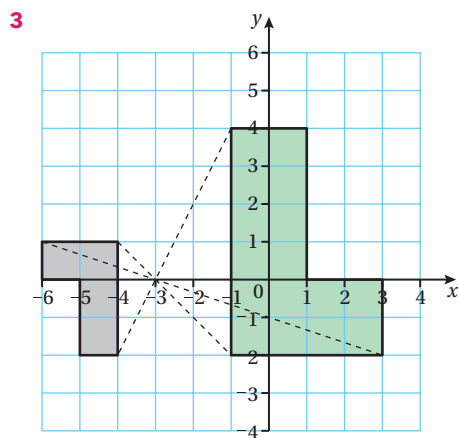
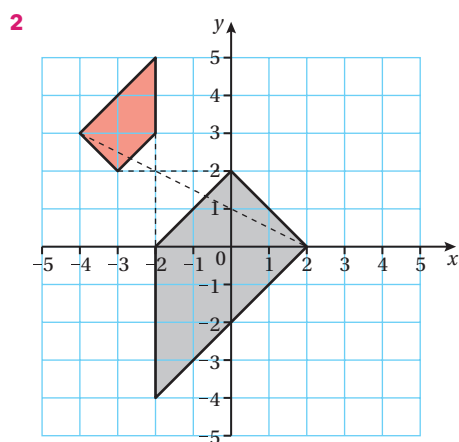
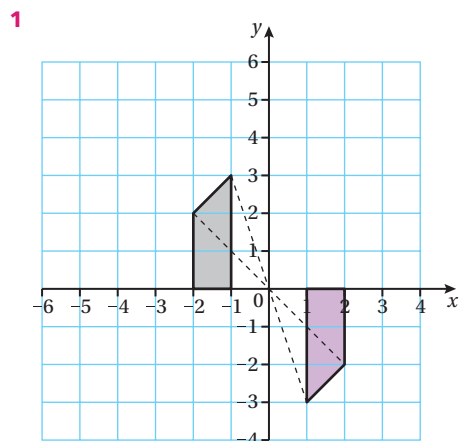
**EXERCISE 35C**







**EXERCISE 35D**



### WORK IT OUT 35.2

Ellie got the answer correct. Always check that each side has been increased by the same scale factor, this doesn't necessarily mean the same number of extra squares. Sides are increased in proportion to each other.

### EXERCISE 35E

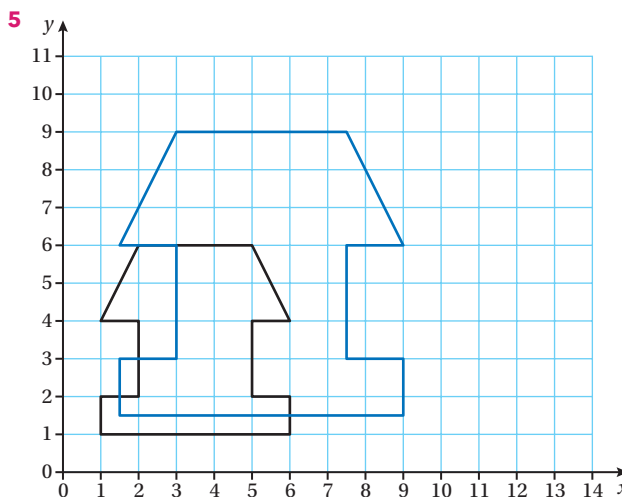
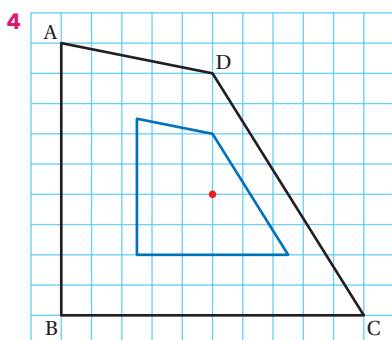
- 1 Option B
- 2 C  $(\frac{1}{2})$ , D (2), F  $(1\frac{1}{2})$
- 3 a Enlargement scale factor 2 centre (-4, -4)  
 b Enlargement scale factor 4 centre (-1, -4)  
 c Enlargement scale factor 3 centre (-4, 4)  
 d Enlargement scale factor  $\frac{1}{2}$  centre (2, 5)  
 e Enlargement scale factor  $\frac{1}{2}$  centre (4, 2)  
 f Enlargement scale factor  $\frac{3}{2}$  centre (-5, 4)
- 4 a Enlargement scale factor -2 centre (2, 1)  
 b Enlargement scale factor  $-\frac{1}{2}$  centre (4, 2)

### EXERCISE 35F

- 1 Option D
- 2 a Yes      b No      c No      d Yes
- 3 a Yes similar    b no      c Yes
- 4 a = 8 cm      b 5.6 cm
- 5 Scale factor is 0.9. LG = 4.5; GH = 3.6; HI = 2.7; IJ = 7.2; JK = 11.25
- 6 GH = KJ = 7.5 cm all other sides = 10.5 cm
- 7 Option A
- 8 1 : 64
- 9 a 3.5  
 b Base of 28 units and height 42 units
- 10 a 7.996 cm (3 dp)  
 b 22.389 cm tall and volume 3446.464 ml
- 11 a Volume 134.041 cm<sup>3</sup> (3 dp), surface area 162.663 cm<sup>2</sup> (3 dp)  
 b Volume 3619.115 cm<sup>3</sup> (3 dp), surface area 1463.963 cm<sup>2</sup> (3 dp)
- 12 a Original surface area 2123.717 cm<sup>2</sup> (3 dp),  
 new surface area 33979.466 cm<sup>2</sup> (3 dp)  
 b Original volume 9202.772 cm<sup>3</sup> (3 dp), new volume  
 588977.413 cm<sup>3</sup> (3 dp)

### CHAPTER REVIEW

- 1 Option B
- 2 a VWX = VYZ (corresponding), VXW = VZY (corresponding),  
 WVX = YVZ (common), angles are the same, therefore similar  
 b 3.75 cm
- 3 6.5 m



- 5 Yes - sides all the same length, internal angles all 120 degrees, so they are enlargements.
- 7 No - different angles are possible in different rhombuses.
- 8 48 cm
- 9 50 cm<sup>3</sup>
- 10 864 cm<sup>3</sup>
- 11 Enlargement scale factor  $-\frac{1}{2}$  centre (-2, 2)

## 36 Congruence

### BEFORE YOU START ...

- 1 a AB and DE or BC and EF or AC and DF  
 b Angle EDF  
 c ACB and DFE or CBA and FED
- 2 a C                      b B                      c A
- 3 a true                  b false                  c false                  d true
- 4 45°

### LAUNCHPAD

- 1 a No; the corresponding sides are not equal.  
 b Yes; three equal sides.  
 c Yes, three equal angles and one equal side.
- 2 a Triangles share the side AC, CD = CB and AB = AD. Hence by SSS the triangles are congruent.  
 b AC is a line of symmetry hence angle ADC is equal to angle ABC.
- 3 OPR is an isosceles triangle since it is made up of two radii of a circle.  
 Hence side OP = side OR.  
 Triangles OPQ and ORQ are congruent (SSS) share side OQ, side PQ = side RQ, and side OP = side OR.  
 Hence angle OQP = OQR, since along a straight line each is 90°.

### WORK IT OUT 36.1

Option C is correct.  
 Option A is wrong because for SAS the angle has to be between the two sides.  
 Option B is wrong because not enough information has been written down to complete the proof.

## EXERCISE 36A

- 1 a RHS      b SSS      c ASA or AAS      d SAS
- 2 Option B
- 3 a Can't tell.  
b SSA so can't tell.  
c Calculate the missing angle and then you can see they are congruent by ASA.  
d RHS
- 4 Angle BCA is opposite to angle DCE and hence equal, hence angle CBA and CED are equal.  
Triangles are congruent by ASA.
- 5 SAS - DF shared, angle DFG = angle DFE, and side EF = side FG.  
ASA, because DEF and DGF are the same angle (isosceles triangle), and they both have a 9 m side and a right angle.
- 6 Angle RTS is opposite to angle PTQ and hence equal, angle TRS and TQP are alternate angles and therefore equal. Triangles are congruent by ASA.
- 7 Angle EAB and DCB are both right angles, since they are co-interior angles with AED and CDE. Side BE = BD. AE = CD. Hence congruent (RHS).
- 8 RHS - ABC and ADC are right angled triangles AB = AD and BC = CD  
SSS - share AC, AD = AB (isosceles triangle) and BC = BD (AC bisects base of the triangle).
- 9 SAS - PR = SU, shared angle at Q, QU = QR
- 10 a Kite so AB = BC and AD = CD, triangles share side BD - SSS congruency.  
b Kite so AD = CD, share side ED, angle AED = CED = right angle - RHS congruency.
- 11 SSS = triangle ADC and triangle ABC, SSS = triangle BAD and triangle BCD,  
ASA = triangle EAB and triangle ECD.
- 12 Side PO = side QO, radii of smaller circle, angle POM = QON, opposite angles are equal, side OM = side ON, radii of larger circle, by SAS congruent triangles.

## EXERCISE 36B

- 1 Triangles share side JL, JK = JM and right angled. Hence congruent and KL = ML.
- 2 a Angle AED = angle BEC opposite angles are equal, third angles in triangles EBC = EDA, so triangles AED and CEB are congruent, hence side AE = side CE.  
Angle AEB = angle DEC opposite angles are equal, side DE = BE, side AE = CE, so triangles ABE and CDE congruent, hence angle ABE = angle EDC.  
b Angle DAC = angle BCA so AD and BC are parallel, from part a angle ABE = EDC so AB and DC parallel hence ABCD has two pairs of parallel sides and is a parallelogram.
- 3 QPR = 56° (base angle, isosceles triangle); PRS = 56° (alternate); RPS = 56° (base angle, isosceles triangle); thus PSR = 180 - 2 × 56 = 68°
- 4 Angle PRS = 180 - angle QRP and angle PSR = 180 - angle PST (angles on a straight line add up to 180).  
PST = PRQ so PRS = PSR and base angles of an isosceles triangle.
- 5 Angle ABE = angle CBE, BE is a shared side and the angle AEB = angle CEB, as they are both complements of the same angles below (AED and CED) and by ASA AEB and CEB are congruent. This means EA and EC are the same length and ED

is a shared side so by SAS triangle EAD and triangle ECD are congruent, and angle EAD = angle ECD.

- 6 AB = CD, BC = DA and the angle ABC = BCD (both 90°), hence triangles ABC and BCD are congruent (SAS) and AC = BD.
- 7 a 110°  
b AB = DC and parallel (parallelogram), angle BDC = ABD (alternate angles). DX = AD. = BC = BY, hence triangles CDX and ABY are congruent (SAS) and CX = AY.  
c As with b, AX = CY, angle CYX = AXY = 110° and triangles share side XY hence congruent (SAS).  
d AY = XC and since CYX = AXY = 110°, alternate angles AY and XC are parallel - pair of equal sides are parallel - AYCX is parallelogram.
- 8 Draw diagonals BP and CQ of rhombus BCPQ. ∠AQB = ∠QAB (Δ ABQ isosceles), ∠QBC = 2∠QAB (exterior angle), ∠QAB = ∠PBC (diagonal of rhombus bisects angle), BP ∥ AQ (corresponding angles equal), similarly QC ∥ PD. QC and BP meet at right angles at X (diagonals of a rhombus are perpendicular). AR is perpendicular to DR.
- 9 a 130° and 110°  
b area =  $5 \times (30 - 5 \times (30 - \frac{5}{\tan 50^\circ} - \frac{5}{\tan 70^\circ})) \div 2 = 59.96 \text{ cm}$
- 10 a square, rectangle or rhombus  
b square  
c kite, rhombus, rectangle or square
- 11 a i ∠ABP = ∠CBP = ∠ADQ = ∠CDQ = 45° (diagonals of square ABCD meet each side at 45°),  
BP = DQ (given), AB = BC = CD = DA (equal sides of square ABCD), so ABP ≅ Δ CBP ≅ Δ ADQ ≅ Δ CDQ (SAS).  
ii AP = CP = AQ = CQ (matching sides of congruent triangles), so APCQ is a rhombus.  
b ∠FMA = ∠GAM (alternate angles, AG ∥ FM). Also AF = FM (in isosceles Δ AFM).  
So 2 adjacent sides of parallelogram AFMG are equal. So AFMG is a rhombus.
- 12 Angle CDB = 36°, angle BDA = 82°, angle ABD = angle DAB = 49°, angle ABC ≠ 90° so AB and DC are not parallel so ABCD is not a trapezium.

