MONTEL SUBSPACES OF FRÉCHET SPACES OF MOSCATELLI TYPE

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In this note we show that every complemented Montel subspace F of a Fréchet space E of Moscatelli type is isomorphic to ω or is finite-dimensional; the last case always occurs when E has a continuous norm. To do this, we first study the topology induced by E on its Montel subspaces, extending a result on Fréchet-Montel spaces of Moscatelli type in [**4**].

We recall that the Fréchet spaces of Moscatelli type were introduced and studied by J. Bonet and S. Dierolf in [**4**]; the general idea behind the construction of such spaces was due to V. B. Moscatelli [**7**].

The paper has three sections. The first one is devoted to the necessary definitions and preliminaries; in the second we prove our main result and in the third we apply it to some concrete function spaces of Fréchet–Sobolev type.

1. Definitions and preliminaries. Let $(\lambda, \|\ \|)$ be a normal Banach sequence space, i.e. a Banach space satisfying

(α) $\varphi \subset \lambda \subset \omega$ algebraically and the inclusion $(\lambda, \|\ \|) \hookrightarrow \omega$ is continuous

(β) $\forall a = (a_k)_k \in \lambda$, $\forall b = (b_k)_k \in \omega$ such that $|b_k| \leq |a_k|$, $\forall k \in \mathbf{N}$, we have $b \in \lambda$ and $\|b\| \leq \|a\|$.

Of course, every projection

$$s_n : \omega \to \omega, \qquad (a_k)_k \to ((a_k)_{k \leq n}, (0)_{k > n})$$

onto the first n-coordinates induces a $\|\ \|$-decreasing endomorphism on λ. We shall introduce the following property on $(\lambda, \|\ \|)$:

$$\lim_{n\to\infty} \|a - s_n(a)\| = 0, \qquad \forall a \in \lambda. \tag{ε}$$

Typical examples of $(\lambda, \|\ \|)$ are the spaces $(l^p, \|\ \|_p)$, where $1 \leq p \leq \infty$, $(c_0, \|\ \|)$ and their diagonal transforms. In particular, the spaces $(l^p, \|\ \|)$, where $1 \leq p < \infty$, and $(c_0, \|\ \|)$ satisfy (ε).

Now, let $(\lambda, \|\ \|)$ be a normal Banach sequence space and let $(Y_k, \|\ \|_k)_k$ be a sequence of Banach spaces; then the Banach space $\lambda((Y_k, \|\ \|_k)_k)$ is defined as the linear space

$$\lambda((Y_k, \|\ \|_k)_k) = \left\{ (y_k)_k \in \prod_{k\in\mathbf{N}} Y_k : (\|y_k\|_k)_k \in \lambda \right\},$$

endowed with the norm $(y_k)_k \to \|(\|y_k\|_k)_k\|$.

Let $(X_k, |\ |_k)_k$ be another sequence of Banach spaces and, for every $k \in \mathbf{N}$, let $f_k : X_k \to Y_k$ be a linear map such that $\|f_k(x)\|_k \leq |x|_k$, for every $x \in X_k$.

Then, following J. Bonet and S. Dierolf [**4**, Definition 1.3 and Proposition 1.4], we define the Fréchet space E of Moscatelli type with respect to $(\lambda, \|\ \|)$, $(X_k, |\ |_k)_k$,

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$(Y_k, \|\ \|_k)_k$ and $(f_k)_k$ as the space

$$E = \left(\prod_{k\in\mathbf{N}} X_k\right) \cap \lambda((Y_k, \|\ \|_k)_k) = \left\{(x_k)_k \in \prod_{k\in\mathbf{N}} X_k;\ (f_k(x_k))_k \in \lambda((Y_k, \|\ \|_k)_k)\right\} \tag{0}$$

with the intersection topology given by the sequence of seminorms

$$p_n((x_k)_k) = r_n((x_k)_k) + r((x_k)_k) \qquad ((x_k)_k \in E),$$

where

$$r_n((x_k)_k) = \sup_{k\le n} |x_k|_k \quad \text{and} \quad r((x_k)_k) = \|(\|f_k(x_k)\|_k)_k\|.$$

For more about such spaces the reader is referred to, for example, [**4**] and [**7**].

