

Pattern Speeds in Barred Galaxies

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Abstract. The ratio \mathcal{R} of the corotation radius to the bar radius ranges from 1 to 1.8 in theoretical models that fit the observed dynamics of gas or stars in the bar and spiral regions of galaxies. Most models cluster around a ratio $\mathcal{R} = 1.2$ for early type galaxies. This ratio confirms the bar-aligned orbit theory long held by Contopoulos and others, which requires that $\mathcal{R} > 1$, but it also suggests that bars do not end *exactly* at corotation, as is often assumed, but significantly inside. The difference between $\mathcal{R} = 1$ in the old interpretation and $\mathcal{R} = 1.2$ or 1.4 in the fitted models is critical for our interpretation of bar-spiral morphologies. It appears that the bar-to-spiral transition in early Hubble types occurs between the inner 4:1 resonance and corotation, in the range corresponding to $\mathcal{R} = 1.55$ to 1 for a flat rotation curve, because of orbit stochasticity where the $2m : 1$ orbits in the x_1 family cluster together in space. Corotation then occurs relatively far out in the arms, at an angle from the bar end that may exceed 60° for tightly wrapped spirals. This position sometimes corresponds to the crossing of a dust lane from inside the arms to outside (NGC 1300, NGC 1365) or to a spiral arm bifurcation as in non-barred galaxies. The relatively high value of \mathcal{R} is also consistent with the identification of inner rings with the inner 4:1 resonance, outer rings with the outer Lindblad resonance, and offset bar dust lanes and nuclear rings with the inner Lindblad resonance. In contrast to this clear interpretation for early type galaxies, there is little consensus on the location of corotation in late type galaxies, which usually lack an inner Lindblad resonance interior to the bar. It is also not known whether late type spirals corotate with their bars, although this appears to be the case for early type galaxies based on the ubiquity and location of outer spirals and rings.

1. Introduction

The pattern speed of a bar in a galaxy, and of the spiral that surrounds the bar, are among the most important parameters that govern the galaxy's dynamics and the morphology of its structure. Yet throughout the entire era when Hubble, Sandage, de Vaucouleurs, van den Bergh and others developed morphological classification systems for bar and spiral structures, there was little understanding of bar pattern speeds. The important breakthrough came with the development of orbit theory, particularly by Contopoulos and Lynden-Bell, and with the

demonstration by Contopoulos (1980) that bars end inside corotation. Since that time, corotation has generally been assumed to lie near the end of the bar (see review in Sellwood & Wilkinson 1993).

Today, we have both direct and indirect indicators of bar pattern speeds that basically confirm this theoretical result. These indicators, and some of the results they obtain, are reviewed here. There are two important new developments. The first is that increasing precision in the determination of bar pattern speeds has led to the realization that bars do not end at *exactly* corotation, but inside by 20%-40%, which is a significant distance in terms of our understanding of why bars end and whether the surrounding spirals actually contain, and perhaps reveal, the corotation resonance. The second is the increasing perception that late Hubble type bars are very different morphologically from early type bars, and that most pattern speed determinations have been for early type bars only. This situation leaves open the possibility that corotation can be at a different location relative to the bar in late type galaxies than in early type galaxies.

2. Direct Indicators of Bar Pattern Speeds

The most direct indicator of a bar pattern speed Ω_p is that proposed by Tremaine & Weinberg (1984), who write $\Omega_p = \int_{-\infty}^{\infty} I v dx / \int_{-\infty}^{\infty} I x dx$ for intensity I assumed proportional to density, line of sight velocity v and position x along a line parallel to the galaxy major axis. This method has been applied to the non-barred galaxy M81 (Westpfahl 1995), and to the barred galaxy NGC 936 (Kent 1987; Merrifield & Kuijken 1995). If we denote the ratio of the corotation radius to the bar length by

$$\mathcal{R} = \frac{R_{\text{corotation}}}{R_{\text{bar}}}, \quad (1)$$

then for NGC 936, $\mathcal{R} = 0.54 - 1.2$ in Kent (1987) and 1.0-1.8 in Merrifield & Kuijken (1995). The Tremaine & Weinberg method favors certain bar-major axis alignments and only works if intensity is really proportional to density. Thus it fails for gas that is turned rapidly into stars or invisible molecular states, or is easily shocked into unresolvable structures.

