COLLISIONAL, COLLECTIVE AND RESONANCE PHENOMENA IN PLANETARY RINGS

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1. Particle Collisions in the Rings

1.1. The physical meaning of the outer boundary of the weak self-gravitating rings

Let us consider two particles of the same radius a on the neighbouring orbits (i.e. with radii R and $R+2a$, $R >> a$, Fig.1). The relative velocity of the centers of particles is $\Delta v_c = (GM_{pl}/R)^{1/2} - [GM_{pl}/(R+2a)]^{1/2} \approx \Omega a$. The velocities of small particles ΔV relative to the colliding particles A and B are dependent on ΔV_c. Let $\Delta V = \alpha \Delta V_c$, where α is a numerical coefficient obtained by the experiment. If this velocity ΔV is more than the first space velocity $\sqrt{Gm/a}$ on the surface of each colliding particle A and B with the mass m (G is the gravitating constant),

$$\alpha \Omega a > \sqrt{\frac{Gm}{a}}, \tag{1}$$

then small particles will go out from colliding particles A and B. Hence, as a result of each collision, masses of colliding particles will decrease, i.e., colliding particles will be destroyed. The inequality (1) can be transformed identically in the following inequality:

$$R \leq R_{cr} \equiv \gamma (\frac{M_{pl}}{\rho_{rm}})^{1/3} \tag{2}$$

where

$$\gamma \equiv 0.62 \alpha^{2/3} \beta^{-1/3}, \tag{3}$$

β is the packing inside every particle, i.e., $\beta = \rho/\rho_m$, ρ is the volume density of the particle in the rings, ρ_m is the density of the material of the composition of the rings, M_{pl} the mass of the central planet. The unknown numerical coefficient γ can be found from the formula (2), where the magnitude of the density of the material in the Saturn's ring was obtained by means of Voyager's spectral analysis: $\rho_{mSat} \approx 0.9 \mathrm{g/cm^3}$. Putting

$$R_{cr} = R_r \tag{4}$$

we find γ for the rings of Saturn:

$$\gamma = 1.59. \tag{5}$$

If we assume that γ is the universal number for all rings, we can calculate ρ_m for the rest of the rings (see Table I). The calculated value of ρ_m corresponds to the

S. Ferraz-Mello (ed.), Chaos, Resonance and Collective Dynamical Phenomena in the Solar System, 75–82.

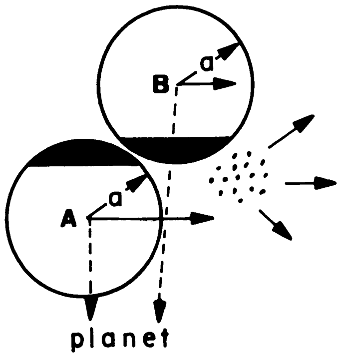

Fig. 1. Collision of two particles of the typical size *a* in the rings. The velocity of the movement of the colliding particles A and B and of small particles appeared as a result of the collision of A with B are shown by arrows.

TABLE I

Planet	Mass of planet M_{pl} in $M_{\oplus}$ $= 6.10^{27}$g	Radius of rings R_r in 10^3km	Density of the material in the rings, ρ_{rm} in g/cm	Composition of rings	Albedo
Jupiter	317.83	129.2	3.6	stone	0.1
Saturn	95.147	136.78	0.9	ice	0.6
Uranus	14.54	51.156	2.6	carbon chondrites	0.05
Neptune	17.23	62.9	1.7	?	?

Voyager's observational data. The physical meaning of the equality (4) consists in the fact that the area inside the rings is characterized by the strong destroy of particles as a result of collisions. The traditional point of view for the region of the rings as being in the Roche zone is a misunderstanding (Gor'kavij and Fridman, 1990).

1.2. The size distribution of the particles

Let us consider the accretion growth of a particle having the size a and the density of the material ρ_m. Let it be moving in the medium of particles of the size r and the integral surface density of the medium be σ. Then, the velocity of the growth of the radius of the particle is (Gor'kavij and Fridman, in preparation):

$$(\frac{da}{dt})^+ \approx \frac{\sigma\Omega}{4\rho}(1+\frac{r}{a})^2, \tag{6}$$

where

$$\sigma = \int_0^{a_{max}} \sigma(a)da. \tag{7}$$

$\sigma(a)$ is the differential density of the disk. The collisional destruction of particles in tangential collisions may be described by the following formula (Gor'kavij and Fridman, 1990):

$$(\frac{da}{dt})^- \approx 9.6(\frac{\rho}{\mathcal{E}_v})^{3/2}\Omega^4 a^6 n(a) \tag{8}$$

where $\mathcal{E}_v$ is the energy needed to fragment a unit volume of a particle, $n(a)$ is the surface differential concentration of the disk.

