Linear Strange Modes in Massive Stars

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Abstract. The occurrence and properties of strange modes and associated instabilities in massive stars are reviewed. If applicable, strange modes may be classified by the maximum of the opacity they are associated with. Whether they are still present in the limit of vanishing or infinite thermal timescales – which corresponds to the NAR and adiabatic approximations respectively – is another criterion for classification. A model for the instability mechanism of strange mode instabilities is discussed.

1 Methods and assumptions

Strange modes and associated instabilities can be identified on the basis of *standard* stellar physics (see, e.g., Baker & Kippenhahn 1962, Cox 1980 and Unno et al. 1989). Models to be tested for stability are either taken from ordinary stellar evolution calculations or may be stellar envelopes with given luminosity, effective temperature and mass. Stability of these models is tested with respect to infinitesimal (both radial and nonradial) perturbations, where - apart from the mechanical equations - also the equations of energy conservation and transport are taken into account, i.e., a linear nonadiabatic analysis (LNA) of the stability is performed.

Even if the input physics and the basic equations describing the problem are quite common, concerning the identification of strange modes (SMs) and associated instabilities (SMIs) for its solution not all standard methods are appropriate. In particular, relaxation methods relying on an adiabatic initial guess for the solution fail in general. A robust technique which allows for a reliable and accurate determination of any eigenfrequency (not only those of strange modes) is the Riccati method which was adapted to stellar stability problems by Gautschy & Glatzel 1990a (for the treatment of nonradial perturbations see Glatzel & Gautschy 1992).

2 Occurrence and definition of strange modes

Strange modes (and associated instabilities) are a common phenomenon in the envelopes of luminous stars (see, e.g., Saio & Jeffery 1988, Saio et al. 1984 and Gautschy 1993 for the case of less massive objects and Glatzel & Kiriakidis 1993a and 1993b for massive stars). The essential parameter which controls their occurrence and properties is the ratio L/M of luminosity to

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mass. More or less independent of the particular stellar model strange modes together with instabilities having growth rates in the dynamical range occur, if the value of L/M exceeds $\approx 10^4$ (in solar units). Thus the mass - luminosity relation becomes important, if domains of instability in the HRD are to be determined. In the upper HRD, for massive stars and given luminosity masses taken from stellar evolution calculations tend to be considerably higher than those indicated by observations. The results discussed are based on conservative high masses which favour stability. Up to now there is no precise definition of the term "strange mode". Some studies on the instability mechanism even suggest that an unambiguous definition might not be possible at all (see Glatzel, 1994). Therefore strange modes may in a loose way be regarded as additional modes not anticipated from experience with adiabatic spectra and neither fitting in the "ordinary spectrum" nor following its dependence on stellar parameters.

3 Classification

Phenomenologically, for massive stars SMs and SMIs come in three groups apparently related to the three opacity maxima (see, e.g., Iglesias et al. 1992 and Rogers & Iglesias 1992) caused by the contribution of heavy elements ("Fe") and the ionization of helium ("He") and hydrogen/helium ("H/He") respectively (see Kiriakidis et al. 1993). The run of the growth rates of the instabilities directly reflects the run of the opacity. Accordingly the opacity bump related to a strange mode may be used for classification. This seems to be justified also by the fact that some properties of SMs vary systematically within this classification scheme:

- *Sensitivity to the treatment of convection:*
  Fe-SMs: sensitive; He- and H/He-SMs: insensitive
- *Dependence on metallicity:*
  Fe-SMs: strongly dependent; He- and H/He-SMs: independent
- *Dependence on the harmonic degree l* (see Glatzel & Mehren 1996):
  Fe-SMs: instabilities up to $l \approx 10$, maximum growth rates are attained around $l \approx 0$;
  He- and H/He-SMs: instabilities up to $l \approx 500$, maximum growth rates are attained around $l \approx 100$
- *Instability mechanism:*
  Fe-SMs: partially κ- mechanism; He- and H/He-SMs: no κ- mechanism
- *Domain of existence in terms of the effective temperature:*
  Fe-SMs: $\log T_{\rm eff} > 4.3$; He- and H/He-SMs: $\log T_{\rm eff} < 4.3$
- *Adiabatic counterpart:*
  Fe-SMs: yes; He- and H/He-SMs: no
- *NAR counterpart (see Section 7):*
  An NAR counterpart exists in any case.