A more common method for determining pattern speeds in barred galaxies is to compare observations of gas velocities and densities with numerical models of gas flows. This is a very powerful method if the gas code is reasonably well representative of real interstellar gas, but of course many gas processes, particularly heating, cooling, and the role of clumpiness and magnetic fields, are not well known. The models could therefore introduce unknown systematic effects. Nevertheless, this method has been applied to perhaps a dozen galaxies getting acceptable fits to the observations, and most of what we know about bar pattern speeds comes from the results.

One of the first applications of this method was by Sanders & Tubbs (1980) who used published optical and HI velocity data with a least-squares fit to each observed point in the bar to find the pattern speed in NGC 5383. They obtained $\mathcal{R} = 1.2$. Teuben et al. (1986) modelled the residual H α velocity field in the bar of NGC 1365 to get $\mathcal{R} = 1$. Recent studies of the same galaxy by Jørsæter & van Moorsel (these proceedings) give $\mathcal{R} = 1.4$. A slightly different

method, fitting velocities around the inner ring in NGC 1433, was used by Buta (1986b) to obtain $\mathcal{R} = 1.3$ for that galaxy; Ryder, Buta & Toledo (these proceedings) reported $\mathcal{R} = 1.5$. Buta (1986b) determined, by the way, that the inner ring in NGC 1433 had the proper gradient in velocities to place it at the inner 4:1 resonance, with corotation near the end of the parallel-to-bar elongation of this ring. The galaxy studied by Kent (1987) using Tremaine & Weinberg's method was studied again by Kent & Glauddell (1989) using a different method of comparing the rotation curve along the bar major axis with a model; they obtained for NGC 936 $\mathcal{R} = 1.25 - 1.6$. It is probably significant from a historical perspective that all of these corotation determinations fit only the velocities inside the bar regions.

Other galaxies were studied by the Florida group using a beam scheme without self-gravity to fit VLA HI observations of NGC 3992 (Hunter et al. 1988), NGC 1300 (England 1989), NGC 1073 (England, Gottesman & Hunter 1990), and NGC 1359 (Ball 1992). All derived $\mathcal{R} \sim 1$, explaining that larger \mathcal{R} would make the bar-generated spirals too weak, and move the ring in NGC 1073 or the gas bar in NGC 3359 out too far. These studies also found that the spiral arms could not be driven purely by the observed bar. Other galaxies in this survey are discussed in the review in these proceedings by Hunter.

A group led by Combes has used a slightly different technique to derive pattern speeds in M51 (Garcia-Burillo, Combes, & Gerin 1993), NGC 4321 (Garcia-Burillo, Sempere & Combes 1994) and NGC 7479 (Sempere, Combes & Casoli 1995b). They derive a potential from the total optical light, including both the bar and the spiral arms, and then let a sticky-particle gas simulation respond to this potential at various pattern speeds. When the structure and velocity of the gas response matches the observations, the associated pattern speed is assumed to be correct. For the two barred galaxies in their program, they got $\mathcal{R} = 1.8$ in NGC 4321 and $\mathcal{R} = 1.1$ in NGC 7479. In the first case, this large value of \mathcal{R} places corotation in the middle of the optical arms, close to the position where optical resonance indicators placed it (Elmegreen, Elmegreen & Montenegro 1992), and significantly beyond the end of the bar. The Garcia-Burillo et al. study of NGC 4321 was more sensitive to the spiral pattern than the bar pattern, so it may not have revealed a difference between the bar and spiral pattern speeds if there is one (e.g., \mathcal{R} for the bar alone could be less than 1.8 if the bar and spirals do not corotate). In the second case, NGC 7479, \mathcal{R} is close to 1 but the spiral arms are so tightly wrapped that nearly all of the spiral system is within corotation.