## CHAPTER REVIEW

- 1 a Congruent by SAS. ABC, FED. b Congruent by SAS. GHI, JKL.  
c Not enough information. d Congruent by SSA. TUV, WXY.
- 2 a option A      b option C
- 3 Let angle QPS = a  
Let angle PQS = b  
Triangle PQR three angles are a, b + angle SQR and angle QRP.  
Angle QST = angle QPS = a.  
Angle TQR = angle PQS = b.  
Triangle QST three angles are a, b + angle SQR and angle QTS.  
Hence angle QRP = angle QTS.
- 4 Angle ABE = angle DCE (angles in a triangle add up to 180°), hence angle EBC = angle ECB (complementary to ABE and DCE).
- 5 Let angle ABC = a.  
Angle ABC = CAB = a base angles of an isosceles triangle.  
Angle ACB = 180 - 2a.  
Angle ACD complementary to angle ACB so angle ACD = 2a.  
Angle ACD = angle ADC = 2a base angles of an isosceles triangle.

Angle  $ADC = EAD = 2a$  alternate angles are equal.

Hence angle  $EAD = 2 \times$  angle  $ABC$ .

**6** Since triangle  $UVW$  and  $UZY$  congruent:

Triangle  $XVZ$  is a right angled isosceles triangle, angle  $WVU =$  angle  $YZU$ .

Side  $WX = YX$ .

Side  $WU =$  side  $UY$ .

Angle  $UWX =$  angle  $UYX$ .

So by SAS triangles  $UWX$  and  $UYX$  are congruent and  $UWXY$  is a kite.

**7**  $MNP$  congruent to  $NPQ$  so angle  $NPQ$  is right angle and  $MN = NP = QP$ .

Therefore  $MQ$  must be equal to  $NP$  and all angles right angles.

Four equal sides and four equal angles means shape is a square.

## 37 Pythagoras' theorem

### BEFORE YOU START ...

**1 a** i Yes    ii No    iii Yes    iv Yes

**b** i Yes    ii No    iii Yes    iv Yes

**2 a** acute angle

**b** obtuse angle

**c** right angle

**d** reflex angle

**e** straight line, which is two right angles

**3** 6 units<sup>2</sup>

**4**  $x$  and  $y$  are equal because it is an isosceles triangle

### LAUNCHPAD

**1** 8.94 m (2 dp)

**2 a** No, it is not a right-angled triangle

**b** 11.18 cm (2 dp)

**3** 6.79 m (2 dp)

### WORK IT OUT 37.1

Option C is correct

Option B is wrong because the numbers have been substituted in the equation wrongly. Option A is wrong because the numbers are multiplied by 2 instead of squared

### EXERCISE 37A

**1** Option A

**2 a** 10 cm    **b** 13.42 cm (2 dp)    **c** 2.59 cm (2 dp)

**d** 1.62 cm    **e** 7.21 m

**3 a** 2.80 cm (2 dp)    **b** 4.47 cm (2 dp)    **c** 4.28 cm (2 dp)

**d** 8.54 km (2 dp)    **e** 10.39 cm (2 dp)    **f** 8.06 cm (2 dp)

**4** 93.67 m

**5 a** 8.54    **b** 21.26    **c** 13.42    **d** 7.30    **e** 10.58    **f** 118.76

### WORK IT OUT 37.2

Option C is correct

### EXERCISE 37B

**1 a** yes    **b** yes    **c** no

**d** yes    **e** yes    **f** yes

**2** Any smaller than that are not whole numbers

**3** There is no limit as numbers are infinite

**4 a** yes    **b** no    **c** no

**d** yes    **e** yes

**5 a** yes    **b** no    **c** no

### EXERCISE 37C

**1** 4.33 cm (2 dp)

**2** 7.42 cm (2 dp)

**3** 14.14 cm (2 dp)

**4** yes

**5 a**  $1^2 + 10^2 = 101$ ,  $\sqrt{101} = 10.05$  cm (2 dp) = 10.1 cm to 1 dp

**b** No

**6** 15.26 (2 dp)

**7 a** 6.5    **b** 6.00 (2 dp)    **c** 6.10 (2 dp)

**8** 69.45 units<sup>2</sup> (2 dp)

**9** 10.2 (2 dp); perimeter is 36.19 units (2 dp)

**10** 7.07 mm (2 dp)

**11**  $3\sqrt{2}$

**12 1** By Pythagoras:

$$BN^2 = AB^2 + AN^2$$

$$\text{But } AN = 2AC$$

$$\text{So } BN^2 = AB^2 + 4AC^2$$

**2** By Pythagoras:

$$CM^2 = AC^2 + AM^2$$

$$\text{But } AM = 2AB$$

$$\text{So } CM^2 = AC^2 + 4AB^2$$

**3** Substituting:

$$BN^2 + CM^2 = AB^2 + 4AC^2 + AC^2 + 4AB^2$$

$$BN^2 + CM^2 = 5AB^2 + 5AC^2 = 5(AB^2 + AC^2)$$

**4** By Pythagoras:

$$BC^2 = AC^2 + AB^2$$

$$\text{So: } BN^2 + CM^2 = 5BC^2$$

Q.E.D.

### EXERCISE 37D

**1** 17.55 cm (2 dp)

**2** Yes, because it is 28.72 cm high (2 dp)

**3** 42.96 m to the nearest cm

**4** No, the longest that will fit is 38.48 cm

**5** 95.4 mm

### EXERCISE 37E

**1** 16.16 (2 dp)

**2 a** 36.06 inches (2 dp)

**b** No, because it is 69.28 inches wide

**3** Yes, because the longest length is 7.81 m

**4** B

**5** 9.54 cm (2 dp)

**6** No,  $30^2 + 30^2 = 1800$ , length of the brace would be  $\sqrt{1800} = 42.43$  cm

**7** 8.08 cm (2 dp)

**8** 0.68 m

- 9 a 16.97 m (2 dp)    b 7.89 m (2 dp)  
 10 a Perimeter 54.64 m and 173.2 m<sup>2</sup> to 2 dp and 1 dp  
       b 18 laps  
 11 a 141.4 cm (1 dp)                      b 6 full rows  
 12 No, it is 3.1 m high so will not fit into the display area

### CHAPTER REVIEW

- 1  $x = 12.81, y = 11.32$   
 2 Pythagoras' theorem states  $a^2 + b^2 = c^2$   
 If this triangle is right angled then  $(5.8 \times 10^2)^2 + (1.16 \times 10^3)^2 = (580\sqrt{5})^2$   
 $1\ 682\ 000 = 1\ 682\ 000$   
 Therefore the triangle is right angled  
 3 95.16 cm  
 4 6.77 mm (2 dp)  
 5 2 m

## 38 Trigonometry

### BEFORE YOU START ...

- 1 a  $\sqrt{305} \approx 17.5$  (3 sf)                      b  $2\sqrt{6} \approx 4.90$  (3 sf)  
 2 7.28  
 3 9.71  
 4  $\frac{35}{AC} = \frac{5}{3}$  so  $AC = 21$  cm

### LAUNCHPAD

- 1 5.78 cm (3 sf)  
 2 15.5°  
 3 a  $\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$     b 1                      c  $\sqrt{3}$   
 4  $\left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{2}\right)^2 = \frac{3}{4} + \frac{1}{4} = 1$   
 5 24.3 (1 dp)  
 6 a  $PQ = 10.1$  cm (3 sf)                      b  $P = 29^\circ, Q = 51^\circ$   
 7 9.9 m (1 dp)  
 8 a 7 cm                      b 51°

### EXERCISE 38A

- 1 Option C  
 2 a 0.530                      b 0                      c 0.445  
       d 0.5                      e 19.081                      f 0.656  
       g 0                      h 0.848                      i 1  
 3 All Sine ratio to find opposite  
       a 7.5 cm                      b 10.8 cm (3 sf)                      c 2.54 cm (3 sf)  
       d 5.51 cm (3 sf)                      e 6.72 cm (3 sf)                      f 29.0 cm (3 sf)  
       g 7.99 cm (3 sf)                      h 14.1 cm (3 sf)  
 4 a 18.25 cm                      b 7.83 cm                      c 9.66 mm  
       d 13.16 cm                      e 4.1 m                      f 14.00 mm

### WORK IT OUT 38.1

Option C is correct

### EXERCISE 38B

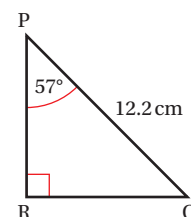
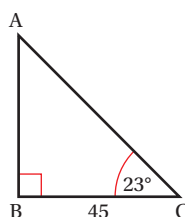
- 1 a 44.4°                      b 44.5°                      c 5.7°  
       d 47.1°                      e 82.9°                      f 85.2°  
 2 a 43.0°                      b 27.0°                      c 68.0°  
       d 15.0°                      e 75.8°                      f 38.3°  
 3 a 25.9°                      b 44.9°                      c 69.5°  
       d 79.6°                      e 26.9°                      f 11.5°

4 24.6°

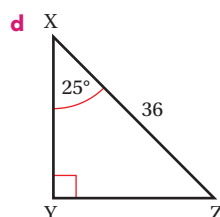
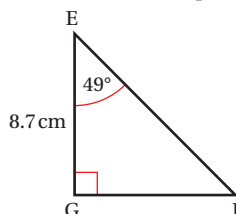
5 42.1°

6 a  $AB = 19.1$  units

b Length  $QR = 10.23$  cm (2 dp)



c  $GF = 10.01$  cm (2 dp)



i  $XY = 32.63$  units

ii  $YZ = 15.21$  units

### EXERCISE 38C

1

Angle $\theta$	$\sin \theta$	$\cos \theta$	$\tan \theta$
0°	0	1	0
30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$
45°	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1
60°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
90°	1	0	$\tan 90^\circ$ is undefined

2 a 1                      b  $\sqrt{2}$                       c  $\sqrt{3}$

Therefore  $\sin x$  will equal  $\cos (90 - x)$  as they are complementary angles that add to 90°

3 a  $x = 4, y = 4\sqrt{2}$

b  $x = \frac{4}{\sqrt{3}}$

c  $x = 16$

d  $x = \frac{20}{3}\sqrt{3}, z = \frac{10}{3}\sqrt{3}$

e  $x = 5, y = 5\sqrt{3}$

f  $x = \frac{8}{3}\sqrt{3}, y = \frac{16}{3}\sqrt{3}$

### EXERCISE 38D

- 1 a 11.20      b 8.58      c 25.27  
 2 a  $x = 10.65$  cm      b  $x = 5.73$  cm      c  $x = 9.06$  cm  
 d  $x = 5.32$  cm      e  $x = 6.46$  cm      f  $x = 155$  mm  
 3 a  $54.65^\circ$       b  $66.82^\circ$       c  $69.75^\circ$   
 d  $25.31^\circ$       e  $52.70^\circ$       f  $50.52^\circ$   
 4 a The ratio of sides to opposite angles is the same (according to the sine rule) and the side opposite Y is shorter than that opposite X.  
 b  $Y = 30.95^\circ$  and  $Z = 109.05^\circ$   
 c 22.1 cm  
 5 a  $51^\circ$       b  $52^\circ$       c 32.35 mm

### EXERCISE 38E

- 1 a 14 cm      b 8.62 cm      c 20.30 cm  
 2  $53.8^\circ$   
 3 a 18.7 m (3 sf)      b  $T = 52.9^\circ$ ,  $U = 32.1$  (3 sf)  
 4  $X = 60^\circ$  (3 sf),  $Y = 32.2^\circ$  (3 sf),  $Z = 87.8^\circ$   
 5 a 14.42 km (2 dp)      b  $296.3^\circ$

### EXERCISE 38F

- 1 a  $5$  cm<sup>2</sup>      b  $3.27$  m<sup>2</sup> (2 dp)  
 c  $22.5$  cm<sup>2</sup>      d  $298.58$  mm<sup>2</sup> (2 dp)  
 e  $5.79$  cm<sup>2</sup> (2 dp)      f  $25.96$  m<sup>2</sup> (2 dp)  
 2 To 2 dp  
 a  $8.16$  cm<sup>2</sup>      b  $19.34$  cm<sup>2</sup>  
 c  $16.01$  cm<sup>2</sup>      d  $30.21$  cm<sup>2</sup>  
 3  $39.09$  cm (2 dp)