An alternative criterion for classification can be derived by considering various approximations. One of them implying an infinite thermal timescale is the adiabatic approximation. The opposite implies a vanishing thermal timescale and has been denoted by NAR approximation (see Section 7). Whether an adiabatic or NAR counterpart exists for a given mode may be used for its classification. Moreover, consideration of these limits can be helpful to identify its physical origin and the mechanism of an associated instability.

4 Strange modes in the adiabatic approximation

Stable adiabatic counterparts of strange modes are found only for the group associated with the opacity maximum due to the contribution of heavy elements (Fe-SMs). In this case the existence of strange modes can be attributed to the particular run of the sound speed which – together with the density – exhibits an inversion at the position of the opacity maximum. This inversion implies an acoustic barrier and splits the stellar envelope into two cavities, each of which develops an independent acoustic spectrum. The spectrum of the outer cavity is identified with strange modes, that of the inner cavity with ordinary modes. A shift of the barrier during stellar evolution causes the spectra to cross thus leading to multiple resonances which in an adiabatic environment unfold into avoided crossings (see Kiriakidis et al. 1993).

Except for extremely low irrelevant effective temperatures the stability analysis based on the adiabatic approximation does not reveal any instability (Glatzel & Kiriakidis 1998; see, however, Stothers & Chin 1993, 1994, 1995). Moreover, even if an instability in the adiabatic approximation would be present, this would require a confirmation by the nonadiabatic analysis, since the adiabatic approximation does not hold for the objects considered which are characterized by high L/M ratios and therefore by short thermal timescales.

5 Domains of instability in the HRD

On the basis of a standard LNA analysis for radial perturbations domains of instability in the HRD have been determined by Kiriakidis et al. (1993). Instabilities associated with He-SMs and H/He-SMs define a largely metallicity independent unstable region in the upper right corner of the HRD, whose boundaries approximately coincide with the observed Humphreys - Davidson limit (Humphreys & Davidson 1979). For solar metallicity (and above) it is continuously connected with the β Cepheid instability strip by an additional instability domain associated with Fe-SMs. It extends to the ZAMS for luminosities above $\log L/L_{\odot} \approx 5.9$ (corresponding to a ZAMS mass of $\approx 80 M_{\odot}$) and encompasses, e.g., the region of line profile variable O stars as well as the position where LBVs are found (see Fullerton et al. 1996).

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6 Wolf - Rayet stars

With respect to growth rates and the number of simultaneously unstable modes the most extreme examples for SMIs are found in models for Wolf - Rayet stars (Glatzel et al. 1993, Kiriakidis et al. 1996). Largely independent of metallicity and opacity instability prevails for masses above 3...5$M_{\odot}$. In this case the NAR approximation may not only be used in a qualitative way to classify modes or to identify the instability mechanism: It provides even quantitatively correct results.

7 The NAR approximation

The NAR approximation (where NAR stands for Non - Adiabatic - Reversible, see Gautschy & Glatzel, 1990b) consists of neglecting the time derivative of the entropy in the perturbation equations. We emphasize that this approximation does not necessarily imply adiabatic changes of state which may be seen by considering the differential of the entropy s expressed in terms of the differentials of the density ρ and the pressure p:

$$ds = C\frac{\alpha}{\delta}\left(-\Gamma_1\frac{d\rho}{\rho} + \frac{dp}{p}\right) \tag{1}$$

