THE LOVE NUMBERS OF THE ANELASTIC EARTH

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We calculate corrections to the Love numbers resulting from the small frequency-dependent variations in the shear modulus, μ, arising from anelasticity in the Earth. The corrections are of the order of a percent, which in some cases exceeds the observational error. We point out the possibility of using these corrections to study the Earth's anelasticity at long (tidal) periods.

At present the slight dissipation, Q(r) in the mantle is studied through the attenuation of seismic and surface waves as well as free oscillations of the Earth, which correspond to a range of periods from ~1 sec to 1 hour. At longer periods the dissipation in the mantle is studied, so far as it is known, through the phase lag of the Earth tides relative to the tide-generating forces and through the damping of the free wobble (periods from 12 hours to 430 days). However, the phase lag of the Earth tides caused by mantle anelasticity is approximately a hundred times smaller than the average shift caused by the ocean tide and that is why the former cannot be observed. The attenuation of the free wobble is possibly to a great extent also dominated by the phase lag of the 430-day tide in the ocean described by the static theory. Thus the observations of the free wobble attenuation (Currie, 1975) cannot be used for the determination of the dissipative function in the mantle at the Chandler frequency. In the present report another possibility is proposed for investigating the dissipative function, Q(r), in the very-long-period range.

In the paper of Akopjan, Zharkov and Lubimov (1975), attention was drawn to the fact that for anelastic Earth models the shear modulus has an essential dependence on frequency. As a result, Earth models inferred from the travel times of seismic body waves with periods ~1 sec, must be corrected for the frequency dependence of the shear modulus when used at the longer, free oscillation periods.

It is obvious that corresponding corrections must also be introduced when computing the Earth tide deformations of longer periods and the

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deformations caused by the free wobble. Let's turn to the calculation of these effects.

At present the best agreement with observations of the attenuation of the Earth's free oscillation, and the surface and body waves are obtained with Lomnitz's creep function (Lomnitz, 1957). For this function the dependence of the dynamic shear modulus upon frequency in a layer with given μ_i and Q_i is expressed by the formula:

$$\Delta\mu_{\mathcal{D}i}(\omega) = -\frac{2\mu_i}{\pi Q_i}\ln\frac{\omega_o}{\omega} \tag{1}$$

where ω_o is the frequency of the body waves corresponding to the period T ~ 1 sec. It is assumed here that the initial Earth model is based on body waves with periods T ~ 1 sec and the dynamic shear modulus for all long-period Earth oscillations is taken from this standard high-frequency model.

Assuming that the dissipative function at very long periods is described by the same creep function, let's determine the influence of this dynamic modulus upon the Earth tide amplitudes. The frequency dependence of Love numbers, because the dynamic shear modulus is different at different frequencies, can be found by a perturbation method with the help of tables of derivatives for the Love numbers (Molodensky, 1976). For the trial distributions obtained by Zharkov et al. (1974) from the attenuation of free oscillations and surface waves the corrections to Love numbers and to the gravimetric factor $\delta = 1 + h - (3/2)K$ for semi-diurnal waves are as follows:

Model N°	6	7	15	2	3	4	16	17	19
$\Delta k \cdot 10^3$	1,8	3,0	4,7	6,1	5,6	4,5	4,5	4,3	4,4
$\Delta h \cdot 10^3$	2,7	5,5	8,7	11,5	10,8	8,4	8,3	8,0	8,2
$\Delta\delta \cdot 10^3$	0,0	1,0	1,7	2,4	2,4	1,7	1,5	1,5	1,6

For the diurnal waves the given corrections in accordance with (1) should be multiplied by 1,1; for a fortnightly -1,3; and for Chandler frequency -1,7.

Let's examine the possibility of observing effects of this order with modern measurements, and thus the possibility of investigating the dissipative function Q(r) at very long periods. Now the highest relative accuracy is achieved in the measurement of the gravimetric factor δ and the period T_{Ch} of the free wobble, which is connected with the Love numbers k at a period of 14 months. According to Currie (1975)

$$T_{Ch} = \frac{2\pi C}{(C-A)[1-(K/K_o)]\omega} = 434 \pm 1 \quad \text{sidereal days} \qquad (2)$$

where C, A are the principal moments of inertia of the mantle, ω is the rate of the Earth's rotation, K is the Love number for the real Earth, and K_o is the Love number for a liquid Earth covered with oceans. Formula (2) determines K with an accuracy slightly above 1%. At the same time corrections to K at the Chandler frequency caused by dynamic shear modulus as it follows from the table given above for various trial models, is 1-3%. However, determining the exact theoretical value for an elastic Earth at the Chandler frequency is complicated by the necessity of taking into account the ocean tide. Because of the absence of cotidal charts for such long periods the necessary corrections are usually computed by using the equations of static equilibrium of the oceans, and the real accuracy is still not quite clear.

For the gravimetric factor, δ, corrections due to the influence of the ocean for semidiurnal and diurnal waves at present are more reliable, so let's consider the possibility of investigating the corrections caused by the dynamic shear modulus from the gravimetric tide observations. The observed values of δ in Western Europe for the wave M_2 are given by Pertzev and Ivanova (1975). After averaging the results from 16 stations, after removing the ocean tide influence and introducing corrections for the Earth's ellipticity, it was found that

$$\delta = 1.160 \pm 0.002 \quad .$$

For comparison of the observed value of δ with theoretical ones we use the static Love numbers for Gilbert and Dziewonsky's Earth models, which are given by Dahlen (1976),

$$\left.\begin{aligned} h &= 0.60967 \\ k &= 0.30088 \end{aligned}\right. \quad \text{for Model 1066a}$$

$$\left.\begin{aligned} h &= 0.60927 \\ k &= 0.30097 \end{aligned}\right. \quad \text{for Model 1066b}$$

from which

$$\delta_{1066a} = 1.1584$$
$$\delta_{1066b} = 1.1578 \quad .$$

Comparing these values with the results observed for the wave M_2 in Western Europe, we see that the difference between the observed and the static theoretical gravimetric factors δ is equal to the average error of observation

$$\delta_{M_2,\text{Europe}} - \delta_{1066} = (2\pm2)\cdot 10^{-3} \quad .$$

The difference has the same sign and value as the corrections in the table given above caused by the dynamic shear modulus for semidiurnal waves for the greatest number of trial models.

So, though now the effect of the influence of dynamic modulus upon the value of δ is not above the error of observation, a reasonable improvement in measurement accuracy will be sufficient to pick out the effect from gravimetric observations and for investigating of the dissipative function of the mantle for very long periods.

Even now it is possible to say that nonlogarithmic creep functions which will give a stronger frequency dependence of the dynamic modulus than (1) might cause noticeable disagreement between the theoretical and experimental Earth tide data.

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