MODELS OF PLANETARY NEBULAE: GENERALISATION OF THE MULTIPLE WINDS MODEL

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ABSTRACT. According to the multiple winds model a planetary nebula forms as the result of the interaction of a fast wind from the central star with the superwind that had previously been emitted by the progenitor star. The basic theory which deals with the spherically symmetrical case is briefly summarised. Various improvements are then considered in turn. A better history is clearly needed of the way that the central star becomes hotter, it is unrealistic to make the assumption that the superwind is spherically symmetrical, and finally there are likely to be important instabilities at some of the interfaces in the PN, notably that between the shocked superwind and the HII layer. These changes in the theoretical description produce a better understanding of the conditions in the outer parts of a PN and of the nature of its general shape, and they should lead to an explanation for the occurrence of high speed motions, and of highly ionized species and high excitation spectral lines.

1. INTRODUCTION

The multiple winds model (Sun Kwok 1982, 1983, 1988; Kahn 1983) gives a very good description of the overall properties of planetary nebulae. In this paper it will be discussed first in its simplest form, and then various restrictions will be removed. The basic model, with spherical symmetry, has its uses and makes it possible to establish various important physical parameters, as well as the relations between them. However, it fails to allow for the fact that the shapes of most PNe are far from spherically symmetrical and it is based on idealised assumptions concerning the onset of the fast wind from the central star which constitutes the planetary nebula nucleus (PNN). Finally there is a stability problem for those PNe which are ionization bounded. The HII layers here are squeezed between the hot shocked stellar wind (HSSW) inside and the non-ionized gas outside, deriving from the superwind (SW); such configurations are in general unstable, and in a wide range of cases the growth time of disturbances is short enough for finite amplitude wave motions to develop within the lifetime of the PN. If so, there is a strong likelihood that the cool gas deriving from the SW will mix with hot gas from the HSSW,

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and that the shell of non-ionized gas will fragment.

2. THE BASIC MODEL

The superwind is assumed to carry away mass at rate $\dot{M}$ from the progenitor of the central star up to time t = 0. It has terminal speed u, and is spherically symmetrical. At time t = 0 the fully fledged PNN is assumed to appear. It has luminosity L, blows a fast wind with terminal speed V(>> u). The wind carries off mechanical energy at rate $L_W \equiv \eta L$, and the star emits Lyman continuum photons at rate $S_* \equiv jL$.

Two shocks develop as the result of the interaction between the winds. On the outside a shock is driven into the gas from the SW, which it compresses into a thin, well-cooled shell at radius r. Closer to the PNN there is another shock facing into the fast wind. The HSSW fills the region between this shock and the HII layer, which lines the inside of the cool shell at radius r.

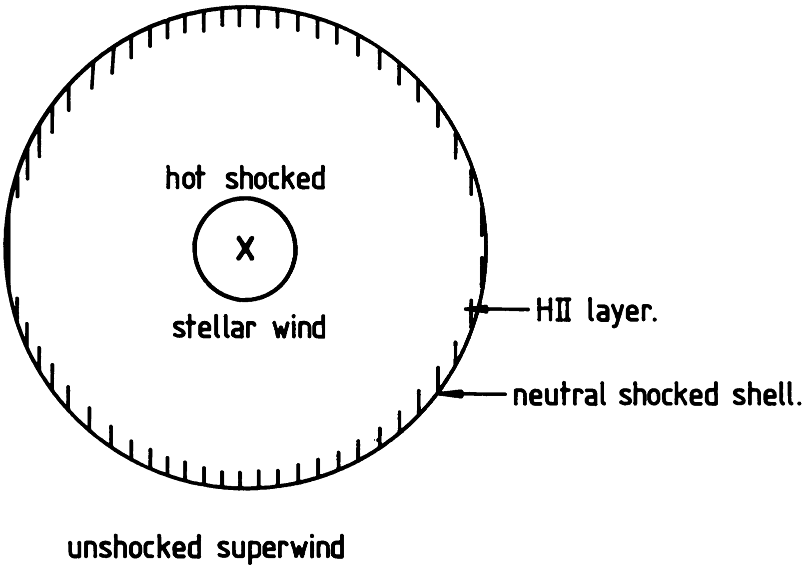

Figure 1. The various regions in the multiple winds model.