where C, α, δ and Γ_1 denote the specific heat, the logarithmic derivative of the density with respect to the pressure at constant temperature, the negative logarithmic derivative of the density with respect to temperature at constant pressure, and the adiabatic index respectively. ds vanishes either, if the term in brackets is zero – which corresponds to the adiabatic limit – or, if the specific heat vanishes (NAR limit). Accordingly, the latter representing the opposite of the adiabatic limit is justified, if matter cannot store heat efficiently, e.g., in a tenuous, extended stellar envelope. The ratio of thermal and dynamical timescales varies from infinity in the adiabatic limit to zero in the NAR limit. The NAR approximation implies several consequences:

- As the matter cannot store heat, the luminosity perturbation has to vanish.
- Any instability mechanism relying on a Carnot - type process, which requires a finite specific heat – such as the classical κ- and ε- mechanisms – is excluded in the NAR limit.
- Due to vanishing thermal timescales thermal modes cannot exist in the NAR limit. Essentially only the mechanics of the system is considered, whereas its thermodynamics is disregarded. Although the NAR limit represents the opposite of the adiabatic limit, both limits are similar in this respect.
- The system is reversible with respect to time, which is common for purely mechanical configurations.

- Closely related to reversibility is the property that eigenfrequencies come in complex conjugate pairs.

Thus, due to its listed properties, the NAR approximation is a useful tool for the classification of modes and the identification of various instability mechanisms.

8 The mechanism of strange mode instabilities

Models for the mechanism of strange mode instabilities have meanwhile been proposed in three studies by Glatzel (1994), Papaloizou et al. (1997) and Saio et al. (1998). These investigations seem to agree upon the following issues:

- The strange mode phenomenon is found both for radial and nonradial perturbations. Geometry is therefore not essential for its occurrence and a plane - parallel model is sufficient for a physically correct description.
- Cowling's approximation does not have a significant influence on the strange mode phenomenon. For a model, the perturbation of the gravitational potential may therefore be neglected.
- As strange mode instabilities do also exist in the NAR limit, a model can be based on this approximation. In particular and as a consequence, the luminosity perturbation can be assumed to vanish. The existence of SMIs in the NAR limit proves them not to be related to a Carnot - type process. In particular, the classical κ- mechanism is thus excluded as their origin.
- A finite value or a particular run of the logarithmic derivative κ_T of the opacity κ with respect to temperature T is not crucial for the occurrence of SMIs. This again excludes the classical κ- mechanism as their origin.
- All objects suffering from SMIs are characterized by high ratios of luminosity to mass. Apart from small thermal timescales this has the consequence that radiation pressure contributes significantly to the total pressure in the envelopes of these stars. Therefore all models are based on considering the limit of small ratios β of gas pressure to total pressure: $\beta \rightarrow 0$.

Using these findings as basic assumptions of a model (for its description we closely follow Glatzel 1994), the mechanical equations (mass and momentum conservation) may be condensed into an acoustic wave equation:

$$\frac{\partial^2 \tilde{\rho}}{\partial t^2} - \frac{\partial^2 \tilde{p}}{\partial r^2} = 0 \tag{2}$$

where $\tilde{p}$ and $\tilde{\rho}$ denote pressure and density perturbation respectively. By adopting the NAR approximation the equation of energy conservation is satisfied. Moreover, within this approximation and in the limit $\beta \rightarrow 0$ of dominant radiation pressure the diffusion equation for energy transport reads:

$$\frac{\partial \tilde{p}}{\partial r} - \beta \frac{\bar{p}}{\bar{\rho}} \frac{\partial \tilde{\rho}}{\partial r} = \frac{\partial \bar{p}}{\partial r} (\kappa_\rho \frac{\tilde{\rho}}{\bar{\rho}} + 4\kappa_T \frac{\tilde{p}}{\bar{p}}) \tag{3}$$

where $\bar{p}$ and $\bar{\rho}$ denote pressure and density in the hydrostatic configuration respectively. κ_ρ and κ_T are the logarithmic derivatives of the opacity with respect to density ρ and temperature T. Equation (3) provides the – in general differential – relation between pressure and density perturbation, which is needed to close the wave equation (2).