We can see from formulae (6),(8) that there is the equality

$$(\frac{da}{dt})^+ = (\frac{da}{dt})^- \tag{9}$$

at the same size of particles $a = a_{cr}$, which is equal to several meters for $\mathcal{E}_v \approx 10^{-2}$ erg/cm^3 (Gor'kavij and Fridman, 1990). Hence, the size distribution may be represented in the sum of the two parts:

1) at $a < a_{cr}$ the accretion dominates and one may account

$$(\frac{da}{dt})^+ >> (\frac{da}{dt})^-;$$

2) at $a > a_{cr}$ the accretion growth is compensated by the collisional destruction – as a result, the equality (9) is fulfilled for every a.

For large particles we obtain from (6)–(8)

$$n(a) = Aa^{-6}(1 + \frac{r}{a})^2, \qquad \text{where} \qquad A \approx 0.03\frac{\sigma\mathcal{E}_v^{3/2}}{\Omega^3\rho^{5/2}}.$$

We may neglect the factor $(1 + \frac{r}{a})^2$ at $\frac{a}{a_{cr}} >> 1$ and then obtain $n(a) \sim a^{-6}$, which corresponds to the observational data.

In the region $a < a_{cr}$, the time of the accretion growth t_a of particles from 0 to a_{cr} must be equal to the time of the collisional destruction t_a at $a > a_{cr}$. Using this condition of the stationary spectrum, $T_a \approx t_d$, we may write

$$t_a \simeq t_d \simeq t_{coll} \sim [2\Omega n(a_{cr})\pi a_{cr}^2]^{-1}.$$

We find t_a integrating the formula (6) and assuming $r \sim a_{cr}$:

$$\frac{\sigma\Omega}{4\rho}t_a \simeq 0.1a_{cr}.$$

Substituting in (7) the definition $\sigma(a) = m(a)n(a)$ where $n(a) = n_0(a/a_0)^q$ we obtain the equality for q:

$$(\frac{a_{cr}}{a_0}^q) \simeq 0.6\frac{(4+q)n(a_{cr})}{n_0 a_{cr}}$$

whence we find

$$q \simeq -(2.7 - 3.1) \tag{10}$$

for

$$\begin{aligned} n(a_{cr}) &= 10^{-3} - 10^{-4}\mathrm{m}^{-2}/\mathrm{m} \\ n_0 &\simeq 3.10^3\mathrm{m}^{-2}/\mathrm{m} \\ a_{cr} &= 5\mathrm{m} \\ a_0 &= 0.01\mathrm{m}. \end{aligned}$$

2. Collective Phenomena

Now we shall qualitatively consider the following instabilities:
a) gravitational,
b) instability of waves with negative energy,
c) negative diffusion,
d) accretion,
e) ellipse.

The instabilities a)–c) cause a short scale structure in rings: the typical scale is of the order of a few thicknesses of the ring, i.e., $\sim$ 100 meters (for Saturn's rings). The accretion instability causes a large scale structure ($\sim$ 800 km for Saturn's rings). The ellipse instability causes the eccentricity of the narrowest ringlets.

2.1. Gravitational instability

Let the original disk be infinitely thin. We choose one of the rings into which we broke up the originally uniform disk of particles. Let a test particle of unit mass be located at a distance δ from the closest point of the ring with width d, a moreover, $\delta >> d$. Then one can consider the ring as an infinitely thin gravitating filament whose potential is $\Psi \sim \ln(1/\delta)$, and the attractive force for the test particle is: $\partial\Psi/\partial\delta \sim 1/\delta \rightarrow \infty$, $\delta \rightarrow 0$. Obviously, the last condition can be fulfilled only for infinitely narrow ring, $d \rightarrow 0$, which is, in principle, allowed by the approximation of an infinitely thin disk. However, if the disk has the original thickness h, then $(\partial\Psi/\partial\delta)_{max} \sim 1/h$; the thinner the disk, the larger is the destabilizing force. Consequently, the dimensionless destabilizing factor is r/h, where r is the radius of the disk, and the dimensionless stabilizing factor is M/m_r, where M and m_r are the masses of the central body and the disk, respectively. The meaning of the stabilizing factor M/m_r lies in the fact that, as it increases, the relative influence of the central body also increases. When the force of attraction of the particles towards the central mass exceeds the force of the particle's mutual attraction, the system is stable for the same reason which insures that a point revolving in a central field is stable (here we do not allow for other interactions besides gravitational). In other words, a system turns out to be unstable if the destabilizing factor exceeds the stabilizing factor, i.e.,

$$\frac{r}{h} > \frac{M}{m_r} \text{ or } Q < 1, \quad Q \equiv \frac{M}{m_r}\frac{h}{r} \tag{11}$$

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The parameter Q is called the Tooter margin coefficient. The condition of disk instability in the form of expression (11) is valid for very short wavelength perturbations with wavelength $\lambda \sim h$. In this case, the disk is broken up into rings with widths of $d \sim h$. But if $\lambda \sim d > h$, then it follows from our arguments that one must replace h by d in condition (11); the larger the widths of the rings, the more difficult it is to fulfill the instability criterion $Q(d) = (M/m_r)(d/r) < 1$. According to the results of processing the Voyager data, $Q \approx 2$ for the B ring of Saturn, i.e., the B ring is near the limit of gravitational instability.