The equations to describe the motion are

$$L_W \equiv \eta L = \frac{d}{dt} 2\pi r^3 P + 4\pi r^2 P\dot{r} \tag{1}$$

$$P = \rho_o(\dot{r} - u)^2 + \frac{M_s \ddot{r}}{4\pi r^2} \tag{2}$$

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$$M_s = \frac{\dot{M}}{u}(r - ut) \quad , \tag{3}$$

and here

$$\rho_o = \frac{\dot{M}}{4\pi u r^2} \tag{4}$$

is the density in the SW just ahead of the outer shock, P is the pressure in the HSSW and M_S is the mass of the shell at radius r.

Equation (1) shows how the mechanical energy supplied by the fast wind goes partly to heat the HSSW and partly to push back the shell outside; equation (2) relates the pressure behind the shell to the ram pressure at the outer shock and the force per unit area needed to accelerate the shell; equation (3) is self-evident, but is based on the assumption that the fast wind starts up at the moment that the SW stops.

The solution of the equations leads to the results

$$r = \lambda ut \ , \quad \dot{r} = \lambda u \ , \quad P = \frac{\dot{M}(\lambda - 1)^2}{4\pi\lambda^2 ut^2} \tag{5}$$

where λ satisfies

$$\lambda(\lambda - 1)^2 = \frac{2}{3}\frac{\eta L}{\dot{M}u^2} \tag{6}$$

Balancing the ram pressure of the fast wind against the pressure in the HSSW shows that the inner shock is at radial distance

$$r_i = \left(\frac{2L_W u}{\dot{M}V}\right)^{\frac{1}{2}} \frac{\lambda t}{\lambda - 1} \tag{7}$$

and therefore

$$\frac{r_i^3}{r^3} = \left(\frac{3\lambda u}{V}\right)^{3/2} \tag{8}$$

is the ratio of the volumes enclosed respectively by the inner and the outer shocks. Here λu is the expansion speed of the PN, typically 30 km/s, and the fast wind speed V is typically 3000 km/s, so that the ratio of volumes becomes about 1:200.

The temperature in the HSSW is found from

$$\frac{kT}{m} = \frac{P}{\rho_H} = \frac{V^2}{9} \tag{9}$$

where

$$\rho_H = \frac{3M_H}{4\pi r^3} = \frac{2L_W t}{V^2} \tag{10}$$

is the density, and M_H the total mass in the HSSW. The fast wind is typically raised to a temperature of 7 x 10^7K on passing through the inner shock. It is easy to verify that radiative cooling of the HSSW will be quite negligible under realistic conditions. Further the sound speed in the HSSW is typically 1300 km s^{-1}, and the expansion of the PNe therefore takes place very subsonically with respect to the hot gas. The pressure will consequently be (almost) uniform in the HSSW bubble.

For illustrative purposes the values of important parameters are here taken to be

$$u = 10 \text{ km s}^{-1} , \quad \dot{M} = 9 \times 10^{-5} M_\odot/\text{year} ,$$

$$L = 6300 \, L_\odot = 2.5 \times 10^{37} \text{ erg/s} , \quad L_w = \eta L = 10^{35} \text{ erg/s}$$

$$S_* \equiv jL = 2.5 \times 10^{47} \text{ photons/s}$$

and are (almost) consistent with a value $\lambda = 3$, and an expansion speed $\dot{r} = 30$ km/s for the shocked shell.

A PN will be optically thick as long as the mass M_s of the shell exceeds the mass M_i of ionized gas in the HII layer. The number of electrons in this layer is

$$N_e = M_i/m_a \tag{11}$$

where m_a is the mass of gas per free electron, and the electron density, at pressure P, is

$$n_e = P/m_a c_i^2 , \tag{12}$$

and here c_i is the isothermal sound speed in the HII region. If b ($\doteq 2 \times 10^{-13}$ cm^3/s) is the recombination coefficient, then, after some algebra, it is found that

$$M_i = m_a N_e = m_a S_*/bn_e \equiv jL \, m_a/bn_e$$

$$= \frac{6\pi j \lambda^3}{\eta b} \, m_a^2 c_i^2 u^3 t^2 \quad . \tag{13}$$

The ratio of masses is

$$M_i : M_s = t : t_* \tag{14}$$

where

$$t_* = \frac{\eta b \dot{M}(\lambda - 1)}{6\pi j \lambda^3 m_a^2 c_i^2 u^3} \tag{15}$$

or, with the present typical values, 15000 years. A PN that is younger will be optically thick. In many cases the PNN evolves through its hot phase in a rather shorter time than 15000 years, so that greater ages cannot occur. Evolution rates for central stars have been discussed in detail by Schönberner (1983, 1986, 1988).

