ON THE ADVANTAGES OF THE MODERN METHODS OF COMPUTATION IN LIFE ASSURANCE CALCULATIONS.

To the Editor of the Assurance Magazine.

SIR,—At page 98, vol. i., of the *Assurance Magazine*, in a letter of your talented correspondent, Mr. P. Gray, "On the comparative Advantages of the Old and New Methods of Computation," is given a problem in illustration of his subject, which has the recommendation of being one which actually occurred, the consideration of which led to the preparation of his remarks. The two following problems, which have also arisen in practice, and which are further illustrative of the same subject, may not be deemed unworthy of a place in your Journal.

I am, Sir,
Your obedient servant,
SAMUEL L. LAUNDY.

Eagle Life Office,
30th September, 1858.

PROBLEM.

A person, aged x, is assured for £m, payable at the age $x+n$, or at previous death, at an annual premium, P, payable until age $x+n-1$. Being desirous of converting his assurance into another of similar amount, payable at death, he requires to know what premium, P′, he should pay for the rest of life.

Benefit terms.

x receives—1st, an assurance of £m payable at death, the present value of which, multiplied by D_x, is . . . . $m.M_x$;

2nd, remission of the premium P for the next n years, the present value of which, multiplied by D_x, is . . . . $P(N_{x-1}-N_{x+n-1})$.

Payment terms.

x gives up his present assurance, the value of which, multiplied by D_x, is . . . $m(D_{x+n}+M_x-M_{x+n})$,

And pays a premium, P′, for the remainder of life, the present value of which, multiplied by D_x, is . . . . . . $P'(N_{x-1})$.

Equating the sum of the benefit terms to that of the payment terms,

$$P'(N_{x-1})+m(D_{x+n}+M_x-M_{x+n})=P(N_{x-1}-N_{x+n-1})+m.M_x;$$

$$\text{whence } P'=\frac{P(N_x-N_{x+n-1})-m.D_{x+n}-M_{x+n}}{N_{x-1}}.$$

PROBLEM.

A person, at age x, assured for £m, at a premium P_x. At age $x+n$ he is desirous of knowing for what amount he can have a free policy in consideration of the premiums already paid.

Benefit terms.

1st, a free policy for £m', the present value of which, multiplied by D_{x+n}, is . . . . . . $m'.M_{x+n}$;

2nd, remainder of premium P_x for the remainder of life, the present value of which, multiplied by D_{x+n}, is $P_x.N_{x+n-1}$.

Payment term.

He gives up the reversion to £m, payable at his decease, the present value of which, multiplied by D_{x+n}, is . . . . $\Big\}$ $m . M_{x+n}$.

$$\text{Equating, } m' . M_{x+n} + P_x . N_{x+n-1} = m . M_{x+n};$$

$$\text{whence } m' = \frac{m . M_{x+n} - P_x . N_{x+n-1}}{M_{x+n}} = m - \frac{P_x . N_{x+n\ 1}}{M_{x+n}}.$$

$$\text{Let } m=1, \text{ then } m' = 1 - \frac{P_x . N_{x+n-1}}{M_{x+n}};$$

dividing by N_{x+n-1}, we have $\dfrac{P_x}{M_{x+n} \div N_{x+n-1}}$ and $\dfrac{M_{x+n}}{N_{x+n-1}} = P_{x+n}$;

$$\therefore\ m = 1 - \frac{P_x}{P_{x+n}},$$

which agrees with the expression given by Mr. Sprague (*Assurance Magazine*, vol. vii., p. 59), derived from the assurance and annuity values.

It is to be observed, that the foregoing Problems include only the case where the premium is just due, and which is the one that will generally occur in practice. For the case where the premium has just been paid, the formulæ will have to be modified by the omission of -1 from all the terms of column N.

FORMULÆ FOR THE ANNUAL PREMIUM FOR A TERM ASSURANCE ON TWO JOINT LIVES.

To the Editor of the Assurance Magazine.

SIR,—It is probably very seldom that an assurance is effected for a term of years on the joint duration of two lives; and when such a case occurs, it will often be thought desirable to employ some method of approximation in order to determine the proper premium for the assurance. One such method may be mentioned. If a policy is to be effected for t years, on the joint duration of two lives, let it be calculated what would be the surrender value of a policy for the whole duration of the same lives, after it has been t years in force, or what percentage of the premiums paid would be returned; then if that percentage be deducted from the Office premium for an assurance for the whole duration of the lives, the remainder will be the premium for the term assurance. But it is desirable to show how such a premium may be calculated exactly, and the suitable formulæ will probably be new to many of the readers of the *Assurance Magazine*, as they are not given in David Jones's *Treatise on Annuities*, nor in any other work of which I am aware.

Let (a) denote the value of an annuity of £1 on the joint lives of the last v survivors of the lives m, m_1, m_2, &c.; and (A) the value of an assurance of £1 on the same lives. Then, it is proved by Jones (Art. 197), that

$$(A)_{\overline{t|}} = r\{1 + (a)_{\overline{t-1|}}\} - (a)_{\overline{t|}}.$$

Also, let (P) denote the annual premium for the same assurance;

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