In the following we perform a local analysis of equations (2) and (3), i.e., we consider solutions of the form $\exp(i\omega t + ikr)$ characterized by the frequency ω and the wavenumber k.

In the limit of high wavenumbers the r.h.s of equation (3) may be neglected and we are left with an algebraic relation between $\tilde{p}$ and $\tilde{\rho}$, where the factor of proportionality involves the isothermal sound speed c_T. Its consequence is the dispersion relation of isothermal sound waves:

$$\tilde{p} = \beta \frac{\bar{p}}{\bar{\rho}} \tilde{\rho} = c_T^2 \tilde{\rho} \quad ; \quad \omega^2 = (c_T k)^2 \tag{4}$$

In this limit pressure and density perturbation are in phase and the perturbation equations reduce to a self-adjoint problem of second order which does not allow for any instabilities.

In an intermediate range of wavenumbers k ($1 < k \cdot H_p < 1/\beta$; H_p: pressure scale height), which exists only for sufficiently small β, the second term on the l.h.s. and the second term on the r.h.s. of the diffusion equation (3) may be neglected. As a consequence, we obtain a differential relation between pressure and density perturbation and a dispersion relation of the form (g: gravity):

$$\frac{\partial \tilde{p}}{\partial r} = \frac{1}{\bar{\rho}} \frac{\partial \bar{p}}{\partial r} \kappa_\rho \tilde{\rho} = g \kappa_\rho \tilde{\rho} \quad ; \quad \omega^4 = -(g \kappa_\rho k)^2 \tag{5}$$

A phase lag of $\pi/2$ is now found between pressure and density perturbation, the perturbation operator is no longer self-adjoint and the dispersion relation (5) represents oscillatory damped and unstable modes with growth rates in the dynamical range. The latter resemble the strange mode instabilities. We emphasize that the perturbation problem is still of second order.

In general an unstable mode is characterized by a phase lag between pressure and density perturbation. In the model for SMIs presented here it is caused by the differential relation between pressure and density perturbation which is determined by the diffusion equation for energy transport. The model thus shows that in a highly nonadiabatic environment involving low heat capacity and short thermal timescales a phase lag leading to SMIs in an intermediate range of wavenumbers is inevitable, if radiation pressure is dominant ($\beta \to 0$) provided gravity and κ_ρ do not vanish, as indicated by equation (5).

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The model presented was proposed by Glatzel (1994) and essentially confirmed by Saio et al. (1998). Although there are many correspondences also with the study of Papaloizou et al. (1997), some differences need to be noted: Essential in their model of the instability is the coupling of isothermal sound waves to a thermal mode which can be found only in the original third order NAR perturbation problem. On the other hand, the approach discussed here relies on second order perturbation problems only which do not allow for the existence of a third wave and the corresponding coupling. Moreover, in the isothermal limit instabilities do not exist here, whereas isothermal sound waves are involved in the instability mechanism proposed by Papaloizou et al. (1997).

References

Baker, N.H., Kippenhahn, R. (1962): Z. Astrophys. **54**, 114

Cox, J.P. (1980): *Theory of Stellar Pulsation* (Princeton Univ. Press, Princeton)