The thickness of the HII layer is also physically important; it equals

$$\Delta_i = \frac{M_i}{4\pi r^2 \rho_i} = \frac{M_i c_i^2}{4\pi r^2 P}$$

and the sound travel time across the HII layer is

$$t_c = \frac{\Delta_i}{c_i} = \frac{6\pi j}{b\eta} \frac{\lambda^3}{(\lambda - 1)^2} \frac{m_a^2 c_i^3 u^2 t^2}{\dot{M}} \tag{16}$$

At a typical age t, say 3000 years, or 10^{11} s, it is found that

$$t_c = 10^{10} \text{ s}$$

and is thus smaller by an order of magnitude than t. The ratio t_c:t increases linearly with time.

3. A MORE REALISTIC START TO THE EXPANSION

One obvious shortcoming of the basic model lies in the assumption that the fast wind starts to blow at the moment that the superwind stops. Computed models of the evolution of the central star allow intervals of between 1000 and 10000 years for the transition from the AGB to a hot PNN, depending on the mass of the star concerned. There is as yet no detailed description of the history of such a transition period. At present the best that can be done is to make an estimate of how the model will change if an interval of length t_o elapses between the end of the superwind and the sudden appearance of a PNN, with a copious photon output in the Lyman continuum and a fast stellar wind.

At time t_o a cavity, with radius ut_o, will have formed in the distribution of gas from the superwind. The fast wind will fill this volume with hot shocked gas, whose pressure at time $t_o + \tau$ will be

$$P = \frac{L_w \tau}{2\pi u^3 t_o^3} = \frac{3}{4\pi} \frac{\lambda(\lambda-1)^2 \dot{M}\tau}{u t_o^3} \tag{17}$$

The pressure builds up quite rapidly and a shock propagates into the surrounding material, forming a dense shell of cool gas. If v is the speed of the shell, then the rate of growth of its mass is given by

$$\frac{dM_s}{d\tau} = \frac{\dot{M}}{u} (v - u) \tag{18}$$

and the equation for v is

$$P = \frac{M_s \dot{v}}{4\pi r_o^2} + \frac{\dot{M}(v-u)^2}{4\pi r_o^2 u} . \quad (19)$$

These equations apply early on during the acceleration, before the radius of the shell has increased much beyond r_o. It follows that

$$v - u = \frac{3}{2}\lambda^{\frac{1}{2}}(\lambda - 1)u\left(\frac{\tau}{t_o}\right)^{\frac{1}{2}} \quad (20)$$

and

$$M_s = \lambda^{\frac{1}{2}}(\lambda - 1)\,\dot{M}\,\frac{\tau^{3/2}}{t_o^{\frac{1}{2}}} . \quad (21)$$

Both the mass of the shell and its outward velocity build up quite rapidly towards the values that they would have at radius r_o according to the basic model. The mass of gas in the HII layer is again given by

$$M_i = N_e m_a = jLm_a^2 c_i^2/bP$$

so that now

$$M_i = \frac{4\pi jLm_a^2 c_i^2 u t_o^3}{3\lambda(\lambda-1)^2\dot{M}\tau} = \frac{2\pi j}{\eta}\,\frac{m_a^2 c_i^2 u^3 t_o^3}{b\tau} . \quad (22)$$

The model is consistent only once M_s exceeds M_i; the HII shell is then optically thick, and this applies when

$$\frac{\tau}{t_o} > \left(\frac{2\pi j}{\eta b\dot{M}}\right)^{2/5} \frac{m_a^{4/5} c_i^{4/5} u^{6/5} t_o^{2/5}}{\lambda^{1/5}(\lambda-1)^{2/5}} . \quad (23)$$