### EXERCISE 38G

- 1 Option A  
 2 3.74 m (to 2 dp)  
 3 35.83 m (to 2 dp)  
 4 34.34 m (to 2 dp)  
 5 a 27.6 m (to 1 dp)  
 b It will decrease to  $46.1^\circ$  (to 1 dp)  
 6 a 6.99 m (to 2 dp)      b 4.72 m (to 2 dp)  
 7 44.48 m (to 2 dp)  
 8 a Use  $10^\circ \tan 20^\circ$  and add her height up to eye level.  
 b It would decrease to  $10.3^\circ$   
 9 a  $020^\circ$       b 281.91 m (2 dp)  
 c 98 668 m<sup>2</sup> to the nearest square metre  
 10  $\frac{5a^2}{\tan 36^\circ} \approx 6.88a$  m<sup>2</sup>  
 11  $15.5^\circ$  (1 dp)  
 12 a 7.92 m (2 dp)      b  $56.1^\circ$   
 13 a  $51.8^\circ$  (1 dp)      b  $38.2^\circ$  (1 dp)

### CHAPTER REVIEW

- 1 Option A  
 2 10.94 m (2 dp)

- 3 4.33 m (2 dp)  
 4  $78.5^\circ$  (1 dp)  
 5 56.41 m (2 dp)  
 6 Because the scale factor is Triangle PQR =  $\frac{3}{2}$  Triangle ABC so PQ = 6 cm and hence  $\sin x = \frac{4.8}{6} = 0.8$  as required. In addition because the triangles are similar  $\frac{AC}{AB} = \frac{PR}{PQ}$  and hence  $\sin x = \frac{AC}{AB} = \frac{3.2}{4} = 0.8$ .  
 7 138 miles  
 8  $35.3^\circ$   
 9 7.4 cm  
 10 74.1 or 105.9 degrees  
 11 9 cm

## 39 Graphs of other functions and equations

### BEFORE YOU START ...

- 1 a 3      b (0, 1)      c  $x = -5$   
 d  $2y - 6x = -4$  becomes  $y = 3x - 2$  (make y the subject and divide both sides by 2). The gradient is 3 identical to  $y = 3x + 1$  therefore, the two lines are parallel.

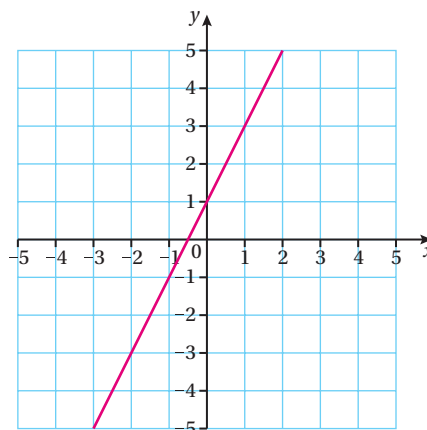
2

x	22	21	0	1	2
y	13	4	1	4	13

- 3 a  $x = -4$ ,  $x = 2$       b  $x = -1$ ,  $x = -4$   
 4  $(x + 2)^2 - 10 = 0$   
 $x = \pm \sqrt{10} - 2$

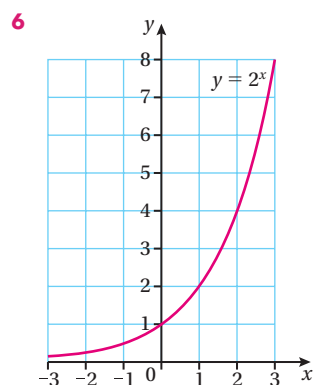
### LAUNCHPAD

- 1 Three points will give a check point, although a straight line requires only 2 points to be defined.  
 2  $y = 2x + 1$  is a linear equation with a gradient of 2 and a y-intercept of 1; using two points on the line (0, 1) and (1, 3).



- 3 a  $y = -x^2 + 1$   
 b The coefficient of  $x^2$  is negative, which means the parabola will have a vertex that is a maximum.  
 c Maximum  
 d Vertex (0, 1)  
 e  $x = -1$  and  $x = 1$

- 4 a Cubic equation  
 b At least 5 values, including half values of  $x$ .  
 5 a  $\frac{1}{0}$  cannot be calculated, undefined  
 b  $y$  gets smaller      c  $\frac{1}{60}$



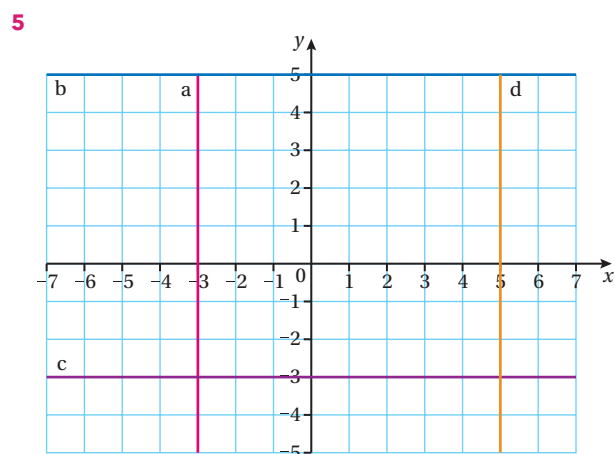
The blue graph is  $y = \sin x$  and the red graph is  $y = \cos x$

### WORK IT OUT 39.1

Option C. None of the functions has a constant value, so they all pass through the origin.

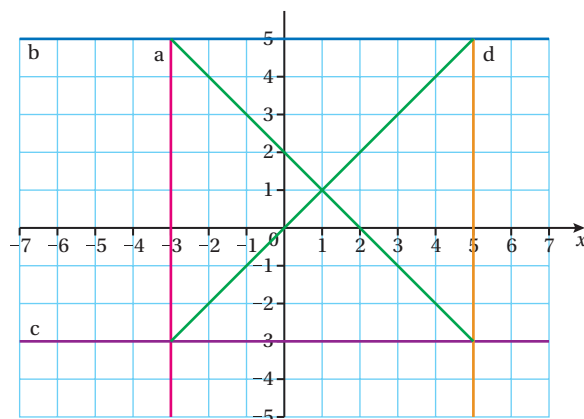
### EXERCISE 39A

- 1 Option B  
 2 a D    b B    c B, C, D    d C    e A  
 3 a  $y = x - 6$     b  $y = -x$     c  $y = 6x$     d  $y = x$   
 4 a A  $x = -6$   
 B  $y = 7$   
 C  $y = -3$   
 D  $y = -4$   
 E  $x = -2$   
 F  $x = 4$   
 b  $y = 2$



- 6 A square is created where the lines intersect, because all four sides are 8 units long, there are two pairs of parallel lines and four angles of  $90^\circ$ .

- 7 a  $x = 1$  and  $y = 1$   
 b mirror line (line or axis of symmetry is acceptable)  
 c i



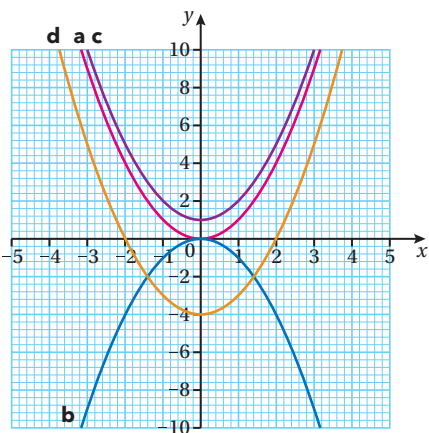
- ii  $y = x$  and  $y = -x$

- 8 a      b   
 c      d   
 e

### EXERCISE 39B

1 Option B

<b>x</b>	-3	-2	-1	0	1	2	3
<b><math>y = x^2</math></b>	9	4	1	0	1	4	9
<b><math>y = -x^2</math></b>	-9	-4	-1	0	-1	-4	-9
<b><math>y = x^2 + 1</math></b>	10	5	2	1	2	5	10
<b><math>y = x^2 - 4</math></b>	5	0	-3	-4	-3	0	5



3  $y = x^2 + 2x - 3$   $-4 \leq x \leq 2$

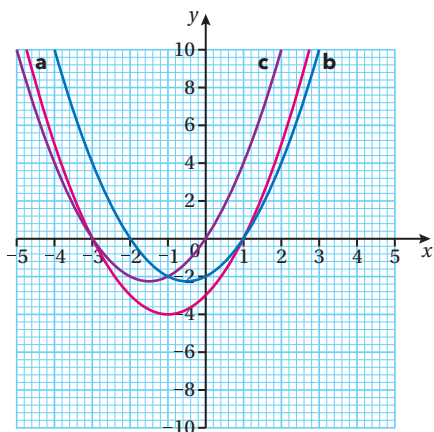
<b>x</b>	-4	-3	-2	-1	0	1	2
<b>y</b>	5	0	-3	-4	-3	0	5

$y = x^2 + x - 2$   $-3 \leq x \leq 2$

<b>x</b>	-3	-2	-1	0	1	2
<b>y</b>	4	0	-2	-2	0	4

$y = x^2 + 3x$   $-4 \leq x \leq 1$

<b>x</b>	-4	-3	-2	-1	0	1
<b>y</b>	4	0	-2	-2	0	4

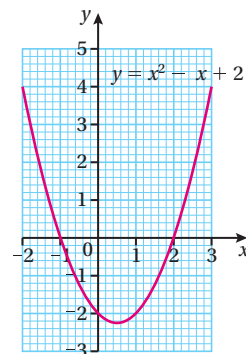


- 4 **a** If the coefficient of  $x^2$  is greater than 1 the impact on the shape of the parabola is to increase the steepness (gradient).  
**b** If the coefficient of  $x^2$  is a value between 1 and 0, the impact on the shape of the parabola is to decrease the steepness (gradient). The parabola widens out.  
**c** If a constant value is added to the graph of an equation such as  $y = ax^2$  becomes  $y = ax^2 + c$ , the graph is moved up the  $y$  axis with a positive value of  $c$  and down for a negative value.

**d** When the coefficient of  $x^2$  is negative the basic parabola  $y = x^2$  is reflected about the  $x$ -axis and has a maximum turning point through the origin.

5  $y = x^2 - x - 2$  for  $-2 \leq x \leq 3$ .

<b>x</b>	-2	-1	0	1	2	3
<b>y</b>	4	0	-2	-2	0	4



- a**  $x = 2$  and  $x = -1$   
**b** These are the values for  $x$  when  $y = 0$  - the values where the graph crosses the  $x$  axis.