Fullerton, A.W., Gies, D.R., Bolton, C.T. (1996): ApJS **103**, 475

Gautschy, A. (1993): MNRAS **265**, 340

Gautschy, A., Glatzel, W. (1990a): MNRAS **245**, 154

Gautschy, A., Glatzel, W. (1990b): MNRAS **245**, 597

Glatzel, W. (1994): MNRAS **271**, 66

Glatzel, W., Gautschy, A. (1992): MNRAS **256**, 209

Glatzel, W., Kiriakidis, M. (1993a): MNRAS **262**, 85

Glatzel, W., Kiriakidis, M. (1993b): MNRAS **263**, 375

Glatzel, W., Kiriakidis, M. (1998): MNRAS **295**, 251

Glatzel, W., Mehren, S. (1996): MNRAS **282**, 1470

Glatzel, W., Kiriakidis, M., Fricke, K.J. (1993): MNRAS **262**, L7

Humphreys, R.M., Davidson, K. (1979): ApJ **232**, 409

Iglesias, C.A., Rogers, F.J., Wilson, B.G. (1992): ApJ **397**, 717

Kiriakidis, M., Fricke, K.J., Glatzel, W. (1993): MNRAS **264**, 50

Kiriakidis, M., Glatzel, W., Fricke, K.J. (1996): MNRAS **281**, 406

Papaloizou, J.C.B., Alberts, F., Pringle, J.E., Savonije, G.J. (1997): MNRAS **284**, 821

Rogers, F.J., Iglesias, C.A. (1992): ApJS **79**, 507

Saio, H., Jeffery, C.S. (1988): ApJ **328**, 714

Saio, H., Baker, N.H., Gautschy, A. (1998): MNRAS **294**, 622

Saio, H., Wheeler, J.C., Cox, J.P. (1984): ApJ **281**, 318

Stothers, R.B., Chin, C.W. (1993): ApJ **408**, L85

Stothers, R.B., Chin, C.W. (1994): ApJ **426**, L43

Stothers, R.B., Chin, C.W. (1995): ApJ **451**, L61

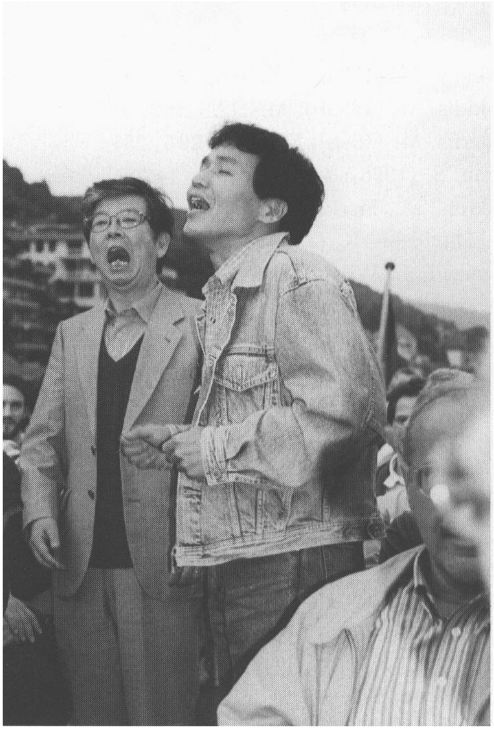
Unno, W., Osaki, Y., Ando, H., Saio, H., Shibahashi, H. (1989): *Nonradial Oscillations of Stars* (Univ. of Tokyo Press, Tokyo)

Discussion

H. Lamers: I understand that the periods you predict for B supergiants and LBVs are on the order of days or less. This is a factor of 10 to 100 shorter than observed. Which effect might be responsible for the long periods?
W. Glatzel: The results discussed are based on rather conservative values for the masses of the objects. If the masses are reduced, the time scales of variability may increase by a factor of 10. Moreover, in the non-linear regime of the evolution of the instabilities, we observe some kind of "period-doubling".

D. Massa: Do these models depend on the interior physics used to construct the model?
W. Glatzel: Strange-mode instabilities have their origin in the envelope of the objects; the interior does not have an influence. The phenomenon is perfectly described on the basis of envelope models.

A. Maeder: What effect limits the growth of the amplitudes of the strange modes in the non-linear regime?
W. Glatzel: The non-linear regime is characterised by the formation of shocks. The associated dissipation seems to limit the growth of perturbations.



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