With the usual values for the parameters of the PN, and setting $t_o = 10^{11}$ s or 5×10^{10} s, the condition becomes, respectively,

$$\frac{\tau}{t_o} > 0.074 \quad \text{or} \quad 0.056 , \quad (24)$$

so that there is a (relatively short) period when the PNN is turned on, but the shell is transparent to Ly c photons. On average about 71 per cent of the photons reach the SW beyond the shocked shell. Using typical parameters and taking t_o to be 10^{11} s or 5×10^{10} s again, the number of Ly c photons thus made available is 1.3×10^{57} and 5×10^{56}, respectively. They are enough to ionize 1.3 $M_\odot$ or 0.5 $M_\odot$ of atomic hydrogen in the wind, or to dissociate and then ionize 0.87 $M_\odot$ or 0.33 $M_\odot$ of molecular hydrogen. This ionized gas in the unshocked SW later recombines, but does so only slowly, so that at time $\Delta\tau$ after the flash of Ly c radiation the electron density still is approximately

$$n_e = (b\ \Delta\tau)^{-1} \quad ,$$

so

$$n_e = 50\ \text{cm}^{-3} \quad ,$$

typically, after 10^{11} s, or 3000 years. Of course the electron density is as large as this only where there is an adequate mass density in the superwind.

Finally to estimate the emission measure of this low ionization halo. Take standard values for the parameters again, and assume that the superwind originally contained hydrogen in molecular form, and that the transition time t_o is 5 x 10^{10} s, then the mass in the halo is 0.33 $M_\odot$ = 6 x 10^{32} gm = M_h and its boundary is at distance

$$r_b = \frac{M_h u}{\dot{M}} = 6 \times 10^{17}\ \text{cm} = 0.2\ \text{pc}.$$

The low ionization halo then has an emission measure

$$EM = n_e^2 r_b = \frac{r_b}{b^2 \Delta\tau^2} = \frac{5 \times 10^{24}}{(\Delta\tau)^2}\ \text{cm}^{-6}\ \text{pc}$$

where $\Delta\tau$ is expressed in seconds. Some 3000 years after the flash the value of EM is 500, so that the low ionization halo is detectable until then, but not much longer.

4. DEPARTURES FROM SPHERICAL SYMMETRY

The multiple winds model must be generalised if it is to describe the variety of shapes of PNe. The most recent observational data are discussed by Balick (1987, 1988) who gives a classification of the different types of structure. He finds that, though there is a large variety among the images of PNe, they can all more or less be accounted for by the multiple winds model, with the simple modification that the superwind is assumed to carry away more mass per unit time in the equatorial regions of the progenitor star than in the polar regions.

An elementary treatment of the dynamics involved has been given by Kahn and West (1985). Their underlying premise is that the superwind contains cool gas, which has a low sound speed, so that any inhomogeneities are smoothed out only slowly. The HSSW, on the other hand, is very hot and has a high speed of sound. It will therefore restore isobaric conditions extremely quickly. Consequently it is much more profitable to study the effects of inhomogeneities in the SW since they are likely to produce a lasting effect on the shape of a PN. The fast wind cannot do so, unless it is very highly collimated and jet-like, and this seems most improbable.

The basic multiple winds model has the useful property that all motions proceed at a uniform rate, and this permits an easy extension of the dynamical treatment to flows which are not spherically symmetrical. Kahn and West deal with axially symmetrical cases where the terminal

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velocity and the mass loss rate in the superwind are constant, but the mass flux depends on the polar angle θ like $1 + \varepsilon \sin^n\theta$. They consider a variety of positive values of ε and n. Increasing ε increases the departure from spherical symmetry, increasing n sharpens the concentration of the mass flux towards the equatorial regions.

The bubble of HSSW will obviously expand more easily in the polar direction where there is least obstruction from the gas released in the superwind. The bubble is often pinched in at the equator, and sometimes the model predicts the existence of a cusp there, and (formally) accumulation of so much material that infinite surface density occurs in the shell along the equator. The reader is referred to the paper for the detailed argument. Here it will be enough to illustrate the variety of possible shapes that can be generated by this model. The important parameters are ε, n and λ; the first two have already been defined, and

$$\lambda \equiv \dot{r}(0)/u \quad ,$$

where $\dot{r}(\theta)$ is the radial velocity in the shocked shell at polar angle θ, and u the speed of the superwind, as before. The polar direction is $\theta = 0$ and is the horizontal direction in the three figures below.

