hinzugefügt sind auch einige Bemerkungen aus der algebraischen Geometrie und hauptsächlich ein Abschnitt über Zentralbild eines Kreises. Für Leser, die sich näher mit darstellender Geometrie beschäftigen wollen, sind Literaturangaben und Anmerkungen gegeben, insbesondere Hinweise auf diejenigen Bücher, in welchen man Beweise im Buche benützter algebraischer und geometrischer Sätze finden kann.

Dieses Buch kann man für Studenten der Hochschulen, aber auch für alle diejenige, die darstellende Geometrie benützen, empfehlen.

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Generalized Functions, Vol. 1: Properties and Operations, by I. M. Gel'fand and G. E. Shilov. (Translated by Eugene Salatan.) Academic Press, New York, 1964. xviii + 423 pages. $12.00.

The first volume in the famous series of Gel'fand and Shilov here appears in a fine translation.

Two important remarks at the outset:

1. "Generalized function" is the same as "Schwartz distribution" throughout Volume 1.

2. This first volume presupposes no advanced training in mathematics (i.e. nothing beyond analytic continuation). It can be read by graduate students, and will especially appeal to Physicists (for whom it is obviously the best source of this material). All complications of functional-theoretic or topological nature are postponed to the second volume. The present volume is therefore not quite self-contained; the omissions, however, are rare, the only essential one being the characterization of generalized functions concentrated at a single point.

A generalized function is a suitably continuous linear functional on the space K of test functions (i.e. infinitely differentiable functions with compact support) defined on a given open subset Ω of Euclidean n-space. Examples: ordinary (locally summable) functions, $f(\varphi) = \int_\Omega f(x)\,\varphi(x)\,dx$ where φ is a test function; Dirac's delta,

$\delta_a(\varphi) = \varphi(a)$ or, in Physicist's notation, $\int_\Omega \delta(x-a)\,\varphi(x)\,dx$.

This concept, first systematically investigated by L. Schwartz ("Theorie des distributions", Vols. I and II, Hermann, Paris, 1957-59), has now permeated almost every branch of analysis. Thus the titles of subsequent volumes in this series: "Theory of differential equations"; "Applications to harmonic analysis"; and "Integral geometry and representation theory".

The main emphasis of the book is on special types of generalized functions, a simple example of which is the function x_+^λ which "agrees" everywhere except at the origin with the ordinary function

$$f(x) = \begin{cases} x^\lambda & (x > 0) \\ 0 & (x < 0) \end{cases} .$$

When suitably regularized at the origin, the generalized function x_+^λ becomes meromorphic in λ with simple poles at $\lambda = -1, -2, \ldots,$ so that $[1/\Gamma(\lambda+1)]\, x_+^\lambda$ is entire. This result may also indicate the fundamental role played by analytic continuation: frequently formulas derived for restricted values of a parameter are readily extended by analytic continuation.

There are three chapters. The first gives the basic definitions and such operations as differentiation (if f is a generalized function, $\frac{\partial f}{\partial x_i}$ is defined by

$$\frac{\partial f}{\partial x_i}(\varphi) = - f\left(\frac{\partial \varphi}{\partial x_i}\right), \quad \varphi \in K\,) ,$$

convolution and regularization. The second chapter is on Fourier transforms, where the emphasis is on calculation of specific transforms rather than on any general theory. A table of some 67 Fourier transforms is given as an appendix.

The third chapter, on distributions concentrated on manifolds, is the most interesting and unusual. It begins with a very lucid and elementary introduction to differential forms. (Uninitiated readers will do well to disregard the remark that this section could be omitted at a first reading.) If $P(x) = 0$ is a smooth manifold, the generalized function $\delta(P)$ is defined by

$$\delta(P)\,(\varphi) = \int_{P=0} \varphi(x)\omega$$

where the differential form ω satisfies

$$dP \cdot \omega = dv \quad \text{(volume element)} .$$

Properties of such functions are developed in considerable detail. In particular, one case in which $P = 0$ is not smooth, namely when

P(x) is a quadratic form, is considered fully. These topics are of special interest to physicists, as is an appendix (transposed from Volume 5 of the Russian edition) on generalized functions of complex variables.

Numerous applications are given, in varying degrees of depth. Examples: evaluation of divergent integrals; elementary solutions of differential equations; the Cauchy problem (including the formulas of Herglotz-Petrovsky for hyperbolic equations); "operational calculus".

The book is written in a beautifully clear and convincing style; the translation is extremely careful and natural - there are very few misprints, those of the original having been mostly rectified. In short, it is difficult to formulate any complaints about the volume at all; readers who find too little mathematical sophistication need only turn to the second volume.

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<u>Generalized Functions and Direct Operational Methods Volume One</u>, by T. P. G. Liverman. Prentice Hall Inc. Englewood Cliffs, N.J., 1964. xii + 338 pages.

The first volume, which is reviewed here, is meant to serve both as an introduction to the theory of generalized functions, via what the author calls the D'_+ space, and to show how this concept can be applied to linear problems in analysis. The applications are devoted primarily to the solution of ordinary differential equations with constant coefficients. As the author states in his preface, the audience at which he aims is applied mathematicians, engineers and physical scientists. The mathematical prerequisites for the mastery of this book are a knowledge of advanced calculus as taught at the third or fourth year level in most U.S. and Canadian universities.

With these rather minimal requirements the author rigorously develops the theory to a considerable extent without clouding over the main ideas. Most of the exposition in this first volume is clear, concise and straightforward. Although there are many clean and neat proofs, there is never any recourse to short obscure proofs. This is one of the book's chief merits. Another is the plentiful supply of graded exercises which allow the reader to develop a mastery of the subject matter. These can, as the author points out in his preface, be broken down into three types:

1. direct applications of material in the text,
2. exercises designed to obtain results by alternate means,
3. exercises designed to develop the theory of other classes of generalized functions by arguments similar to those used for the class D'_+.