The values of \mathcal{R} in these studies are summarized in Table 1. \mathcal{R} is typically larger than 1, usually around 1.2 ± 0.2 . In most cases, this value is large enough to place the bar-to-spiral transition inside corotation, so the spiral arms extend for a significant distance inside corotation. We conclude that bars end inside corotation, possibly between the inner 4:1 resonance and corotation, and that the bar-spiral transition does not occur at corotation. Other studies summarized below are also included in Table 1 and lead to the same conclusion.

2.1. Indirect Indicators of Pattern Speeds

The relative position of corotation and other resonances can often be determined from optical features in the bar-spiral pattern. If there is a rotation curve for

Table 1. Summary of Ratios of Corotation Radii to Bar Radii

Galaxy	\mathcal{R}	Reference
Kinematical Methods		
NGC 936	1.25-1.6	Kent & Glaudell 1989
	1.0-1.8	Merrifield & Kuijken 1995
NGC 1073	~ 1	England et al. 1990
NGC 1300	~ 1	England 1989
NGC 1359	~ 1	Ball 1992
NGC 1365	1.4	Joersæter & van Moorsel, these proceedings
NGC 1433	1.3,1.5	Buta 1986b, Ryder et al. 1995
NGC 3992	~ 1	Hunter et al. 1988
NGC 4314	1	Quillen et al. 1994
NGC 4321	1.8	Garcia-Burillo et al. 1994;
	1.8	Sempere et al. 1995a
NGC 4596	1.25	Kent 1990
NGC 5383	1.2	Sanders & Tubbs 1980
NGC 7479	1.1	Sempere et al. 1995b
NGC 5236	1.4	Kenney & Lord 1991
Morphological Methods		
NGC 613	1	Elmegreen et al. 1992
NGC 1300	~ 1.04	Elmegreen et al. 1992
NGC 1398	2.2?, 1.3?, 1?	Moore & Gottesman 1995
NGC 3351	1.2	Devereux et al. 1992
NGC 3504	1	Kenney et al. 1993
NGC 4321	2	Elmegreen et al. 1992

a galaxy, then the location of any resonance feature can be used to determine corotation, although some resonance features (OLR) may give corotation more accurately than others (ILRs).

The shapes of dust lanes in bars indicate the presence or absence of an inner Lindblad resonance. Early numerical simulations of dust lane shapes by Sanders & Huntley (1976) and Sanders & Tubbs (1980) have recently been extended with high resolution by Athanassoula (1992), who makes the following points: (1) the presence of an offset, leading dust lane on each side of the bar indicates that an ILR is present near the bar center; a single, relatively straight and continuous dust lane through the whole bar, sometimes even straight through the center, indicates there is no ILR. (2) If there is an ILR, then $\mathcal{R} < 1.2$ (for the bar potentials used in that study), because otherwise the offset dust lanes would curve outward as they approached the center, which is contrary to observations. (3) Offset dust lanes that are straight arise in bars with large quadrupole moments, such as long strong bars; offset dust lanes that curve inward as they approach the center arise in bars with relatively small quadrupole moments, such as weak fat bars. (4) offset straight dust lanes have so much shear

that star formation is inhibited, while offset curved dust lanes have little shear so star formation is allowed.

Offset dust lanes tend to extend from the bar end to the ILR at the position of an inner ring (see below) or pseudo-spiral. One can in principle determine corotation from the radius of the inner ring or pseudo-spiral, but in practice such determinations are inaccurate. The problem is that the relative positions of corotation and the inner Lindblad resonance depend sensitively on derivatives of the rotation curve in the inner regions, and such derivatives are often inaccurate. Also, the exact extent of the inner perpendicular stellar orbits (type x_2), which are the real indicators of an ILR, are not accurately determined by the linear theory of epicycles, which is usually used to determine κ in the expression $\Omega - \kappa/2$. Thus even an accurate rotation curve and a known CR resonance may not give the true position of the ILR and inner ring or pseudo-spiral, and conversely an accurate rotation curve with a known ILR radius may not get the true CR radius.