Also, we introduce for every $n \in \mathbf{N}$ the following continuous maps

$$J_n : \prod_{k\le n} X_k \to E,\ (x_1, \ldots, x_n) \to ((x_k)_{k\le n}, (0)_{k>n}),$$

$$S_n : E \to \prod_{k\le n} X_k,\ (x_k)_k \to (x_1, \ldots, x_n),$$

where S_n is the restriction to E of the canonical projection from $\prod_{k=1}^{\infty} X_k$ onto $\prod_{k\le n} X_k$. Clearly, J_n is an isomorphism into and $S_nJ_n = I_{\prod_{k\le n} X_k}$, for every $n \in \mathbf{N}$. Hence the map J_nS_n is a projection from E onto its Banach subspace $J_n\left(\prod_{k\le n} X_k\right)$. Moreover, we recall that the countable product space $\prod_{k=1}^{\infty} X_k$ can be represented as $\text{proj}_n\left(\prod_{k\le n} X_k, S_{n+1,n}\right)$, where $S_{n+1,n}: \prod_{k\le n+1} X_k \to \prod_{k\le n} X_k$ is the canonical projection $(x_1, \ldots, x_n, x_{n+1}) \to (x_1, \ldots, x_n)$.

We shall use standard terminology for the theory of Fréchet spaces, as in [**5**]. In particular, for two Fréchet spaces F and G, we shall write $F \simeq G$ to mean that F is topologically isomorphic to G.

2. Montel subspaces of Fréchet spaces of Moscatelli type. For the sequel, E always denotes the Fréchet space of Moscatelli type with respect to $(\lambda, \|\ \|)$, $(X_k, |\ |_k)$, $(Y_k, \|\ \|_k)$ and $f_k : X_k \to Y_k$, with $\|f_k(x)\|_k \le |x|_k$ for every $x \in X_k$. We shall assume that $(\lambda, \|\ \|)$ satisfies the property (ε). Our proofs rest on the following basic lemma.

LEMMA 1. *Let F be a Montel subspace of E. Then there exist $n_0 \in \mathbf{N}$ and $d > 0$ such that, for every $x \in F$, we have*

$$r(x) \le d r_{n_0}(x). \tag{1}$$

Proof. Suppose that (1) does not hold. Then, we have

$$\forall n \in \mathbf{N},\ \forall d > 0,\ \exists x \in F \quad \text{such that} \quad r(x) > d r_n(x). \tag{$*$}$$

Let $(d_n)_n$ be an arbitrary sequence of positive numbers decreasing to 0 with $d_1 < \frac{1}{2}$ and let $\tau_n = \|((1)_{k\le n}, (0)_{k>n})\|$, for all $n \in \mathbf{N}$. Since $(\lambda, \|\ \|)$ satisfies the property (ε) and we suppose that $(*)$ holds, we can find inductively a sequence $(x_n)_n \subset F$, $x_n = (x_{nk})_k$, and a sequence of integers $1 = k_1 < k_2 < \ldots < k_n < k_{n+1} < \ldots$ such that, for all $n \in \mathbf{N}$,

$$r(x_n) = 1 \tag{2}$$

$$r_{k_n}(x_n) < \frac{d_n}{\tau_{k_n}} \tag{3}$$

$$r(x_n - S_{k_{n+1}} x_n) < d_n. \tag{4}$$

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We claim that the closed linear span $[x_n : n \in \mathbf{N}] \subset F$ of $(x_n)_n$ in E is an infinite dimensional Banach subspace of F which leads to a contradiction because F is a Montel space. To show this we proceed as follows.