### WORK IT OUT 39.2

Option C

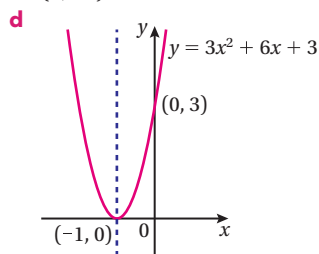
### EXERCISE 39C

1

Graph	Turning point	Axis of symmetry	y-intercept	x-intercepts	Roots of the equation
<b>a</b>	(2, -9) minimum	$x = 2$	(0, -5)	(5, 0) (-1, 0)	$x = 5$ and $x = -1$
<b>b</b>	(-2, -1) minimum	$x = -2$	(0, 3)	(-3, 0) (-1, 0)	$x = -3$ and $x = -1$
<b>c</b>	(4, 16) maximum	$x = 4$	(0, 0)	(0, 0) (8, 0)	$x = 0$ and $x = 8$
<b>d</b>	(0, 1) maximum	$y$ -axis ( $x = 0$ )	(0, 1)	(-1, 0) (1, 0)	$x = -1$ and $x = 1$

2  $y = 3(x + 1)^2 + 0$

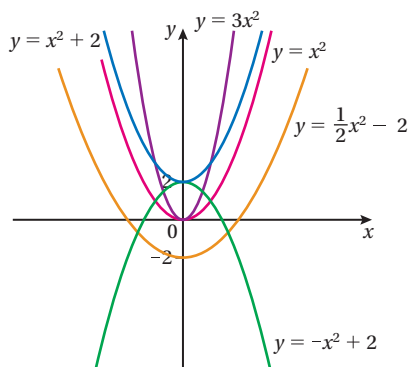
- a** (0, -5)      **b**  $x = -1$  vertex (-1, 0)      **c** (-1, 0)



### EXERCISE 39D

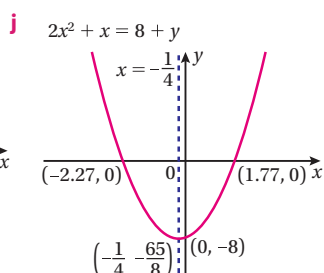
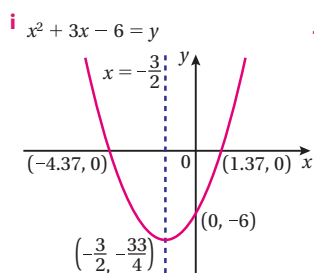
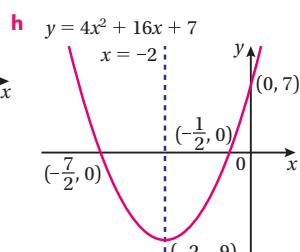
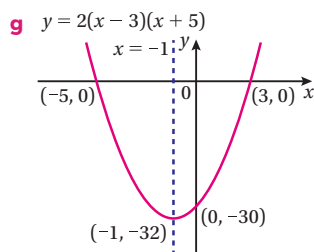
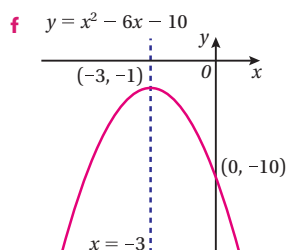
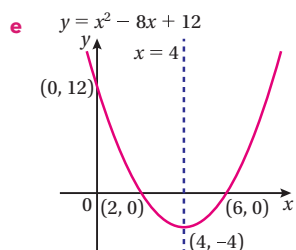
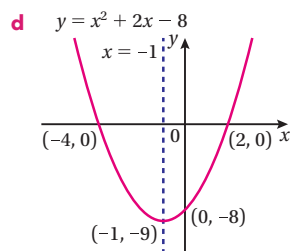
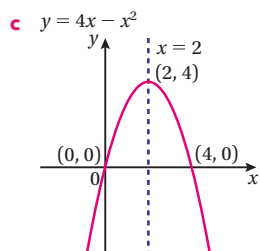
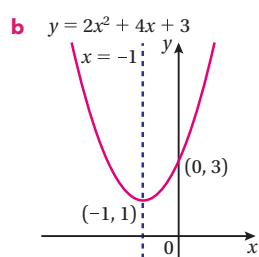
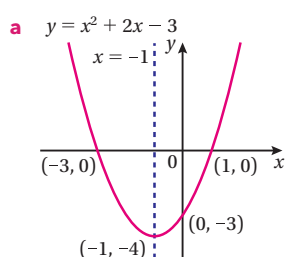
- 1 **a**  $y = 6x^2$       **b**  $y = x^2 + 4$       **c**  $y = \frac{1}{2}x^2$   
**d**  $y = -x^2 + 2$       **e**  $y = x^2 - 1$       **f**  $y = -7x^2$

2





Graph	Equation	y-intercept	x-intercept(s)	Symmetry axis	Turning point
a	$y = x^2 + 2x - 3$	(0, -3)	(-3, 0) (1, 0)	$x = -1$	(-1, -4)
b	$y = 2x^2 + 4x + 3$	(0, 3)	None	$x = -1$	(-1, 1)
c	$y = 4x - x^2$	(0, 0)	(0, 0) (4, 0)	$x = 2$	(2, 4)
d	$y = x^2 + 2x - 8$	(0, -8)	(-4, 0) (2, 0)	$x = -1$	(-1, -9)
e	$y = x^2 - 8x + 12$	(0, 12)	(2, 0) (6, 0)	$x = 4$	(4, -4)
f	$y = -x^2 - 6x - 10$	(0, -10)	None	$x = -3$	(-3, -1)
g	$y = 2(x - 3)(x + 5)$	(0, -30)	(3, 0) (-5, 0)	$x = -1$	(-1, -32)
h	$y = 4x^2 + 16x + 7$	(0, 7)	$(-\frac{1}{2}, 0)$ $(-\frac{7}{2}, 0)$	$x = -2$	(-2, -9)
i	$x^2 + 3x - 6 = y$	(0, -6)	(-4.37, 0) (1.37, 0)	$x = -\frac{3}{2}$	$(-\frac{3}{2}, -\frac{33}{4})$
j	$2x^2 + x = 8 + y$	(0, -8)	(-2.27, 0) (1.77, 0)	$x = -\frac{1}{4}$	$(-\frac{1}{4}, -\frac{65}{8})$

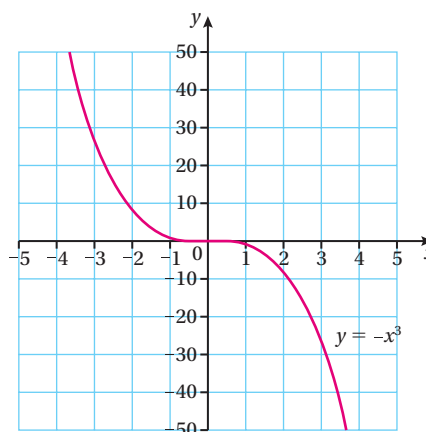


### EXERCISE 39E

1

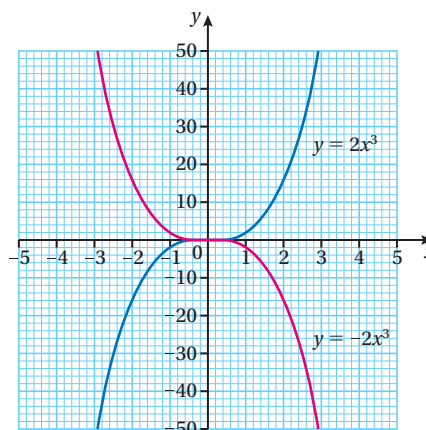
x	-3	-2	-1	0	1	2	3
$y = -x^3$	27	8	1	0	-1	-8	-27

$y = x^3$  ranged from negative values for negative values of  $x$  to positive values for positive values of  $x$  with  $y = -x^3$  ranges from positive values for negative values of  $x$  to negative values for positive values of  $x$ .



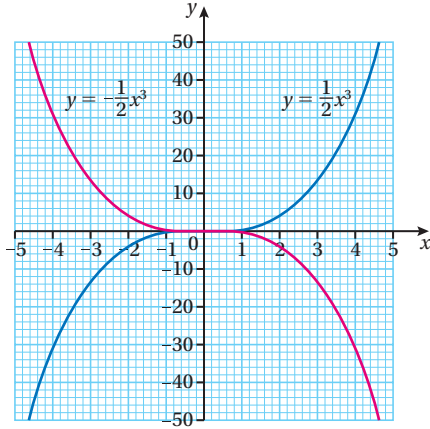
2 a

x	-3	-2	-1	0	1	2	3
$y = -2x^3$	54	16	2	0	-2	-16	-54
$y = 2x^3$	-54	-16	-2	0	2	16	54



**b**

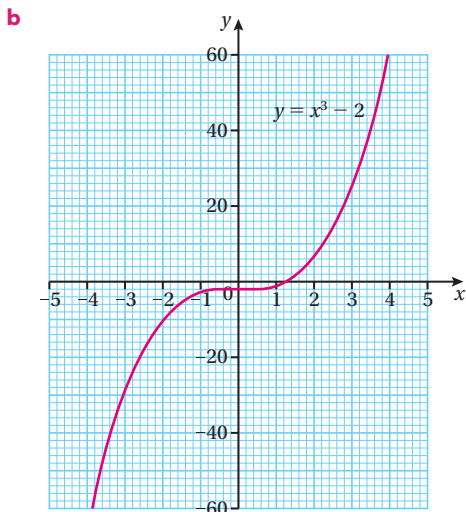
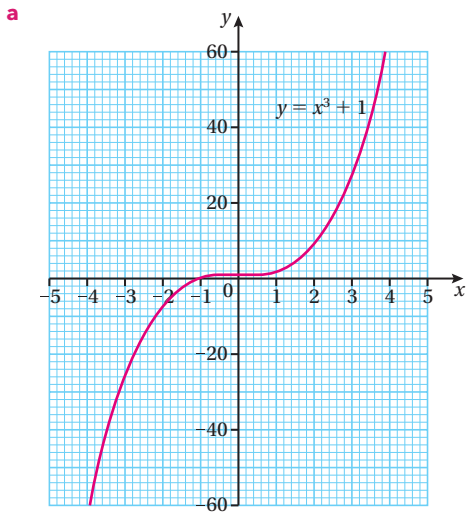
$x$	-3	-2	-1	0	1	2	3
$y = -\frac{1}{2}x^3$	13.5	4	0.5	0	-0.5	-4	-13.5
$y = \frac{1}{2}x^3$	-13.5	-4	-0.5	0	0.5	4	13.5



**3** Given the graph of  $y = 4x^3$  if this graph is reflected about the  $y$ -axis this will produce the graph  $y = -4x^3$

**4**

$x$	-3	-2	-1	0	1	2	3
$y = x^3 + 1$	-26	-7	0	1	2	9	28
$y = x^3 - 2$	-29	-10	-3	-2	-1	6	25



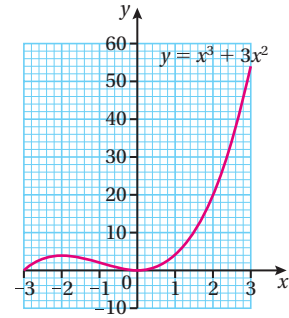
**5 a**  $y = x^3 + 1$ ;  $y$ -intercept  $(0, 1)$

**b**  $y = x^3 - 2$ ;  $y$ -intercept  $(0, -2)$

**6** Line A is  $y = x^3 + 5$       Line B is  $y = x^3 - 6$

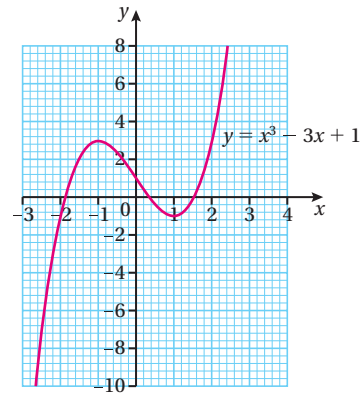
**7 a**  $y = x^3 + 3x^2 - 3 \leq x \leq 3$

$x$	-3	-2	-1	0	1	2	3
$y$	0	4	2	0	4	20	54



**b**  $y = x^3 - 3x + 1 - 3 \leq x \leq 4$

$x$	-3	-2	-1	0	1	2	3	4
$y$	-17	-1	3	1	-1	3	19	53



### EXERCISE 39F

1 a

$x$	-4	-2	-1	1	2	4
a) $y = \frac{2}{x}$	-0.5	-1	-2	2	1	0.5

$x$	-6	-3	-1	1	3	6
b) $y = \frac{6}{x}$	-1	-2	-6	6	2	1

$x$	-10	-8	-6	-4	-2	2	4	6	8
c) $xy = -12$	1.2	1.5	2	3	6	-6	-3	-2	1.5

$x$	-8	-6	-4	-2	1	2	4	6	8
d) $y = \frac{8}{x}$	-1	-1.333	-2	-4	8	4	2	1.333	1

3

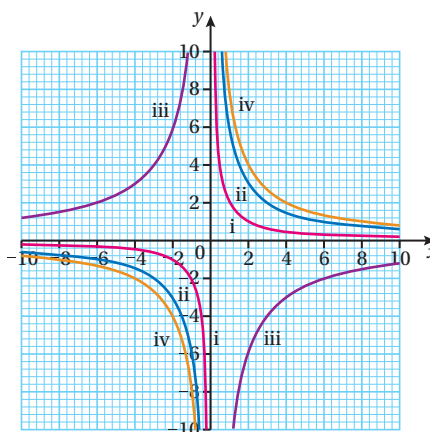
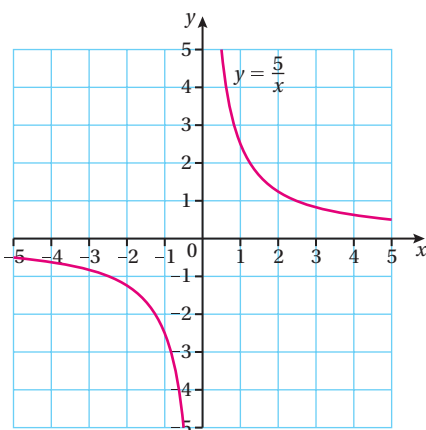
$x$	-5	-4	-3	-2	-1	0	1	2	3	4	5
a) $y = \frac{1}{x}$	-0.2	-0.25	-0.333	-0.5	-1	Not defined	1	0.5	0.333	0.25	0.2
b) $y = \frac{1}{x} + 1$	0.8	0.75	0.6667	0.5	0	Not defined	2	1.5	1.333	1.25	1.2
c) $y = \frac{1}{x} + 3$	2.8	2.75	2.6667	2.5	2	Not defined	4	3.5	3.333	3.25	3.2