Rings can be good indicators of resonances (Kormendy 1979; Athanassoula et al. 1982; Buta 1986a; Buta 1995). Properties of nuclear rings were studied by Buta & Crocker (1993); inner rings were discussed by de Vaucouleurs & Buta (1980); outer resonance rings were investigated by Buta & Crocker (1991), and others. A comprehensive review of ring observations is in Buta (1995).

Dynamical simulations by Schwarz (1981, 1984, 1985) and Byrd et al. (1994) suggest that outer rings occur at the OLR. An outer R_1 ring is perpendicular to the bar and has a semimajor axis close to the OLR. An outer R_2 ring is parallel to the bar and has a semiminor axis close to the OLR. Inner rings have occurred in simulations at CR or at the inner 4:1 resonance, depending on the code used (presumably because of different schemes for dissipation – see the review by Combes in this proceedings). The most likely resonance for an inner ring seems to be the inner 4:1 resonance because here the orbits rapidly change shape with increasing distance; this implies that gas orbits can cross and produce shocks there. Nuclear rings occur near the ILR but the exact position with respect to the ILR is not certain. If there are two ILRs, then the nuclear ring may occur closest to the inner ILR (Combes & Gerin 1985).

If the inner ring is associated with the inner 4:1 resonance, then the bar would seem to end well inside CR, because most bars with inner rings appear to end inside the inner ring and the inner 4:1 resonance is inside CR. However, inner rings tend to be elongated along the bars (Byrd et al. 1994) and their major axes could be close to CR even though they are associated with the inner 4:1 resonance. There is apparently no specific part of an inner ring that can be identified exactly with the inner 4:1 resonance radius.

Considering all the possible rings in galaxies, it seems the best indicator of a resonance that can be used to give corotation and a pattern speed is an R_1 (or R_2) outer ring or pseudoring, whose major (or minor) axis is supposed to be very close to the outer Lindblad resonance. Nuclear rings may give precise locations for inner Lindblad resonances, but corotation is not easily determined from these alone, as discussed above. Inner rings are only approximately associated with a resonance radius.

Outer and nuclear rings are primarily associated with early type galaxies. This is presumably true for outer rings because they take a relatively long time

to form and early type galaxies are much more evolved than late types, i.e., they have gone through a greater number of rotations in a Hubble time. Nuclear rings are more prominent in early type galaxies because these are the types that tend to have inner Lindblad resonances, i.e., for types Sbc and earlier (see below). The ILR results from the steeply rising rotation curve in the inner regions of early type galaxies; such a steep rise results from the bulge.

There are many examples in the literature where offset dust lanes or rings have been used to infer the location of corotation. One recent example is for NGC 3351, in which Devereux, Kenney & Young (1992) find a CO bar and streaming motions in the nuclear regions perpendicular to the main optical bar. There is also a nuclear ring and an HII ring. Devereux et al. suggest that the perpendicular bar and nuclear ring are at the ILR, and that CR is at the outer edge of the HII ring. This gives $\mathcal{R} = 1.2$. Another perpendicular CO bar was found in NGC 4691 by Wiklund, Henkel & Sage (1993).

Kenney, Carlstrom & Young (1993) found a diamond shaped HII ring in NGC 3504 and suggested that this ends at the inner 4:1 resonance. There is also a starburst ring between two ILRs and the outer arms appear to end at the OLR. This gives $\mathcal{R} = 1$.

Inner rings and spiral features are more ambiguous in NGC 1398, for which Moore & Gottesman (1995) suggest several possibilities for resonance locations. If the inner ring is at the inner 4:1 resonance, then the OLR is at the optical edge of the disk, giving $\mathcal{R} = 2.2$. If the outer ring is the OLR, then the inner ring is CR and $\mathcal{R} = 1.3$. If the bar is assumed to end at CR, putting $\mathcal{R} = 1$, then the OLR of the bar is at about CR for the spiral in the outer disk; this may imply some resonance excitation of the outer spiral (Tagger et al. 1987).