Put $\tilde{x}_n = (S_{k_{n+1}} - S_{k_n})(x_n)$, for every $n \in \mathbf{N}$. Then, by (2) and (β), $r(\tilde{x}_n) \leq 1$. Also, by (3) and (β), $r(S_{k_n} x_n) \leq \tau_{k_n} r_{k_n}(S_{k_n} x_n) = \tau_{k_n} r_{k_n}(x_n) < d_n$ (by recalling that $\|f_k(x)\|_k \leq |x|_k$, for every $x \in X_k$ and for every $k \in \mathbf{N}$); also by (2) and (4) $r(S_{k_{n+1}} x_n) > 1 - d_n$. Hence, for every $n \in \mathbf{N}$, we have

$$0 < 1 - 2d_1 \leq 1 - 2d_n < r(\tilde{x}_n) \leq 1. \tag{5}$$

Moreover, by (β), for every $j < i$ and $(a_n)_n \subset \mathbf{R}$, we obtain

$$r\left(\sum_{n=1}^{j} a_n \tilde{x}_n\right) \leq r\left(\sum_{n=1}^{i} a_n \tilde{x}_n\right); \tag{6}$$

hence, $(\tilde{x}_n)_n$ is a basic sequence with respect to the seminorm r with basis constant $= 1$.

Now, if $\tilde{x} = \sum_{n=1}^{\infty} a_n \tilde{x}_n$ converges with respect to r, then from (5) and (6) it follows that, for every $n \in \mathbf{N}$, we have

$$r_{k_i}\left(\sum_{n=1}^{\infty} a_n \tilde{x}_n\right) = r_{k_i}\left(\sum_{n=1}^{i-1} a_n \tilde{x}_n\right) \leq \sum_{n=1}^{i-1} |a_n| \, r_{k_i}(\tilde{x}_n) \leq \frac{2c_i}{1 - 2d_1} r(\tilde{x}) \tag{7}$$

with $c_i = \sum_{n=1}^{i-1} r_{k_i}(\tilde{x}_n) < \infty$. Therefore, $(\tilde{x}_k)_k$ is a basic sequence of E such that its closed span $[\tilde{x}_n : n \in \mathbf{N}]$ in E is an infinite dimensional Banach subspace of E. We observe that from (2), (3), and (4) we then get for every $n \in \mathbf{N}$

$$\begin{aligned} r(x_n - \tilde{x}_n) &= \|((\|f_k(x_{nk})\|_k)_{k \leq k_n}, (0)_{k_n < k \leq k_{n+1}}, (\|f_k(x_{nk})\|_k)_{k > k_{n+1}})\| \\ &\leq \|((\|f_k(x_{nk})\|_k)_{k \leq k_n}, (0)_{k > k_n})\| + \|((0)_{k \leq k_{n+1}}, (\|f_k(x_{nk})\|_k)_{k > k_{n+1}})\| \\ &\leq \tau_{k_n} r_{k_n}(x_n) + r(x_n - S_{k_{n+1}} x_n) < d_n + d_n = 2d_n. \end{aligned}$$

If we take $(d_n)_n$ small enough to have $d = \sum_{n=1}^{\infty} d_n < \frac{1 - 2d_1}{4}$ (e.g., we can take $d_n = \left(\frac{1}{11}\right)^n$), this implies that if $\tilde{x} = \sum_{n=1}^{\infty} a_n \tilde{x}_n$ converges, then $x = \sum_{n=1}^{\infty} a_n x_n$ also converges with respect to the seminorm r and

$$\left(1 - \frac{4d}{1 - 2d_1}\right) r(\tilde{x}) \leq r(x) \leq \left(1 + \frac{4d}{1 - 2d_1}\right) r(\tilde{x}). \tag{8}$$

Finally, from (3), (7) and (8) it follows that, for every $i \in \mathbf{N}$,

$$\begin{aligned} r_{k_i}(x) &\leq r_{k_i}\left(\sum_{n=1}^{i-1} a_n x_n\right) + r_{k_i}\left(\sum_{n \geq i} a_n x_n\right) \\ &\leq \sum_{n=1}^{i-1} |a_n| \, r_{k_i}(x_n) + \sum_{n \geq i} |a_n| \, r_{k_i}(x_n) \\ &\leq \frac{2}{1 - 2d_1} r(\tilde{x}) \left(\sum_{n=1}^{i-1} r_{k_i}(x_n) + \sum_{n \geq i} \frac{d_n}{r_{k_n}}\right) \\ &\leq \frac{2c_i'}{1 - 6d} r(x), \end{aligned}$$