5. INSTABILITIES IN THE HII LAYER

So far the flow in PNe has been taken to be orderly, with each component in the structure moving smoothly. There has been no mixing between the different kinds of gas; in particular it has been assumed that there is negligible thermal exchange across the contact discontinuity between the HSSW and the HII region. One consequence of this assumption is that the HSSW loses negligible amounts of energy by radiation (and none by conduction) and therefore expands adiabatically.

However a closer investigation shows the HII layer to be subject to instabilities, with a short growth time (Kahn and Breitschwerdt 1988). The motion is driven by flows at the adjacent ionization front and in general grows better in regions where the illumination from the PNN is incident obliquely. It is therefore to be expected that the effects of the instability will be most marked in those regions of a PN shell where the distortions from spherical symmetry are largest.

In the simplest treatment of the effect, the HII layer has thickness $\mathcal{Z}$ and the contact discontinuity with the HSSW is along the plane $z = 0$. The layer of non-ionized gas lies beyond the plane $z = \mathcal{Z}$, and is assumed to be so massive that it can be regarded as immobile. The boundary condition at $z = 0$ is determined by the fact that the HSSW is so hot, and has so high a sound speed, that its pressure remains constant throughout any disturbance. Finally the Ly c radiation from the central star is incident on the layer at angle β with the z-direction, and the x-axis is coplanar with Oz and the direction to the PNN.

A sound wave passing along the HII layer propagates with the isothermal sound speed c_i. It has to satisfy the boundary conditions that there shall be no pressure changes on the plane $z = 0$, the interface with the HSSW; on the plane $z = \mathcal{Z}$ there must be (almost) no Ly c flux, because

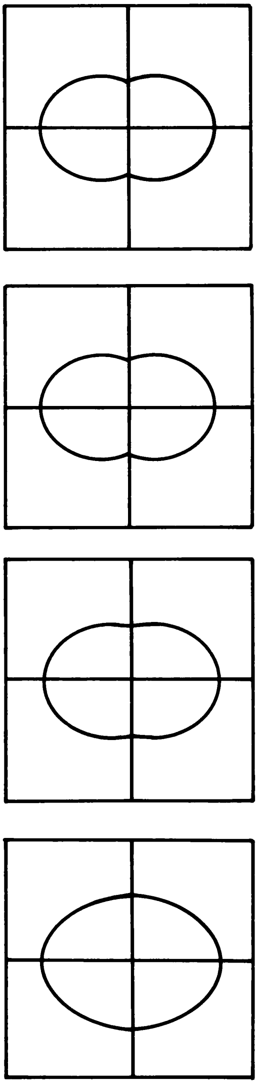

Figure 2. Shape of the nebula for $\varepsilon = 1$, $n = 2$ and $\lambda = 2$ (bottom), 3, 4 and 5, showing the effects of fast expansion.

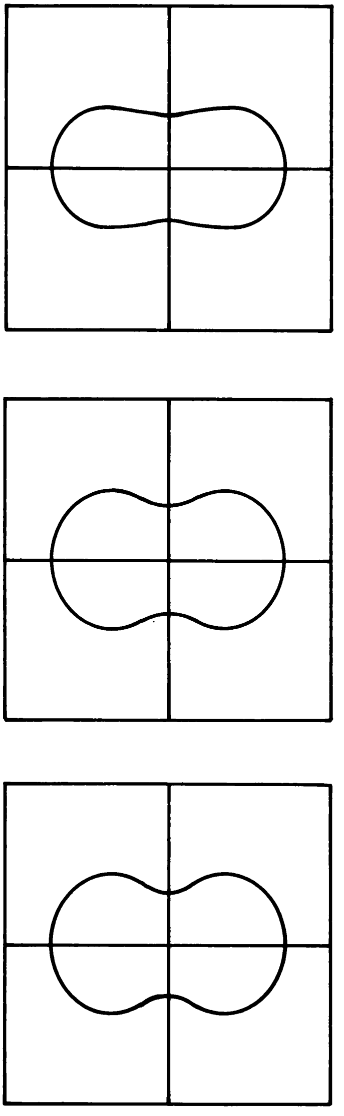

Figure 3. Shape of the nebula for $\lambda = 3$, $\varepsilon = 5$ and $N = 3$ (bottom), 4, 5, showing the effects of increased concentration to the equatorial plane.

