

MASON, L. J. and WOODHOUSE, N. M. J. *Integrability, self-duality and twistor theory* (London Mathematical Society Monographs (New Series) No. 15, Clarendon Press, Oxford, 1996), x + 364pp., 0 19 853498 1, (hardback) £45.

Integrable systems of PDEs have come a long way for a subject where the object of study lacks a definition. In medicine it is common to define a condition by saying that to qualify a patient must exhibit (say) four out of six given symptoms and perhaps we should do something analogous to define integrability of a system of PDEs, the symptoms being things like: the existence of nonlinear “superpositions” of solutions; existence of “sufficient” constants of the motion; existence of a “Lax pair”; satisfying analogues of the Painlevé condition for ODEs; etc. The unifying idea behind this book is the suggestion that one particular feature that is particularly important in integrability is the existence of a “twistor construction” for the system. The authors’ hope (I am perhaps putting words into their mouths here) is that, if one knew how to say this correctly, it might serve as a single feature characterising integrable systems.

The chain of ideas which has led to this is as follows. One example of an integrable system is the self-dual Yang-Mills equations for a connection on a vector bundle over  $\mathbb{R}^4$ . One has a twistor construction for this system (the Ward correspondence) which puts solutions of this system in one-to-one correspondence with a certain set of holomorphic vector bundles over a subset of complex projective 3-space. It is now known that many popular integrable systems of PDEs are equivalent to the Yang-Mills equations with some symmetry group imposed. (The discovery that the KdV and nonlinear Schrödinger equations arise this way is joint work of one of the authors in 1989). This process provides something like a classification scheme for a large family of integrable systems and the authors have done us a favour by presenting this large body of information in a unified way.

The twistor construction for Yang-Mills may also undergo a “symmetry reduction”, leading to a “twistor construction” for the reduced system. The nature of this varies widely depending on the signature of the original metric on  $\mathbb{R}^4$  and the symmetry group, but it leads to some family of objects in holomorphic geometry being in one-to-one correspondence with the solutions of the system. The authors show how many features of the integrable system can be seen and understood in terms of the twistor constructions.

This is a subject which needed a book firstly to draw together a rather large range of ideas and secondly to gather together systematically a lot of things spread through the literature (and a few useful things that have not previously made it into print). The authors have on the whole done a very good job; doubtless the readiness to retreat into coordinate and index notation will be more popular with mathematical physicists than with mathematicians, but parts of the book are rather easier for a mathematician to approach. Anybody working in integrable systems or in twistor constructions will want a copy of this book or at least want it in their Library.

T. BAILEY

KANAMORI, A. *The higher infinite: large cardinals in set theory from their beginnings* (Perspectives in Mathematical Logic, Springer-Verlag, Berlin-Heidelberg-New York-London-Paris-Tokyo-Hong Kong, 1994), xxiv+536 pp., 3 540 57071 3, £77.50.

For some books it is easy to predict a longevity and a wide readership. This is one of them. The first in a several volume series, the book comprehensively provides an account of the theory of large cardinals from their beginnings to the early 70’s, and several of the more important offshoots leading up to the frontiers of current research.

As such it will be invaluable for graduate students and others coming to the field, both for the evolutionary view that it espouses of the various historical confluences to the theory, and the clear insights and perspectives it throws on the pathways to those frontiers.