4 The constant in  $y = \frac{1}{x} + c$  moves the graph  $y = \frac{1}{x}$  the graph in a vertical direction

5 Neo is correct  $y = x$  is a line of reflective symmetry for the graph  $y = \frac{1}{x}$

6 a  $y = \frac{5}{x}$   $-5 \leq x \leq 4$

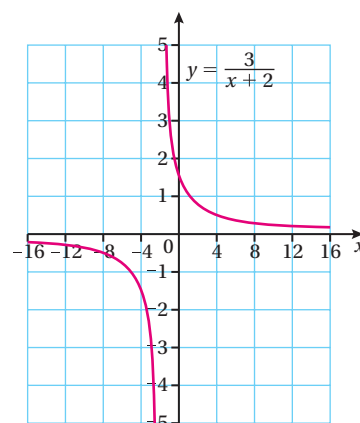
$x$	-5	-4	-3	-2	-1
$y$	-1	-1.25	-1.667	-2.5	-5
$x$	0	1	2	3	4
$y$	Not Defined	5	2.5	1.667	2.5



2 The constant in the equations affects the graph – when it is positive it moves the graph further away from the origin. When it is negative it moves it away from the origin and the orientation changes to the 3<sup>rd</sup> and 4<sup>th</sup> quadrant.

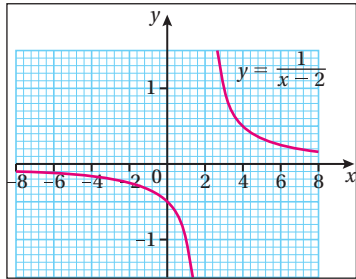
b  $y = \frac{3}{x+2}$   $x = -16, -12, -8, -4, 0, 4, 8, 12, 16$

$x$	-16	-12	-8	-4	0	4	8	12	16
$y$	-0.21	-0.3	-0.5	-1.5	1.5	0.5	0.3	0.21	0.1667

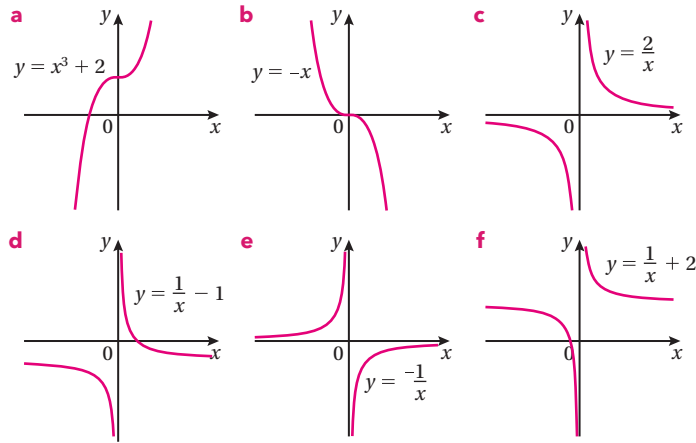


c  $y = \frac{1}{x-2} \quad -4 \leq x \leq 6$

$x$	-4	-3	-2	-1	0	1
$y$	-0.1667	-0.2	-0.25	-0.33	-0.5	-1
$x$	2	3	4	5	6	
$y$	Not Defined	1	0.5	0.33	0.25	



7

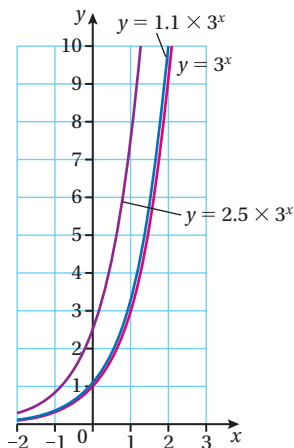


8  $y = \frac{1}{x^2}$

**EXERCISE 39G**

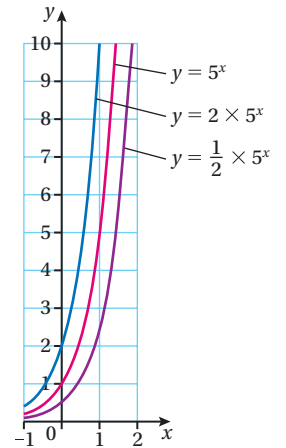
- 1 a i  $y = 3^x$     ii  $y = 1.1 \times 3^x$   
 iii  $y = 2.5 \times 3^x$     values:  $-2 \leq x \leq 3$

$x$	-2	-1	0	1	2	3
i $y = 3^x$	0.111	0.333	1	3	9	27
ii $y = 1.1 \times 3^x$	0.122	0.367	1.1	3.3	9.9	29.7
iii $y = 2.5 \times 3^x$	0.278	0.833	2.5	7.5	22.5	67.5

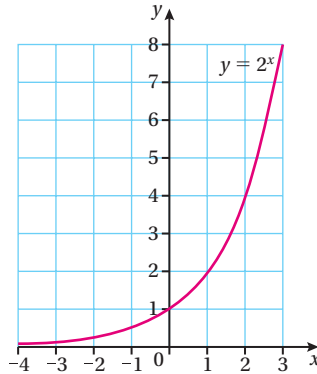


- b i  $y = 5^x$     ii  $y = 2 \times 5^x$   
 iii  $y = \frac{1}{2} \times 5^x$     values:  $-1 \leq x \leq 2$

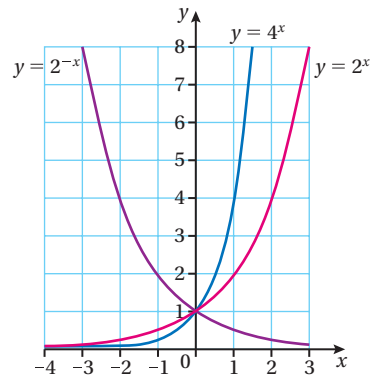
$x$	-1	0	1	2
i $y = 5^x$	0.2	1	5	25
ii $y = 2 \times 5^x$	0.4	2	10	50
iii $y = \frac{1}{2} \times 5^x$	0.1	0.5	2.5	12.5



2 a

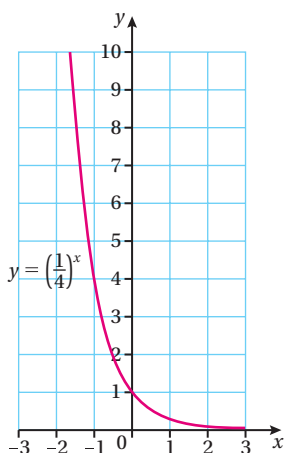


b

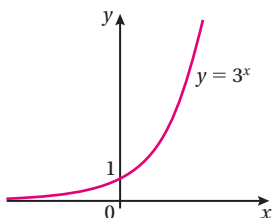


3  $y = \left(\frac{1}{4}\right)^x$  for  $-3 \leq x \leq 3$

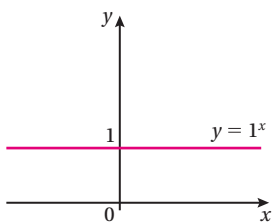
<b>x</b>	-3	-2	-1	0	1	2	3
<b>y</b>	64	16	4	1	0.25	0.0625	0.01563



4 a



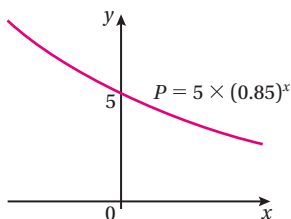
b



5 a A This is a decreasing graph when  $t < 0$  values will be  $> 5$  and when  $t > 0$  values will be  $< 5$  but remain positive.

b The graph cuts the  $y$  axis when  $t = 0$  (0, 5)

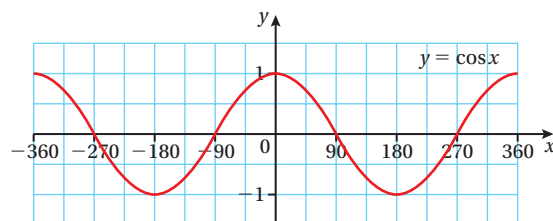
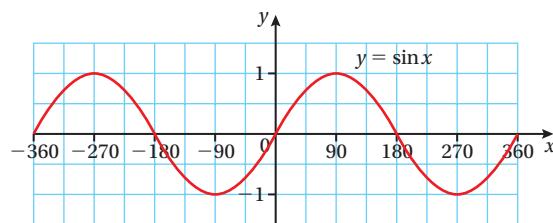
c



### EXERCISE 39H

1 a

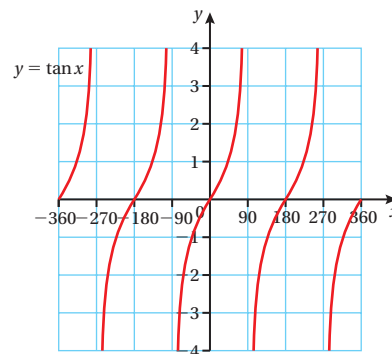
<b>x</b>	-360	-270	-180	-90	0	90	180	270	360
<b>y = sin x</b>	0	1	0	-1	0	1	0	-1	0
<b>y = cos x</b>	1	0	-1	0	1	0	-1	0	1
<b>y = tan x</b>	0	not defined	0	not defined	0	not defined	0	not defined	0



$y = \sin x$  has a maximum value of 1 and repeats every  $360^\circ$

$y = \cos x$  has a maximum value of 1 and repeats every  $360^\circ$

$y = \tan x$  has a maximum value that cannot be defined and it repeats every  $180^\circ$



b  $\sin x = \cos x$  between  $0$  and  $90^\circ$  when  $x = 45^\circ$ ;  $\sin 45^\circ = 0.707$   
 $\cos 45^\circ = 0.707$

2 Graph A most resembles  $y = \sin x$  because it starts at zero and repeats in a wave curve

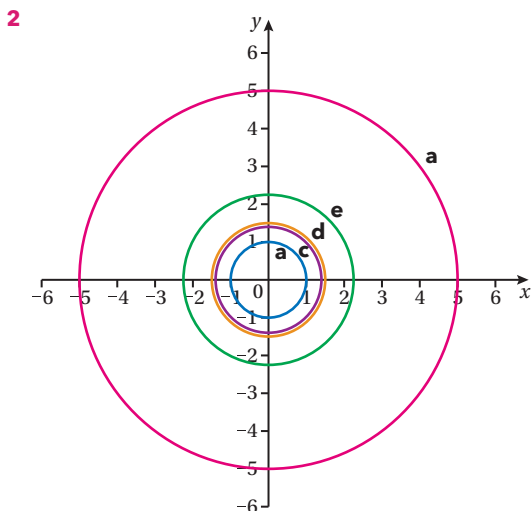
Graph B most resembles  $y = \cos x$  because at zero it has a maximum value and continues in a wave graph, but it does not have negative values.

### WORK IT OUT 39.3

Option B.