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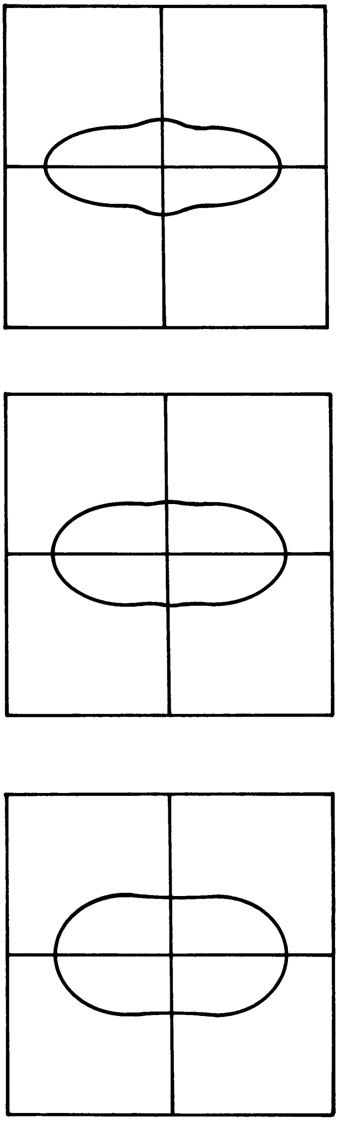

Figure 4. Shape of the nebula for $\lambda = 3$, $n = 2$ and $\varepsilon = 3$ (bottom), 6 and 18 showing the effect of increased departure from spherical symmetry.

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ionization fronts have to be well shielded from the ionizing flux. This second condition applies whenever the timescale of a motion is much longer than the recombination time in the HII region. The properties of such waves are then found from the usual equations of motion, and the condition of (almost) perfect shielding; a wave propagating parallel to the x-axis has the space-time dependence

$$\cos mz \exp i\,(kx - \omega t), \tag{25}$$

with ω, k and m satisfying the relations

$$\frac{\omega^2}{c_i^2} = k^2 + m^2 , \tag{26}$$

and

$$\frac{m}{\mathcal{Z}(k^2+m^2)} = \frac{1 - \exp\{i(m\mathcal{Z}+\alpha)\}}{m\mathcal{Z} + \alpha} \qquad \frac{1 - \exp\{-i(m\mathcal{Z}-\alpha)\}}{m\mathcal{Z} - \alpha} , \tag{27}$$

where $\alpha \equiv k\mathcal{Z} \tan \beta$.

Relation (26) just relates the frequency of the waves to the wave-vector; relation (27) expresses the condition of no Ly c flux at the ionization front. Possible variations of the transmitted flux arise from the changing shape of the interface of the HII layer with the HSSW, and from internal changes in electron density during the passage of the wave. Their combined effects must cancel. The angular frequency ω for such waves is in general complex, with imaginary parts (i.e. growth or decay rates) of order $c_i/\mathcal{Z}$. Instabilities occur for all values of the wave number k, and the growth time is typically the sound crossing time of the HII layer, in the interesting cases therefore smaller by an order of magnitude, or more, than the dynamical time scale of the PN.

In favourable cases the unstable waves will grow to such large amplitudes that turbulence occurs and mixing takes place at the interface between the HSSW and the HII layer. The temperature of the hot gas is sharply reduced by this dilution with cooler gas, but no thermal energy is lost until radiative cooling becomes significant. For electron density n_e the cooling time is

$$t_c = \frac{3kT}{\Lambda(T) n_e} \tag{28}$$

where $\Lambda(T)$ is the usual cooling function (see e.g. Dalgarno and McCray 1972). The electron density can be expressed in terms of the pressure P, and for a PN, in which $\lambda = 3$, it follows that

$$t_c = \frac{54\pi (kT)^2 u t^2}{\Lambda(T)\, \dot{M}} \tag{29}$$

At a representative time, say 10^{11} s, and with the usual parameters, the cooling time equals the dynamical time provided that the mixing of