Other morphological indicators of resonances may be related to the two bar types found by Elmegreen & Elmegreen (1985). Early type bars are flatter and longer than late type bars, and the early types tend to have offset dust lanes and nuclear rings, unlike the late types. This difference suggests that early type bars end near CR, as is usually assumed for bars in general, but that late type bars may not. The short lengths of late type bars are consistent with their locations near or inside the ILRs of the surrounding spirals. This does not necessarily imply that late type bars end at their own ILRs, although this could be the case if the bars and outer spirals corotate. The structure of late type bars also suggests that they contain no ILRs inside of them, i.e., from the straightness of their bar dust lanes and the lack of nuclear rings. Thus there are two reasons why late type bars may end far inside their CR radii; this would place corotation somewhere in the middle of the spiral in the disk. However, late type bars could instead have corotation near their ends, in which case the pattern speeds of late type bars would be approximately twice the pattern speeds of the spirals in the main disks. Such a difference in pattern speeds for bars and spirals has been suggested for barred galaxies in general (Sellwood & Sparke 1988). Evidently, the location of CR for late type bars is unknown.

Hunter (1990) suggested another division of bar types between those which have HI in their bars (Class II) and those which do not (Class I). He notes that Class II bars tend to have star formation along their lengths but no starburst rings, which is consistent with their lack of an ILR. Class I bars may end at CR and sweep out the HI from the bar region; this type would presumably have an

ILR. Another discussion of barred galaxies with HI holes is in van Driel & van Woerden (1994).

There have been several theoretical studies of different bar types that may correspond to different resonance locations. Pasha & Polyachenko (1994) suggest that most bars end near the ILR, and are made by the Lynden-Bell (1979) mechanism in which initially elongated orbits align with each other in the rising part of the inner rotation curve. Athanassoula & Sellwood (1986) argued against this because simulations of this alignment mechanism with initially hot and thin disks gave bars that saturated at amplitudes that were too low to be realistic.

Petrou & Papayannopoulos (1986) suggested that bars are made from inner 1:1 orbits, in which case they could end far inside CR. Inner 2:1 orbits of the x_1 type, occurring between the ILR and CR, tend to dominate inner 1:1 orbits, however.

Combes & Elmegreen (1993) made self-consistent, gas+stellar, 3D models of barred galaxies with early and late type rotation curves and found the two bar types (long-flat and short-exponential) for rotation curves characteristic of early and late type galaxies. The early type bars ended near CR and the late type bars ended near their own ILRs, and contained no ILRs inside of them. Both types had primarily x_1 orbits; the short bars were not formed by the Lynden-Bell mechanism. In the late type galaxy models, the bar actually extended at a weak level out to the vicinity of CR, but the spiral began near the ILR, thus masking the presence of the underlying bar. This numerical result is consistent with the observations, but Sellwood (these proceedings) suggested that edge effects in the late type simulations artificially shortened the bars. Further models are needed to clarify this possibility.

3. Summary

Bars in early type galaxies end near 0.8 times the corotation radius, giving a value of $\mathcal{R} = R_{\text{CR}}/R_{\text{bar}} \sim 1.2 \pm 0.2$. This is the region between the inner 4:1 resonance and the corotation resonance, suggesting that orbit resonances in the crowded region where 2m:1 orbits pile up are responsible for the terminations of bars.

The spiral arms begin well inside corotation and may show dust lane cross-overs or other indicators of corotation midway along their lengths. With $\mathcal{R} = 1.2$, the angle between the end of the bar and the corotation resonance in the spiral arm can be large. For a logarithmic spiral, this angle is given by

$$\Delta\theta = \frac{\log \mathcal{R}}{\tan i} \quad (2)$$

for arm pitch angle i . Setting $\mathcal{R} = 1.2$ and $i = 10^\circ$ gives $\Delta\theta = 60^\circ$, which is more than half of the way around the center from one end of the bar to the other. Corotation should therefore be prominently placed midway out in the spiral arms of barred galaxies.

Bars in late type galaxies, later than Hubble type Sbc, for example, may have their corotation radii well outside the bar ends, possibly at twice the bar radius ($\mathcal{R} \sim 2$) and midway in the main disk. This is uncertain, because there have been very few dynamical studies of late-type bar pattern speeds. More

certain is the observation that late type bars contain no inner Lindblad resonance. This follows from the lack of nuclear rings, ring-like star bursts, and offset dust lanes. Either late type bars end near their own inner Lindblad resonance and possibly corotate with the outer spirals, or they end closer to their own corotation resonance and have a different pattern speed than the disk spirals.