### EXERCISE 39I

- 1 a Radius = 5 units  
 b  $(3, 4) 3^2 + 4^2 = 9 + 16 = 25$      $(-3, 4) (-3)^2 + 4^2 = 9 + 16 = 25$   
 c  $(4, 3)$      $(-4, -3)$      $(-3, -4)$      $(3, -4)$   
 $(0, 5)$      $(0, -5)$      $(-5, 0)$      $(5, 0)$  any four



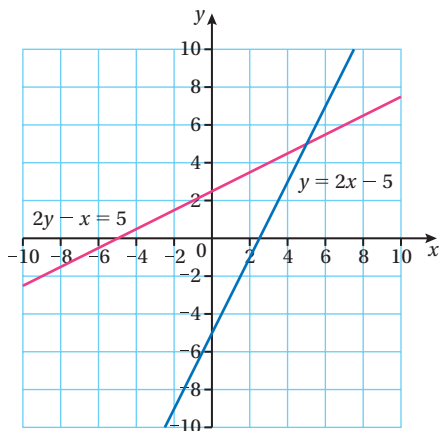
- 3 a  $(6, 8)$      $(-6, 8)$      $(5\sqrt{2}, 5\sqrt{2})$      $(10, 0)$   
 b  $(5, 12)$      $(-5, -12)$      $(-5, 12)$      $(0, 13)$   
 4 a  $x^2 + y^2 = 1$     b  $x^2 + y^2 = 4$     c  $x^2 + y^2 = 9$     d  $x^2 + y^2 = 16$

### EXERCISE 39J

- 1 a Check student drawing.    b Gradient =  $-\frac{4}{3}$   
 c Gradient =  $\frac{3}{4}$     d  $y = \frac{3}{4}x - \frac{25}{4}$   
 2 a  $y = -2x + 5$     b  $y = \frac{1}{2}x + 10$     c  $y = -\frac{1}{3}x + 10$   
 3  $y = 9$  and  $x = 9$

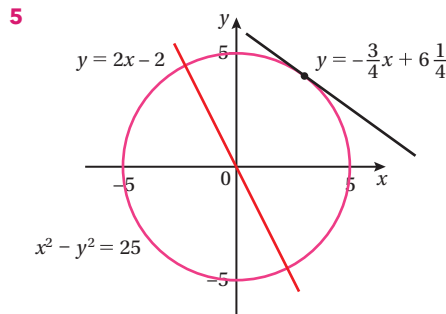
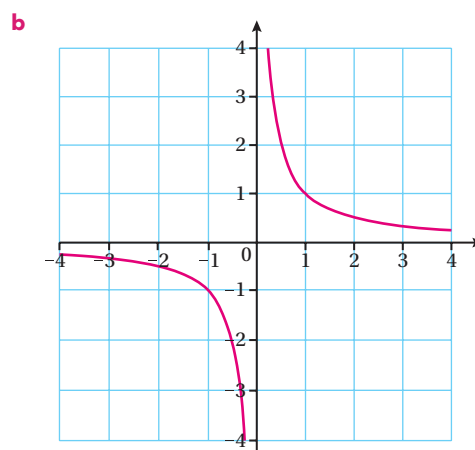
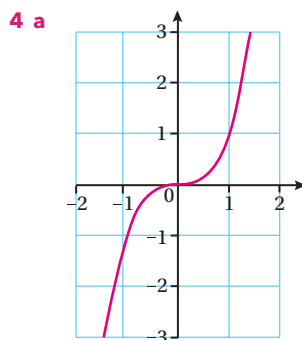
### CHAPTER REVIEW

- 1 Point of intersection of  $y = 2x - 5$  and  $2y - x = 5$  is  $(5, 5)$



- 2 a  $x = -1$  and  $x = 3$   
 b  $y = -(x + 1)(x - 3)$   
 $y = -(x^2 - 2x - 3)$   
 $y = -x^2 + 2x + 3$   
 c  $x^2 - 2x - 3 = 0 \Rightarrow (x - 1)^2 - 4$   
 The turning point is  $(1, -4)$ , axis of symmetry  $x = 1$ .

- 3 a Red curve:  $y = \cos x$  Blue curve:  $y = \sin x$   
 b In the range  $0^\circ \leq x \leq 270^\circ$   $\sin x = \cos x$  for  $x = 45^\circ, 225^\circ$   
 $\sin 45^\circ = \cos 45^\circ$  and  $\sin 225^\circ = \cos 225^\circ$



The point of contact of the tangent  $y = -\frac{3}{4}x + 6\frac{1}{4}$  with the circle  $x^2 + y^2 = 25$  is  $(3, 4)$   
 $y = 2x - 2$  cuts the circle  $x^2 + y^2 = 25$   $x^2 + (2x - 2)^2 = 25$   
 $5x^2 + 8x - 21 = 0$  solving gives  $x = 1.4$  and  $x = -3$   
 $y = 0.8$  and  $y = -8$  the points where the line cuts are  $(1.4, 0.8)$  and  $(-3, -8)$

The point  $(3, 4)$  lies on the circle  $x^2 + y^2 = 25$   $3^2 + 4^2 = 25$   
 $9 + 16 = 25$

$x = 3$  and  $y = 4$  satisfies the equation  $y = -\frac{3}{4}x + 6\frac{1}{4}$

$$y = -\frac{3}{4} \times 3 + 6\frac{1}{4} = -\frac{9}{4} + \frac{25}{4} = \frac{16}{4} = 4$$

- 6 a  $y = x^2 - x - 12$     b  $x^2 + y^2 = 16$   
 c  $y = -x^2$     d  $y = \frac{4}{x}$   
 e  $y = \frac{1^x}{2}$     f  $y = 3 \times 2^x$

## 40 Growth and decay

### BEFORE YOU START ...

- 1 a 0.05      b 1.9      c 0.004      d 0.125  
 2 a \$53.68      b £36.93      c £7.28      d \$156.40  
 3 a 100      b number doubles  
 4 a initial number of bacteria  
 b each hour the number of bacteria is multiplied by 2

### LAUNCHPAD

- 1 a £573.76      b  $500 \times 1.035^n$       c 6 years  
 2 a 3.1 grams (1 dp)  
 b 28 years  
 c  $4 \times 0.975^n$

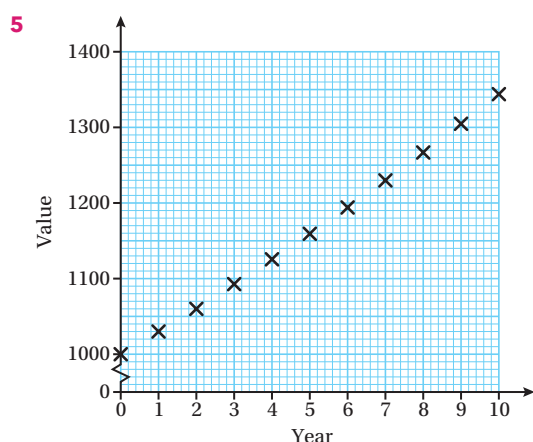
### WORK IT OUT 40.1

Student B and Student C are correct. Student A has used simple percentage increase, but it should be compound.

### EXERCISE 40A

- 1 a £306  
 b £318.36 (2 dp)  
 c £351.50 (2 dp)  
 d  $300 \times 1.02^n$   
 2 Option C  
 3 Option C  
 4

Investment	Interest rate	1 year	2 year	$5\frac{1}{2}$ year	$n$ year
£250	2%	£255	£260.10	£278.77	$250 \times 1.02^n$
£1500	4.5%	£1567.50	£1638.04	£1910.87	$1500 \times 1.045^n$
£50	3%	£51.50	£53.05	£58.83	$50 \times 1.03^n$



- 6 a 256      b  $100 \times 1.04^n$   
 7 a 5 444 617      b 844 617      c 4 447 500  
 d No, population rates vary with many factors, such as economic prosperity, weather etc  
 8 a £668.68      b  $450 \times 1.02^n$

- 9 4 months  
 Month 1  $£200 + 8\% = £216$   
 Month 2  $£216 + 8\% = £233.28$   
 Month 3  $£233.28 + 8\% = £251.94$   
 Month 4  $£251.94 + 8\% = £272.10$

- 10 After 5 years be roughly half a million (4 952 198) and after 10 years there will be well over twelve million million roughly  $1.226 \times 10^{13}$ .

With no outside influences the biologist should be concerned as the predicted spread is large. They should carry out further studies. It is unlikely that the initial 2 mussels are the only ones in the lake.

- 11 a Over 5 years simple interest will get a total of 30%, compound interest 30.70% – compound better  
 b Over 4 years simple interest will get a total of 24%, compound interest 23.88% – simple better

### 12

Investment	Rate	1 year	2 year	3 year	$n$ year
\$600	1.5%	\$609	\$618.14	\$627.41	$600 \times 1.015^n$
£500	6%	£530	£561.80	£595.51	$500 \times 1.06^n$
\$6000	10%	\$6600	\$7260	\$7986	$6000 \times 1.1^n$
£750	20%	£900	£1080	£1296	$750 \times 1.2^n$

- 13 a no      b £213.13  
 c one more month with £200 saving plus interest will cover this (£2789.48)

- 14 a 160%      b £1.69      c £4.39  
 d annually rise of 3.3%      e 2023

- 15 a \$12 889.46 (compound interest \$62 889.46, simple interest \$50 000)  
 b 6.29% (2 dp)

- 16 £296 023.73

- 17 Model 1 12.476% Model 2 12.4864% Model 3 12.3% Model 2 best

- 18 15 hours

### WORK IT OUT 40.2

Student B and Student C are correct, Student A has used simple percentage increase instead of compound.

Student C has used the most efficient method.

### EXERCISE 40B

- 1 Option D  
 2 Option A  
 3 a £10 120      b £8565.57  
 c £5645.41      d  $11\ 000 \times 0.92^n$   
 4

Initial cost	Depreciation rate	1 year	2 year	6 year	$n$ year
£400	2%	£392	£384.16	£354.34	$400 \times 9.8^n$
£2 500	15%	£2125	£1806.25	£942.87	$2500 \times 0.85^n$
£50 000	3.5%	£48 250	£46 561.25	£40 376.98	$50\ 000 \times 0.965^n$

- 5 a 724 g  
 b Mathematically speaking there will always be DDT present, however the actual amount will become negligible and too small to be measured.

6 Graph B

- 7 a 20% drop    b 1342    c 20 hours

8 685.7 Pa (1 dp)

- 9 a 7 137 564    b 10 years

10

Initial cost	Depreciation rate	1 year	2 year	6 year	$n$ year
\$7500	7.5%	\$6937.5	\$6417.19	\$4697.99	$7500 \times 0.925^n$
£650	5%	£617.50	£586.63	£477.81	$650 \times 0.95^n$
\$34 000	11%	\$30 260	\$26 931.40	\$16 897.36	$34\,000 \times 0.89^n$
£12 000 000	7.5%	£11 100 000	£10 267 500	£7 516 776.59	$12\,000\,000 \times 0.925^n$

- 11 5% loss each year, 8 years to cost less than two thirds of today's price

### CHAPTER REVIEW

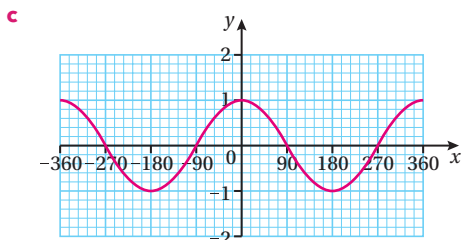
- 1 a £85    b £765    c £153    d £38.25  
 e £153  
 2 £2903.70  
 3 £4188.80  
 4 9  
 5 3 years  
 6 5.7% per year

## 41 Transformations of curves

### BEFORE YOU START ...

- 1 Only  $5y + x = 10$ : sketch is a straight line passing through (0, 2) and (10, 0).  
 $y = x^2 + 9$  is a parabola with a minimum point (0, 9) and  $y$ -axis as the axis of symmetry.  
 $y = \frac{2}{x}$  is a rectangular hyperbola in the 1st and 3rd quadrants.  
 $y = (x + 7)^2 - 2$  is parabola, turning point (-7, -2), line of symmetry  $x = -7$ ,  $y$ -axis intercept (0, 47).