2.2. Instability of waves with negative energy

As was shown by Fridman and Polyachenko (1984), the energy E_K of the k^{th} harmonic of the perturbed gravitating solid-body rotating disk, Ω_0 =const, is

$$E_K = -\frac{K^2|\Psi_K|^2}{4\Omega_0^2}(2\pi G\sigma_0|K| - K^2c_s^2)\frac{\omega_0^2}{2\pi G\sigma_0|K|}, \quad (12)$$

where $\omega_0^2 \equiv 4\Omega_0^2 - 2\pi G\sigma_0|K| + K^2c_s^2)$, σ_0 is the surface density, c_s is the sound speed, K is the wave vector, G is the gravitational constant, Ψ is the gravitational potential.

In the simplest case, the growth rate of the slow dissipative instability is (Morozov *et al*, 1985)

$$\omega = \nu K^2 \frac{2\pi G\sigma_0|K| - K^2c_s^2}{\omega_0^2} \quad (13)$$

where ν is viscosity coefficient. Comparing (12) and (13), we can see that the condition of the dissipative instability

$$2\pi G\sigma_0|K| - K^2c_s^2 > 0 \quad (14)$$

corresponds to the negative energy. The condition (14) may be more easily fulfilled than the condition of the gravitational instability, as in the latter case the "self-gravitating" term $2\pi G\sigma_0|K|$ must be larger than

$$\kappa^2 = 4\Omega_0^2(1 + \frac{r}{2\Omega_0}\frac{d\Omega_0}{dr}),$$

which is large at the presence of the central mass.

2.3. Instability of negative diffusion

Let us create a sinusoidal surface density perturbation the disk: $\sigma \sim \sigma_0 \cos kx$. Let us examine Region 1 with an increased density σ_1 (for $0 < x < x_0$) and region 2 with decreased density σ_2 (for $x_0 < x < x_1$) . The density is unchanged on the boundary at the point x_0. The following amount of material flows across a unit length of the boundary separating Regions 1 and 2: $\sigma_1 v_1 - \sigma_2 v_2$, where v_1 and v_2 are the diffusion velocities, which are proportional to the mean thermal velocities of the particles in Regions 1 and 2 respectively. Instability sets in when

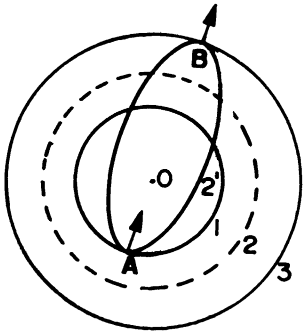

Fig. 2.

the particle density in Region 1 is increased due to migration of particles from Region 2, i.e., $\sigma_1 v_1 - \sigma_2 v_2 < 0$. Since $\sigma_1 > \sigma_2$ then the condition for instability is fulfilled, for example, when $v \sim \sigma^{\alpha-1}$ where $\alpha < 0$. The last condition indicates that the particle velocity must decrease with increasing density of the disk. This is possible in the case of inelastic particles when the frequency of collisions increases with increasing density of the medium and the outflow of kinetic energy increases; the chaotic velocity of the particle decreases. The boundary between Regions 1 and 2 corresponds to the inflection point x_0 of the function $\sigma(x)$, i.e., $\partial^2\sigma(x)/\partial x^2 = 0$ at the point $x = x_0$,. $\partial^2\sigma(x)/\partial x^2 < 0$ in Region 1 and $\partial^2\sigma(x)/\partial x^2 > 0$ in Region 2. It follows from the diffusion equation $\partial\sigma/\partial t = D\partial^2\sigma/\partial x^2$ that, if the diffusion coefficient is negative ($D < 0$) in both regions, the density will increase in Region 1 ($\partial\sigma_1/\partial t > 0$) and will decrease in Region 2 ($\partial\sigma_2/\partial t < 0$). Now it is understood why the instability examined above has the name "negative diffusion instability".

2.4. Accretion instability

The instabilities examined above lead to the growth of short-scale structure. The large-scale structure of the rings can arise as a result of accretion instability connected with the accretion of "external" material, for example, with the flow of the fine dust through the ring system to the planet (because of the Poynting-Robertson effect).