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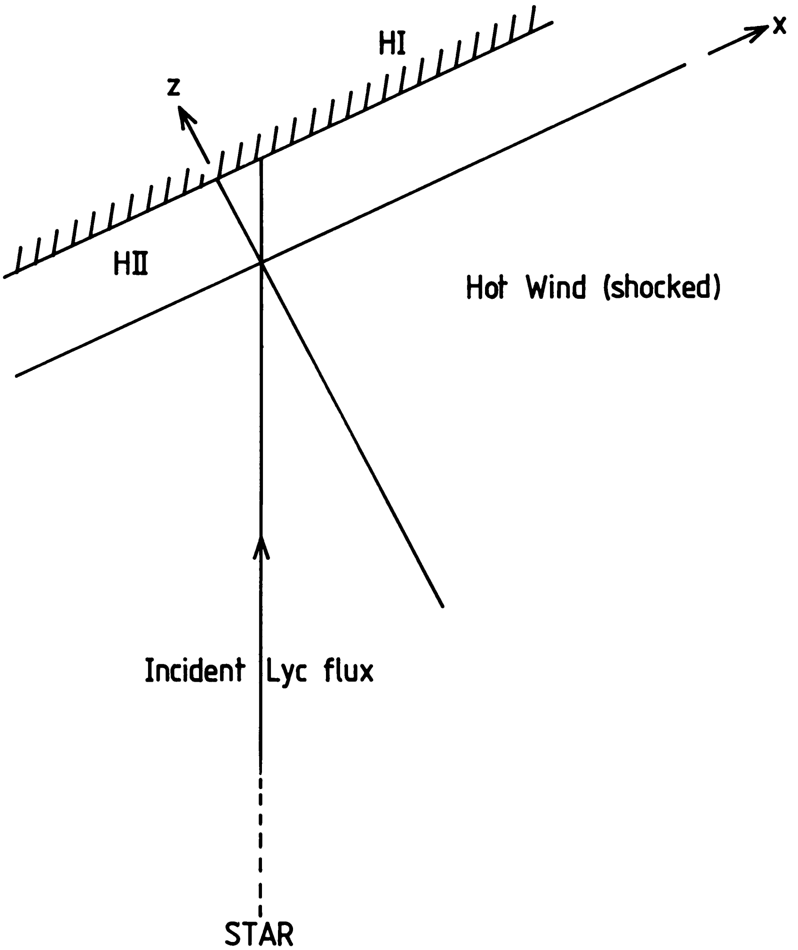

Figure 5. The setting for an unstable sound wave in the HII layer. The Ly c flux from the star is incident at angle β to the normal of the interface.

the gas has reduced the temperature to about 10^6K. The gas begins to cool well at that temperature and should emit photons with typical energies in the range of a few hundred eV.

An absolute upper limit is set to the rate at which such mixing can occur by the condition that the gas in the HII layer cannot flow at a speed faster than c_i. At radius r the maximum rate of addition of mass is therefore given by

$$\dot{M}_{add} < 4\pi r^2 P/c_i$$

$$= \frac{(\lambda-1)^2}{\lambda^2} \frac{\dot{M}r^2}{uc_i t^2} = (\lambda - 1)^2 \frac{\dot{M}u}{c_i} \qquad (30)$$

and the coefficient $(\lambda - 1)^2$ equals 4 when $\lambda = 3$. The actual rate of addition of gas from the HII layer will be $\varepsilon(\lambda - 1)^2\dot{M}u/c_i$, where ε is an efficiency factor. The mass in the HSSW increases at rate $2L_w/V^2$ and the mixture therefore settles to a temperature given by

$$\frac{kT}{m} = \frac{2}{9} \frac{L_w c_i}{\varepsilon(\lambda-1)^2\dot{M}u} = \frac{\lambda}{3\varepsilon} cu_i \qquad (31)$$

after some reduction. Inserting standard values for parameters here leads to the conclusion that the HSSW can be cooled to 10^6K by the mixing process, provided that the efficiency factor ε exceeds 0.01. If this does occur it will be followed by a catastrophic loss of pressure. There are two further consequences: the drop in pressure allows the HII layer to thicken and the instability rate will drop, and there will be large scale motions in the HSSW to compensate for the pressure imbalance. The second of these conclusions is interesting in connection with the observations reported by Lopez et al (1988) of high velocity components, with speeds up to 130 km/s, in NGC 2899.

6. CONCLUSIONS

The multiple winds model continues to provide valuable insight into the nature of PNe. In its simplest form it deals with objects having spherical symmetry and leads to the definition of interesting length and time scales. Real PNe will differ in various respects from the basic models. Some simple modifications have been discussed in this paper, and it has been argued that they can lead to a better understanding of the properties of PNe that are actually observed.

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