In principle, this book should be accessible to a reader with a fairly rudimentary knowledge of group theory, especially part I . The development is fairly rapid and the later parts certainly require the careful attention of the reader. I recommend the book strongly to two types of potential reader: the reader who wishes to see a proof of a beautiful and key theorem in the classification theorem explained by a master and the reader who is already expert in finite group theory and who wishes to gain detailed insight into the current programme of placing the theory of sporadic simple groups in a conceptual framework.

G.R. ROBINSON

HUGHES, B. and RANICKI, A. *Ends of complexes* (Cambridge Tracts in Mathematics Vol. 123, Cambridge, 1996), xxv+353 pp., 0 521 57625 3 (hardback), £45.00 (US$64.95).

The subject of ends is the study of topological properties of spaces determined by the complements of compact subsets. Approaches to the subject of ends include the study of ends as a choice of components in the complements of compact subsets, the study of the number of ends, and the applications of the homotopy theory of proper maps (for which inverse images of compact sets are compact). The book under review concentrates on the topology of the end space of a non-compact manifold, where the *end space* $e(W)$ of a space W is the space of all paths $\omega : ([0,\infty],\infty) \to (W^\infty,\infty)$ such that $\omega^{-1}(\infty) = \{\infty\}$ with the compact-open topology. Since, if W is compact, its one-point compactification W^∞ is disconnected, the end space of a compact space is empty.

In 1965 Siebenmann introduced the notion of a tame end of a manifold satisfying a geometric condition that ensures that the topology of the end space is reasonably well-behaved. The main result obtained in the volume under review describes tame manifold ends in dimensions ≥ 6 and does so in very precise terms, which we may summarise by saying that such ends look like infinite cyclic covers of compact manifold bands. A *band* (W, c) consists of a compact space W with a map $c : W \to S^1$ such that the pullback infinite cyclic cover $\overline{W}$ is finitely dominated (i.e., it is a homotopy retract of a finite CW complex).

The introduction gives fair warning:

"The proof . . . occupies most of Parts One and Two (Chapters 1–20)."

The chapters are short, but we are still offered a trip of 254 pages. This prospect struck fear into the heart of the timorous reviewer, but there was nothing for it but to begin.

In the early chapters the scene is set with some care. The invariants of the end space are introduced along with a swift but thorough account of homotopy limits and colimits – material for which a good introduction is hard to find elsewhere. The authors expect a certain bravado from the reader, who is definitely assumed to be a working rather than a lazy mathematician, but the ingredients are there to be worked upon.

The flavour and delicacy of the subject are best sampled in an example. A singular r-chain in a space W is (as usual!) a formal $\mathbb{Z}$-linear sum $\sum_{\alpha\in\mathbb{Z}} n_\alpha\sigma_\alpha$ of singular r-simplices. A *locally finite* singular r-chain is a formal $\mathbb{Z}$-linear product $\prod_{\alpha\in\mathbb{Z}} n_\alpha\sigma_\alpha$ such that each point of W has a neighbourhood meeting only finitely many of the σ_α. The *locally finite homology* of W is then defined in the obvious way, and the homology at ∞ is defined, via a mapping cone, to measure the difference between the locally finite and ordinary singular homology and to link them in a long exact sequence. If we regard the natural numbers $\mathbf{N}$ as a discrete space, then $H_0(\mathbf{N})$ is of course a free abelian group of countably infinite rank, which we identify with the polynomial ring $\mathbb{Z}[z]$. But the locally finite version $H_0^{lf}(\mathbf{N})$ is the product of countably many copies of $\mathbb{Z}$, identified with the power series ring $\mathbb{Z}[[z]]$. The non-zero quotient $\mathbb{Z}[[z]]/\mathbb{Z}[z]$ is the homology $H_{-1}^\infty(\mathbf{N})$ at ∞. But $e(\mathbf{N}) = \emptyset$, and so the homology at ∞ is not the homology of the end space.

This sort of bad behaviour is eliminated by the imposition of tameness restrictions, and Part One of the book proceeds with a careful development of tameness and its desirable consequences.

Part Two is largely devoted to the other ingredients of the main theorem, the topology of infinite cyclic covers, the properties and construction of bands and their relationship with geometric *ribbons*. The aim is to understand the circumstances in which the infinite cyclic cover of a band is a ribbon and when the technique of *wrapping up* can be applied to a ribbon to get a band. The geometric motivation is kept to the fore, and a number of judicious examples assist the exposition.

The third and final part of the book is a scenic downhill section on the algebraic theory. The emphasis is on the module theory of polynomial rings, which play a key role as the natural coefficient ring for the chain complex of an infinite cyclic cover. The basic homological algebra of mapping cones and mapping tori is developed along with the theory of algebraic bands. If (W, c) is a topological CW band then the chain complex of the universal cover of W is a chain complex band. The discussion of local finiteness requires the extension of scalars to a power series ring, and the end complex is then obtained as a mapping cone of an inclusion. There are some nice examples presented in the discussion of the subtleties of algebraic tameness, essential for the reader who likes to feel solid ground – even non-compact ground – underfoot. The book concludes with the algebraic analogues of ribbons, using bounded algebra (in which modules are graded by a metric space) and bounded topology (in which CW complexes are measured by a cellular map to a metric space).

The book gathers together the main strands of the theory of ends of manifolds from the last thirty years and presents a unified and coherent treatment of them. It also contains authoritative expositions of certain topics in topology such as mapping tori and telescopes, often omitted from textbooks. It is thus simultaneously a research monograph and a useful reference, and can be recommended not only to topologists who want the state of the art in the theory of ends, but also to mathematicians who want to see, in a highly non-trivial way, algebra reflected in the mirror of topology and topology fitted to an algebraic framework built for the purpose.

The book is produced to a high standard by the Cambridge University Press, and the complex typesetting problems are solved in an appealing and error-free manner. Publishers and authors are to be congratulated on the achievement.

N. D. GILBERT

FETTER, H. and GAMBOA DE BUEN, B. *The James Forest* (London Mathematical Society Lecture Note Series Vol. 236, Cambridge University Press, Cambridge, 1997), xi + 255 pp., 0 521 58760 3 (paperback), £27.95 (US$44.95).

A common perception of Banach space theory is that it consists of little more than a string of counterexamples. This is a pity, partly because there are many surprisingly strong theorems about arbitrary Banach spaces, and partly because many of the counterexamples that do exist (and it is undeniable that there are several important ones) have greatly enriched the subject, drawing attention to fascinating and unexpected phenomena and not simply killing off interesting problems.

There are four constructions (at least) that stand out as having an importance that transcends the original problem for which they were constructed. In no particular order they are Tsirelson's space, which introduced the idea of inductive constructions and has led to many examples showing that a general Banach space need not have nice subspaces or non-trivial operators, the Kalton-Peck space, which introduced so-called twisted sums, Gluskin's random finite-dimensional spaces, clever sums of which have distinguished many forms of the approximation property, and James's spaces, which have contributed enormously to our understanding of duality and reflexivity. It is to these last spaces that *The James Forest* is devoted.

The most famous property of James's original space, which we shall call J, is that the natural embedding of J into its double dual has codimension one. It follows immediately that J is not isomorphic to its square $J \oplus J$, and is therefore a counterexample to a problem of Banach. A