At the present time, the best method for determining the corotation radius in a barred galaxy is to compare the velocities and densities of the gas with the velocities and densities in numerical simulations that use a potential obtained from optical or infrared light. These determinations are consistent with morphological indicators of resonances, such as rings or curved inner dust lanes. The best morphological indicator of a resonance seems to be the presence of an outer R_1 or R_2 type ring, whose semi-major or minor axis, respectively, should be close to the outer Lindblad resonance. Nuclear rings clearly indicate the presence of an inner Lindblad resonance, but a casual extrapolation from this resonance to corotation is likely to be inaccurate.

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Discussion

P. Teuben: You listed $\Omega_{\text{bar}} \neq \Omega_{\text{spiral}}$ under late types (cf. Sellwood & Sparke), but their N-body bar is early. The idea probably belongs to both?

B. Elmegreen: I think the evidence for $\Omega_{\text{bar}} = \Omega_{\text{spiral}}$ is good for early types where there are outer rings that seem to require a steady Ω_{bar} and where a high fraction of barred galaxies have grand design spirals. The evidence for this is weak in late types, and if true, then it must be that $\mathcal{R} \geq 2$. Otherwise $\Omega_{\text{bar}} \neq \Omega_{\text{spiral}}$ for these late types.

E. Athanassoula: I have run a large number of simulations with initial rotation curves resembling those of late type galaxies, but I have never found a bar ending at ILR. What ingredient do you have that I miss? How sure are you of your result?

B. Elmegreen: Our main requirement is that the peak in $\Omega - \kappa/2$ is very low, so that at the pattern speed where Ω equals this peak, corotation is so far out in the disk that there are not many stars to absorb the angular momentum from the bar region. Then we find that the bar starts small and stays small, and the gas begins a spiral at the end of the bar near the ILR. Some bar-like distortion persists out to corotation, however, but it is very weak.

A. Zasov: Can you comment on the usual acceptance that the presence of a dust lane running along the inner side of a spiral arm is an indicator of the region inside the corotation circle?

B. Elmegreen: This is not always seen in a clear way. We find that corotation might commonly show some bifurcation in the spiral arms, and this might be accompanied by the dust lane cross over that you suggest. I believe that when it is seen clearly, the cross over point is probably a good indicator of corotation, to within 10% in radius. Barred galaxies like NGC 1300 and NGC 1365 seem to have this in the right place.

G. Contopoulos: I am glad that there is a consensus now that bars terminate at or inside corotation. This fact is explained by the structure of the orbits (e.g., Contopoulos, A&A 81, 198, 1980). But what is the structure of the orbits in the case of bars that terminate near the ILR? In particular, do you consider as plausible the suggestion of Petrou and Papayannopoulos that some bars terminate at the inner 1/1 resonance?

B. Elmegreen: This may be but I think such orbits are dominated by the more common 2/1 orbits. In our simulations, 2/1 orbits showed the strongest response inside about one disk scale length, and they persisted weakly out to corotation.

M. Weinberg: Can you comment on the constancy of pattern speed based on the morphological agreement between observations and theoretically predicted morphology?

B. Elmegreen: The strongest case for constant pattern speeds, even for non-barred galaxies, seems to be that resonance indicators such as spurs at the inner 4:1 resonance, the end of the star formation ridges and bifurcations at the corotation resonance, the end of the main spiral arms at the OLR, and outer rings at the OLR, are pretty well defined and self-consistent in several dozen nearby cases. The longest time scale arises for the outer rings, so it would seem that pattern speeds can drift no more than outer rings over a Hubble time. I think this indicates that the pattern speed is defined by the disk mass distribution, usually placing CR near two disk scale lengths, and not by transient kinematic effects. Strongly interacting galaxies provide an interesting contrast: they usually do not have these resonance indicators, and they probably have young spirals.