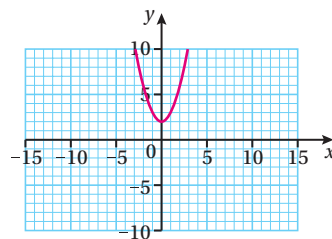
- 2 a  $y = \sin x$     b  $y = \tan x$



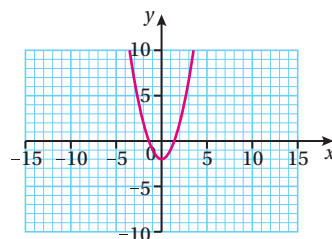
- 3  $3x^2 + 6x + 3 = 3(x + 1)^2 + 4$

### LAUNCHPAD

- 1 a Axis of symmetry  $x = 0$ , vertex (0, 2)

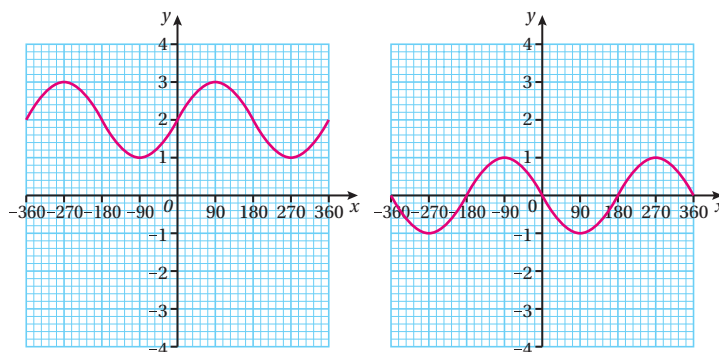


- b Axis of symmetry  $x = 0$ , vertex (0, -2)



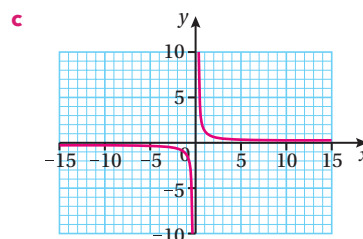
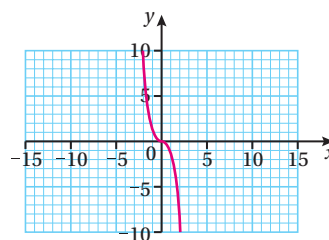
- c  $y = -x^2$

- 2 a  $\sin 90^\circ = 1$ ;  $\cos 90^\circ = 0$ ;  $\cos \theta = 1$  when  $\theta = 360n$  for integer values of  $n$



- 3 a  $y = x^3$

- b  $y = -x^3$



### WORK IT OUT 41.1

Graph A has two solutions.

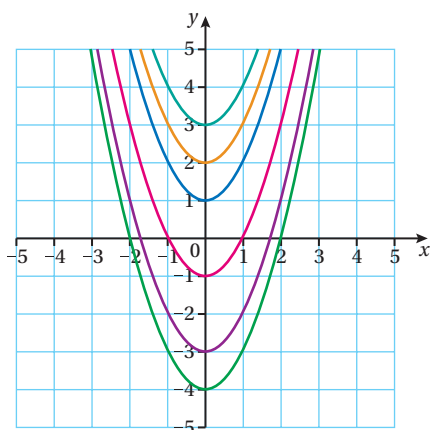
Graph B has no solution and Graph C has one solution.



### EXERCISE 41A

1 Option D

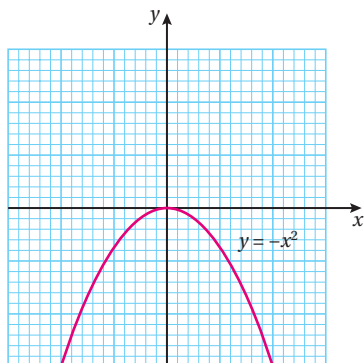
2



Vertices are as follows:

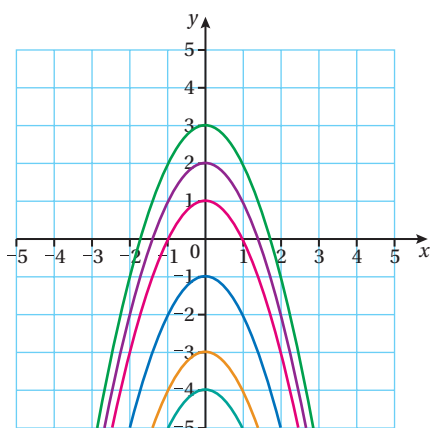
- a** (0, 1)    **b** (0, -1)    **c** (0, -4)    **d** (0, 2)  
**e** (0, -3)    **f** (0, 3)

3 A reflection of  $y = x^2$  about the  $x$ -axis.



4 As  $x$  increases and decreases in value,  $y$  decreases.  $-x^2$  is always negative.

5 The transformation is a reflection for all the curves about the  $y$ -axis (mirror line).



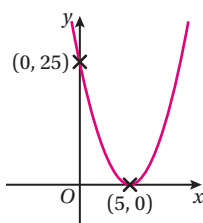
### EXERCISE 41B

1 The graphs are images, reflected in the horizontal line which passes through the vertex.

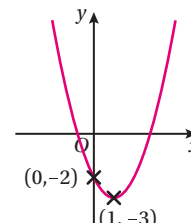
- a**  $y = 3$     **b**  $y = -3$   
**c**  $y = 3$     **d**  $y = -3$

2 Equations a and c won't have real roots.

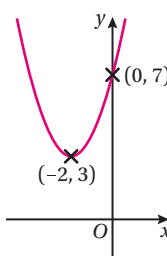
3 a



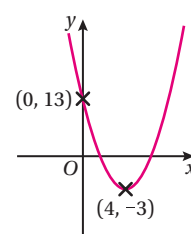
b



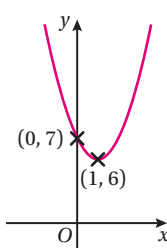
c



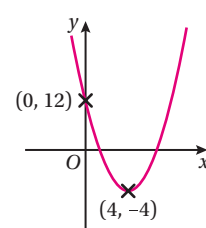
d



e



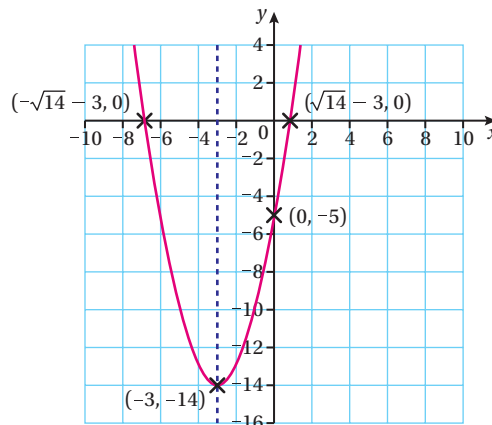
f



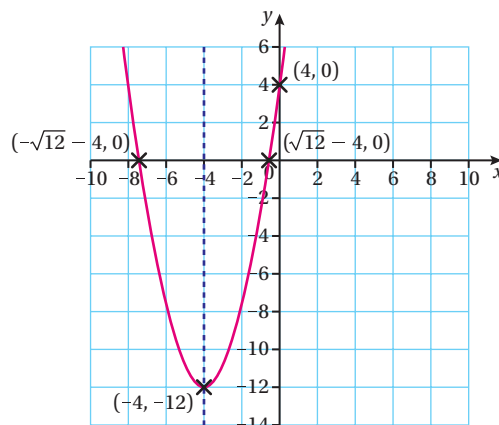
### EXERCISE 41C

1 Option C

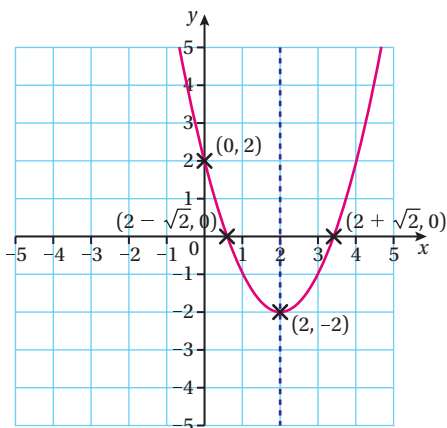
2 a  $y = (x + 3)^2 - 14$



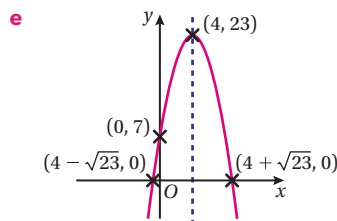
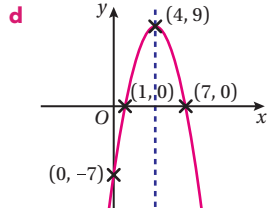
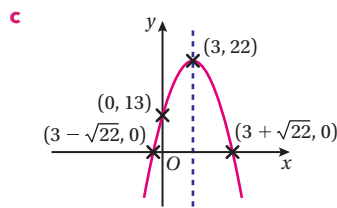
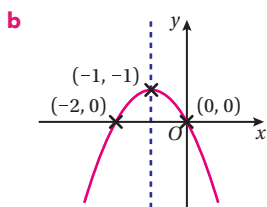
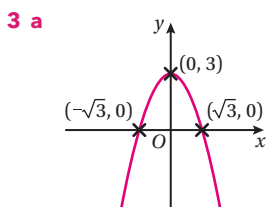
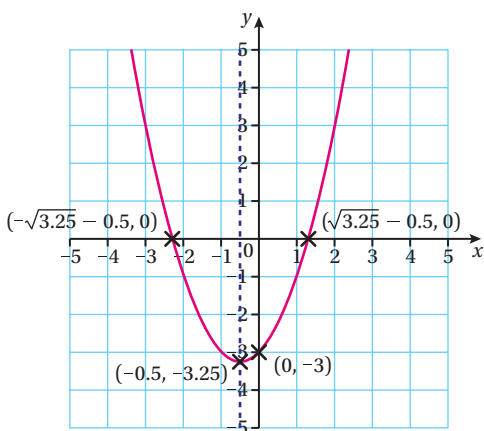
b  $y = (x + 4)^2 - 12$



c  $y = (x - 2)^2 - 2$



d  $y = (x + 0.5)^2 - 3.25$



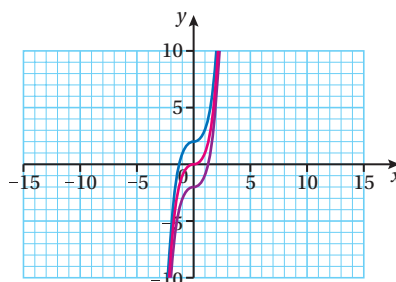
**EXERCISE 41D**

- 1 a  $y = \sin x + 2$  and  $y = \sin x - 2$  are vertical translations. The sine waves move up 2 units and down 2 units.  
 b  $y = \cos x + 2$  and  $y = \cos x - 2$  are vertical translations. The cosine waves move up and down 2 units.

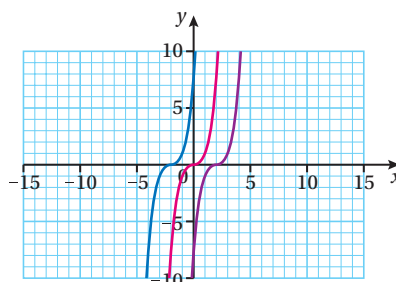
- 2 a  $y = \sin(x + 90^\circ)$  and  $y = \sin(x - 90^\circ)$  are horizontal translations to the left and right by  $90^\circ$   
 b  $y = \cos(x + 90^\circ)$  and  $y = \cos(x - 90^\circ)$  are horizontal translations to the left and right by  $90^\circ$   
 3 a  $y = \sin x + 1$ : a vertical translation up of 1 unit  
 b  $y = \sin(x + 45^\circ)$ : a horizontal translation left of  $45^\circ$   
 c  $y = \cos(x - 45^\circ)$ : a horizontal translation right of  $45^\circ$   
 d  $y = \cos x - 1$ : a vertical translation down 1 unit  
 4  $y = -\sin x$  is a reflection of  $y = \sin x$  in the  $x$ -axis.  
 5  $y = -\cos x$  is a reflection of  $y = \cos x$  in the  $x$ -axis.