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with $c_i' = \sum_{n=1}^{i-1} r_{k_i}(x_n) + \sum_{n\geq i} \frac{d_n}{\tau_{k_n}} < \infty$. (We may always assume that $\tau_n \geq 1$ for every n.) This and (8) imply that $(x_n)_n$ is a basic sequence of E equivalent to $(\tilde{x}_n)_n$, so that $[x_n : n \in \mathbf{N}]$ is an infinite dimensional Banach subspace of F. This completes the proof.

Consequently, we obtain the following result.

THEOREM 2. *Let F be a Montel subspace of E. Then the topology induced on F by E coincides with the one induced by* $\prod_{k=1}^{\infty} X_k$.

REMARKS. 1. By Theorem 2, to construct some examples of Montel subspaces F of a given Fréchet space E of Moscatelli type it suffices to look at the Montel subspaces of $\prod_{k=1}^{\infty} X_k$ for which the only algebraic condition $(f_k(x_k))_k \in \lambda((Y_k, \|\ \|_k)$ is satisfied, for every $x = (x_k)_k \in F$.

2. If we suppose that E is a Montel space, then from Theorem 2 it follows that $E = \prod_{k=1}^{\infty} X_k$. This and inequality (1) imply that $\dim X_k < \infty$ for every $k \in \mathbf{N}$ and there exists k_0 such that $f_k(X_k) = \{0\}$, for every $k \geq k_0$. Therefore, Proposition 2.7 of [**4**] can be seen as a particular case of Theorem 2.

Now we can state and prove our main result.

THEOREM 3. *If F is a complemented Montel subspace of E, then either $F \simeq \omega$ or* $\dim F < \infty$.

Proof. Suppose that F is a complemented Montel subspace of E and $P: E \to F$ is a projection with $\ker P = G$ and $Q = I - P$. Now, by Theorem 2, F is a Montel subspace of $\prod_{k=1}^{\infty} X_k$. Therefore, by Lemma 1.1 of [**6**], $F = \operatorname{proj}_n(S_n(F), \tilde{S}_{n+1,n})$, where $\tilde{S}_{n+1,n}: S_{n+1}(F) \to S_n(F)$, restrictions of the $S_{n+1,n}$'s, are clearly surjective. We shall show that $S_n(F)$ is a closed subspace of $\prod_{k\leq n} X_k$ and hence it is a Banach space. Consequently, F is a quojection; for the definition see [**8**, 8.4.27]. Since F is Montel, it must be either isomorphic to ω or finite dimensional; see, e.g., [**8**, 8.4.31]. Then, we put $H_n = S_n(F) \subset \prod_{k\leq n} X_k$ and we denote by $\bar{H}_n$ its closure in $\prod_{k\leq n} X_k$, for every $n \in \mathbf{N}$. Moreover, we denote again by J_n and $\bar{J}_n$ the restriction of J_n to H_n and $\bar{H}_n$ respectively. Since F is Montel, the composition maps

$$P_n = S_n P J_n : H_n \xrightarrow{J_n} E \xrightarrow{P} F \xrightarrow{S_n} H_n,$$

$$\bar{P}_n = S_n P \bar{J}_n : \bar{H}_n \xrightarrow{\bar{J}_n} E \xrightarrow{P} F \xrightarrow{S_n} H_n$$

are compact for every $n \in \mathbf{N}$, where $\bar{P}_n$ is the compact extension of P_n to $\bar{H}_n$. Now, we observe that if $x \in H_n$, then $J_n x = P J_n x + Q J_n x$, and hence $x = S_n P J_n x + S_n Q J_n x$ (recalling that J_n is a right-continuous inverse of S_n). This implies that $S_n Q J_n x = x - S_n P J_n x = x - P_n x$ belongs to H_n too. Therefore, the composition map