The mechanism of this instability is related to the mechanism for forming sandy dunes in a desert.

Accretion instability generates large scale layering of the rings, since small scale fluctuations do not succeed in gathering into a "dune" as a consequence of rapid diffusion spreading over a time $t \sim \lambda^2$ (λ is the scale of perturbations).

2.5. Ellipse instability

Let us consider a disk with circular orbit of particles $v_d(r) \sim r^{-1/2}$. Let us assume that, as a result of a perturbation, one of the circular orbit 2 was transformed in a slightly elliptic orbit 2' (see Fig. 2). The change of the rotation velocity of a test particle v_t in the elliptical trajectory according to the conservation of linear momentum is: $v_t(r) \sim r^{-1}$. We can see that the velocity of the test particle $v_t(r)$ changes with radius more sharply than the velocity of the disk particles. It means that at a point A the test particle will be decelerated by the slower disk particles and will approach the planet even closer. At the point B the test particle will be accelerated interacting with more quick disk particles and tend to move even further from the planet. One more similar example is the classical Laplace-Maxwell problem of the stability of an absolute solid ring revolving around a planet. At the displacement of the ring all its parts continue to revolve with the same velocity. Therefore, the gravitational force begins to dominate for the parts of the ring closer to the planet and centrifugal forces dominates for the distant part of the ring. As a result, the solid ring at the condition $c^2/v^2 >> 1$ (where c is the speed of sound oscillations in the ring) will fall onto the planet.

Let us note that Maxwell's conclusion on a fall of a hypothetical solid ice ring onto the planet is incorrect, as in this case we have the opposite inequality ($c \approx 3.3$ km/s, $v \approx 16 - 20$km/s) $c^2/v^2 << 1$,; therefore such a ring must be torn to pieces before its fall because of a small-scale instability (Fridman *et al*, 1984). But the Maxwell's corollary that rings of Saturn have the meteoritic structure turned out to be true.

3. Resonance nature of Uranian rings

In 1985 Gor'kavij and Fridman put forwards a hypothesis on the resonance nature of the Uranian rings. According to this hypothesis, a number of undiscovered satellites must exist outside the boundary of the ring area. The localization of these satellites must be connected with the rings localization by means of the following resonance condition

$$q\Omega_r - (q+1)\Omega_s \;\; = 0, \qquad q = 1, 2, 3,$$

where Ω_r, Ω_s are angular velocities of rings and unknown satellites correspondingly. The comparison of the predictions of Gor'kavij-Fridman's hypothesis (1985) and the Voyager-2 discoveries are in table II (next page).

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TABLE II

The hypotheses (Gor'kavij and Fridman, 1985)	The Voyager-2 observations (1986)
1. A series of small satellites exist beyond the outer boundary of the rings of Uranus.	1. 9 of 10 new satellites of Uranus are discovered beyond the outer boundary of the rings.
2. Satellites are not formed inside the ring zone.	2.Only one smallest satellite is discovered in the intermediate zone (near the outer edge of the rings).
3. Ring positions are determined by 1 : 2, 2 : 3 and 3 : 4 type resonances from undiscovered satellites (situated in the 50,000 to 82,500 km zone).	3. 8 of 10 new satellites are discovered in this zone and have resonances of this type in the region of the rings. The correlation coefficient between ring positions and the resonances is very high, $\approx$ 0.84 (Gor'kavij, Taidakova, Fridman, 1988).
4. Each of the 5 predicted satellites determine the positions of two rings simultaneously.	4. Each of the 4 discovered satellites determines the positions of two rings simultaneously; their orbits agree well with the orbits of predicted satellites.

The satellites with two resonances				
Predicted, R_h, km	66450	62470	61860	58600
Discovered, R_s, km	66090	62680	61780	59170
Accuracy of Agreement, $R_h - R_s$, km	+360	-210	+80	-570

The hypotheses (Gor'kavij and Fridman, 1985)	The Voyager-2 observations (1986)
5. The features of the outer ring are explained by the presence of "shepherd" satellites.	5. The ϵ ring is the only one near which "shepherd" satellites have been discovered.
6. The satellites diameters are $\sim$ 100 km.	6. The average diameter of the satellites is $\approx$ 70 km

Discussion

P.Goldreich **– I can't resist pointing out that the outer ϵ-ring shepherd also shepherds the γ ring through its 6:5 resonance and the inner ϵ ring shepherd shepherds the δ ring by means of its 23:22 resonance.**

A.M.Fridman **– Indeed, we can't argue proceeding from the data of Voyager-2 only, that shepherds with the sizes less than 10 km are absent inside the Uranian rings. But the account of the aerodynamic drag makes problematic the existence of small satellites inside the ring.**

P.Goldreich **– I agree that atmospheric drag is a problem when considering the α and β ring.**

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