**EXERCISE 41E**

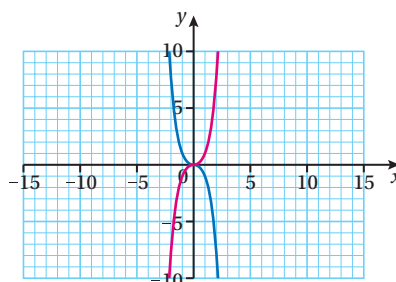
1 a



b



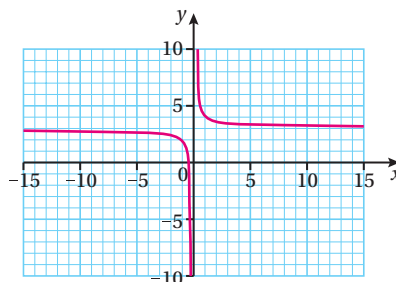
c Line of reflection is  $y = 0$

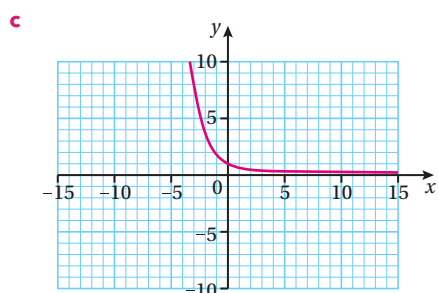
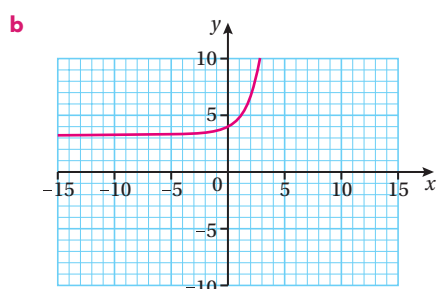
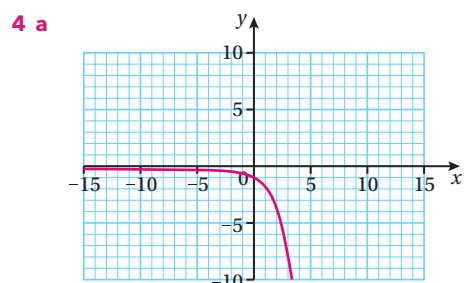
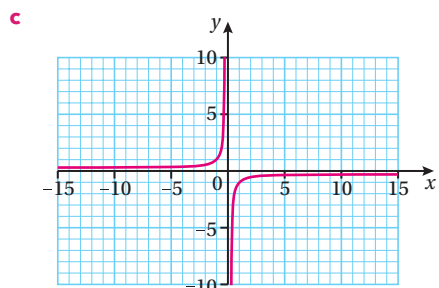
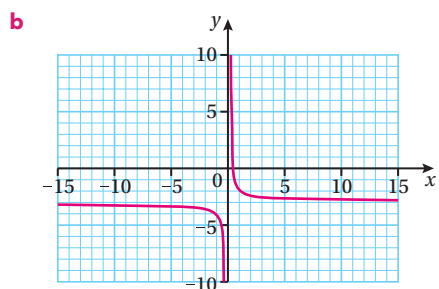


2 Option C

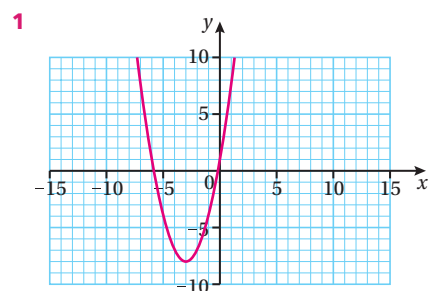
3  $y = \frac{1}{x}$  is not defined, so the curve of  $y = \frac{1}{x}$  is discontinuous at  $x = 0$ .

a





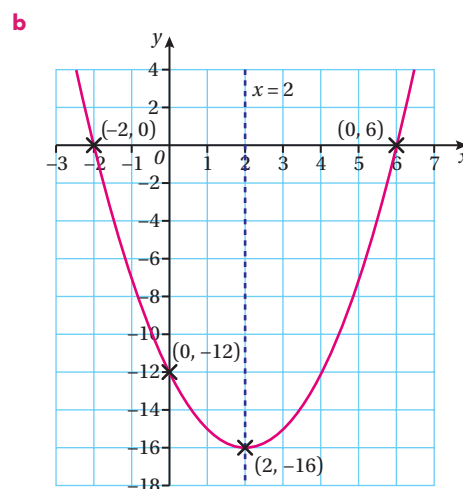
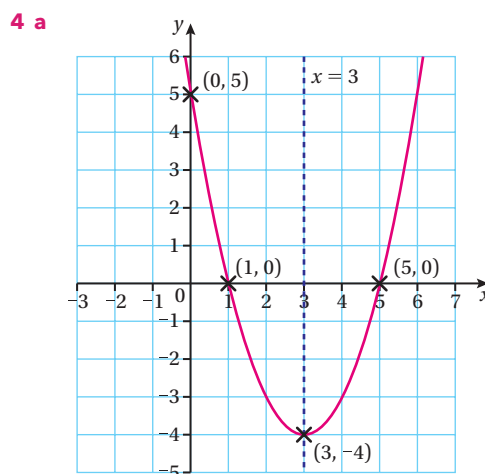
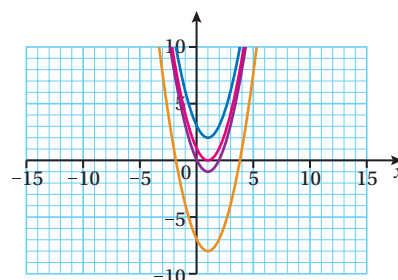
**EXERCISE 41F**



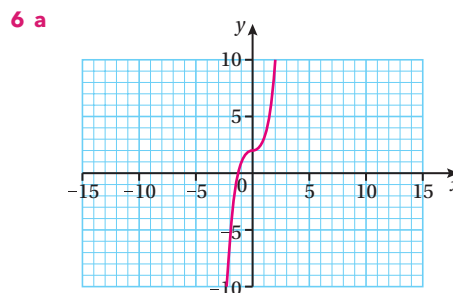
**a**  $y = x^2$       **b**  $y = (x + 5)^2 - 11$

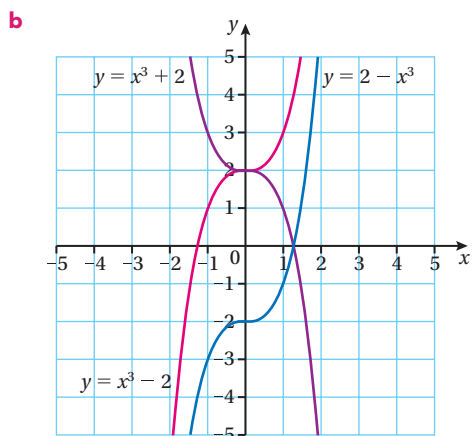
**2** Option A

**3 a**  $a = 0$       **b**  $a = 2$       **c**  $a = -1$       **d**  $a = -8$



**5**  $y = \sin(x + 90^\circ)$





**7**  $y = 2^{(x-3)}$

**8 a** A translation of  $60^\circ$  to the right and vertical translation down of  $-1$

$$y = \cos(x - 60^\circ) - 1 \quad (150^\circ, -1)$$

$$-1 = \cos(150^\circ - 60^\circ) - 1 = 0 - 1 = -1$$

**b** Reflection in the  $y$ -axis

$$y = x^2 - 2x \quad (-1, 3) \quad 3 = (-1)^2 - 2(-1) = 3$$

**c** Translations of 1 unit to the left

$$y = \frac{1}{(x+1)} \quad (-2, -1) \quad -1 = \frac{1}{-2+1} = \frac{1}{-1} = -1$$

$$\left(1, \frac{1}{2}\right) \quad \frac{1}{2} = \frac{1}{(1+1)} = \frac{1}{2}$$

**d** Reflection about the line  $y = 1$

$$y = \frac{1}{(x+1)} + 1 \quad \left(1, \frac{1}{2}\right) \quad \frac{1}{2} = \frac{-1}{(1+1)} + 1 = -\frac{1}{2} + 1 = \frac{1}{2}$$

**e** Vertical translation

$$y = 2^x - 1 \quad (1, 1) \quad 1 = 2^1 - 1 = 2 - 1 = 1$$

**f** Vertical translation

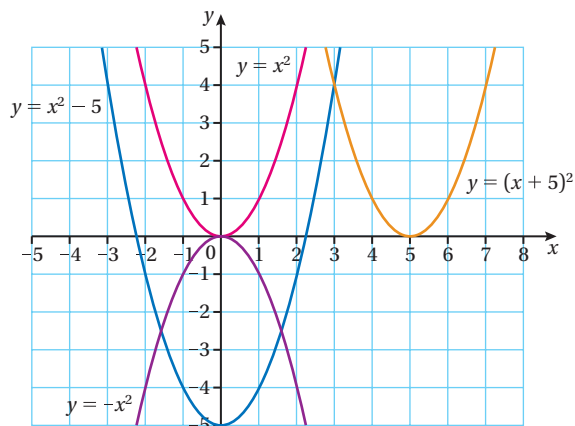
$$y = \tan x - 2 \quad (0, -2) \quad -2 = \tan 0 - 2 = 0 - 2 = -2$$

**9**  $y = \sin(x - 40^\circ) - 2$

### CHAPTER REVIEW

**1** Option B

**2 a**



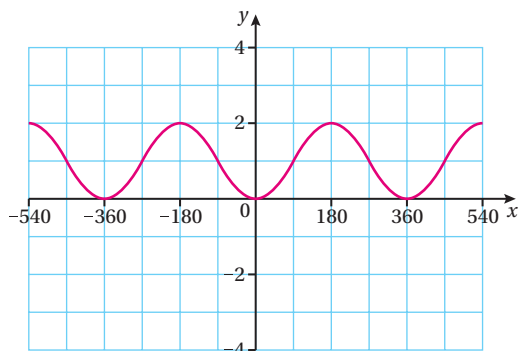
**b** Translation 5 units down

**c** Reflection in the  $x$ -axis ( $y = 0$ )

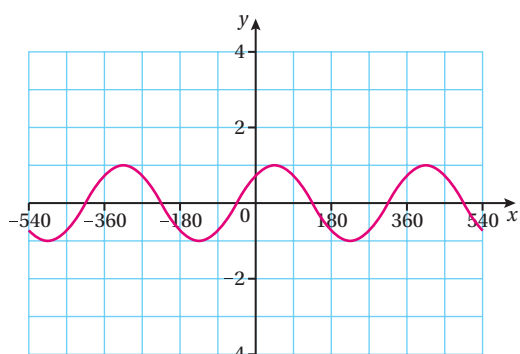
**d** Translation of 5 units to the right

**3**  $y = \sin(x - 45^\circ)$

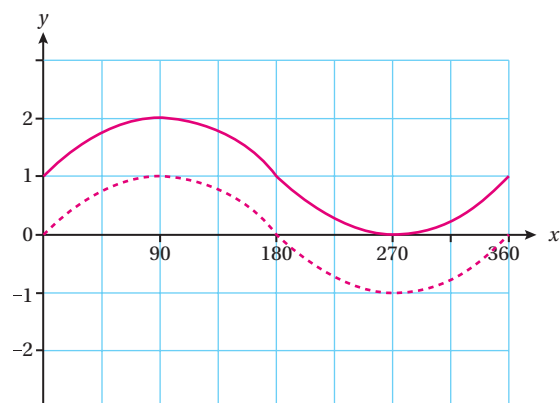
**a**



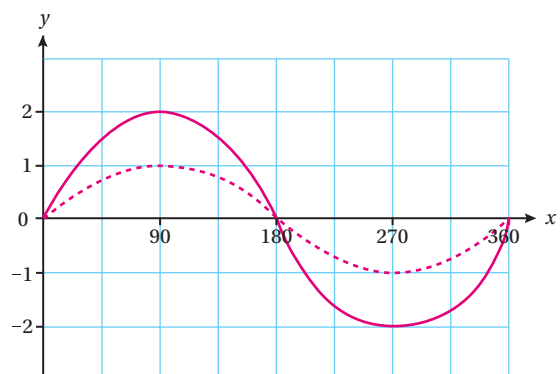
**b**



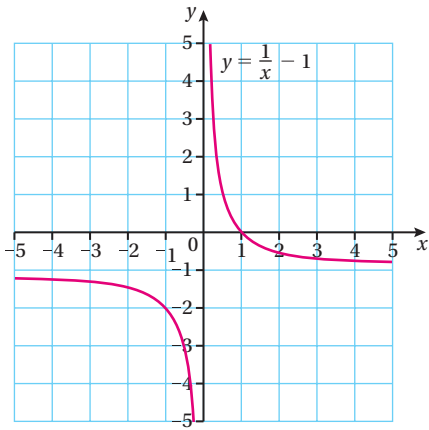
**4 a**



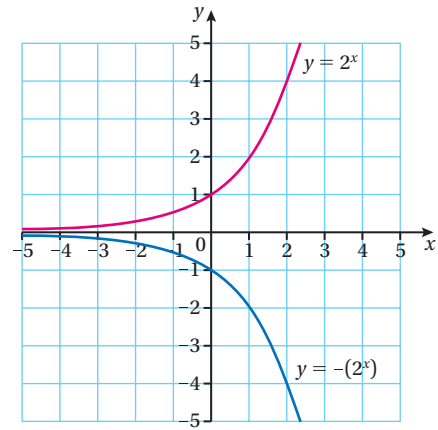
**b**



5 a



b



6 It would move one unit to the left.