$$Q_n = S_n Q J_n : H_n \xrightarrow{J_n} E \xrightarrow{Q} G \xrightarrow{S_n} S_n(G)$$

is such that its range $Q_n(H_n)$ is also contained in H_n. Clearly, $I_{H_n} = P_n + Q_n$ and so $Q_n = I_{H_n} - P_n$. Since P_n is a compact map from H_n into itself, it follows that $Q_n(H_n) \subset H_n$ is a closed subspace of H_n and hence it is a Banach subspace of $\prod_{k\le n} X_k$. This implies that $\bar{Q}_n = S_n Q \bar{J}_n$, which is the continuous extension of Q_n to $\bar{H}_n$, is such that its range $\bar{Q}_n(\bar{H}_n)$ is also contained in H_n. Now, we can prove that H_n is a closed subspace of $\prod_{k\le n} X_k$. In fact, if $(x_j)_j \subset H_n$ and $(x_j)_j$ converges to $x \in \bar{H}_n$ in $\prod_{k\le n} X_k$, then $(P_n x_j)_j$ converges to $\bar{P}_n x \in H_n$ and $(Q_n x_j)_j$ converges to $\bar{Q}_n x \in H_n$. However $x_j = P_n x_j + Q_n x_j$, for every j, and hence, letting $j \to \infty$, we obtain $x = \bar{P}_n x + \bar{Q}_n x \in H_n$. Thus, the proof is complete.

COROLLARY 4. *If E is a Fréchet space of Moscatelli type with a continuous norm, then E does not have an infinite dimensional complemented Montel subspace.*

3. Applications to Fréchet–Sobolev spaces. Let Ω be an open subset of $\mathbf{R}^N$, with $N \ge 1$. For $m, k \in \mathbf{N}$, $k \le m$ and $1 \le p \le \infty$, $C^m(\Omega) \cap H^{k,p}(\Omega)$ is a Fréchet space with its natural intersection topology given by the sequence of norms

$$p_n(f) = \max_{|\alpha|\le k} \left(\int_\Omega |f^{(\alpha)}(x)|^p \, dx \right)^{1/p} + \max_{|\alpha|\le m} \max_{x \in J_n} |f^{(\alpha)}(x)|, \quad \text{for} \quad p < \infty,$$

or

$$p_n(f) = \max_{|\alpha|\le k} \sup_{x\in\Omega} |f^{(\alpha)}(x)| + \max_{|\alpha|\le m} \max_{x\in J_n} |f^{(\alpha)}(x)|, \quad \text{for} \quad p = \infty,$$

where $(J_n)_n$ is a sequence of compact subsets of Ω such that $J_n = \bar{\mathring{J}}_n \subset \mathring{J}_{n+1}$ and $\bigcup_n \mathring{J}_n = \Omega$. Moreover, for $1 \le q < p \le \infty$, the space $L^p_{loc}(\Omega) \cap L^q(\Omega)$ of all q-integrable and locally p-integrable functions in Ω is also a Fréchet space, whose topology is given by the following sequence of norms

$$q_n(f) = \left(\int_\Omega |f(x)|^q \, dx \right)^{1/q} + \left(\int_{J_n} |f(x)|^p \, dx \right)^{1/p}, \quad \text{for} \quad p < \infty,$$

or

$$q_n(f) = \left(\int_\Omega |f(x)|^q \, dx \right)^{1/q} + \sup_{x\in J_n} |f(x)|, \quad \text{for} \quad p = \infty,$$

where $(J_n)_n$ is a sequence of compact subsets of Ω defined as above. Now, by Theorems 1 and 2 of [2] (see also [3]), the Fréchet space $C^m(\Omega) \cap H^{k,p}(\Omega)$ is of Moscatelli type for $k = 0,1$ when Ω is an open subset of $\mathbf{R}^N$ $(N > 1)$ and for every $k \in \mathbf{N}$ when Ω is an open subset of $\mathbf{R}$ or $\Omega = \mathbf{R}^N$. Also, the Fréchet space $L^p_{loc}(\Omega) \cap L^q(\Omega)$, $1 \le q < p \le \infty$, is of Moscatelli type, as it follows from the fact that the map

$$L^p_{loc}(\Omega) \cap L^q(\Omega) \to \left(\prod_{n=1}^\infty L^p(K_n) \right) \cap l^q(L^q(K_n))$$

$$f \to (f_{|K_n})_n,$$

where $K_1 = J_1$ and $K_{n+1} = \overline{J_{n+1} \setminus J_n}$, for every $n \ge 1$, is an isomorphism onto. Finally, we recall that the inclusion map $H^{m,p}(\Omega) \hookrightarrow L^q(\Omega)$ is continuous if $m \in \mathbf{N}$, $1 \le p < q < \infty$, and $N - mp > 0$, $\frac{m}{N} \ge \frac{1}{p} - \frac{1}{q} > 0$ and if Ω is an open subset of $\mathbf{R}^N$ with the cone property, by the

Sobolev imbedding theorem. (See, for example, Theorem 5.4 of [**1**]). Then, the space $H^{m,p}_{\text{loc}}(\mathbf{R}^N) \cap L^q(\mathbf{R}^N)$, with m, p and q satisfying the above conditions, is a Fréchet space with respect to the topology generated by the following sequence of norms

$$r_n(f) = \max_{|\alpha| \le m} \left(\int_{\{x \in \mathbf{R}^N : |x| \le n\}} |f^{\alpha}(x)|^p \, dx \right)^{1/p} + \left(\int_{\mathbf{R}^N} |f(x)|^q \, dx \right)^{1/q}.$$

The same proof of Theorem 1 of [**2**] works to show that the spaces $H^{m,p}_{\text{loc}}(\mathbf{R}^N) \cap L^q(\mathbf{R}^N)$ are of Moscatelli type.

Then, by Theorems 2 and 3 of §2, we can deduce the following results.

COROLLARY 5. *Let E be one of the following three spaces:*

(a) $C^m(\Omega) \cap H^{k,p}(\Omega)$, *with* $k = 0, 1$ *when* Ω *is an open subset of* $\mathbf{R}^N$ $(N > 1)$ *or* $k \in \mathbf{N}$ *when* Ω *is an open subset of* $\mathbf{R}$ *or* $\Omega = \mathbf{R}^N$, *with* $m \in \mathbf{N}$, $m \ge k$, *and* $1 \le p < \infty$;

(b) $L^p_{\text{loc}}(\Omega) \cap L^q(\Omega)$, *with* Ω *an open subset of* $\mathbf{R}^N$ $(N \ge 1)$ *and* $1 \le q < p \le \infty$;

(c) $H^{m,p}_{\text{loc}}(\mathbf{R}^N) \cap L^q(\mathbf{R}^N)$, *with* $m \in \mathbf{N}$, $1 \le p < q < \infty$, *and* $N - mp > 0$, $\dfrac{m}{N} > \dfrac{1}{p} - \dfrac{1}{q} > 0$;

and let F be a Montel subspace of E. Then the topology induced on F by E coincides with the one induces by $C^m(\Omega)$, *by* $L^p_{\text{loc}}(\Omega)$ *and by* $H^{m,p}_{\text{loc}}(\mathbf{R}^N)$ *in the case* (a), (b) *and* (c) *respectively.*

COROLLARY 6. *The Fréchet spaces considered in Corollary* 5 *do not have an infinite dimensional complemented Montel subspace.*

REMARK 3. We note that Corollaries 5 and 6 remain valid also for $C^m(\Omega) \cap H^{k,p}(\Omega)$ with $k > 1$ and Ω an arbitrary open subset of $\mathbf{R}^N$, with $N > 1$, although a representation of type (0) is not available. The proof in this case is similar to the proofs of Theorems 2 and 3 of §2 with some simple changes.

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