COLLAPSE, EQUILIBRIUM, AND FRAGMENTATION OF ROTATING, ADIABATIC CLOUDS

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ABSTRACT

Numerical calculations of the collapse of adiabatic clouds from uniform density and rotation initial conditions show that when restricted to axisymmetry, the clouds form either near-equilibrium spheroids or rings. Rings form in the collapse of low thermal energy clouds and have $\beta = T/|W| \gtrsim 0.43$. When the axisymmetric constraint is removed and an initial m=2 density variation is introduced, clouds either collapse to form near-equilibrium ellipsoids or else fragment into binary systems through a bar phase. Ellipsoids form in the collapse of high thermal energy clouds and have $\beta \lesssim 0.27$. The results are consistent with the critical values of β for instabilities in Maclaurin spheroids, and suggest that protostellar clouds may undergo a dynamic fragmentation in the nonisothermal collapse regime.

1. INTRODUCTION

A major problem in the theory of star formation is determining the forms a collapsing interstellar cloud passes through prior to forming a pre-main-sequence star. While dense interstellar clouds and pre-main-sequence stars have been extensively observed, the intermediate forms have not, because of the presumed rapid evolution in this phase and because of the inherent obscuration of the placental clouds within which stars form.

Dense, cool, interstellar clouds are sufficiently optically thin that the compressional heat generated during collapse to higher densities is radiated away, maintaining the cloud at an isothermal temperature of $\sim$ 10K. A considerable amount of numerical work modelling hydrodynamic collapse in three spatial dimensions (3D) has shown that a gravitationally unstable, rotating, isothermal gas cloud will fragment into a small number of fragments (see Bodenheimer, 1980, for a recent review). The fragments are gravitationally unstable themselves, and so may collapse and dynamically fragment into subfragments. Eventually

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the fragments reach densities ($\gtrsim 10^{-12}$g cm^{-3}) high enough that the compressional heat is trapped within the cloud; the subsequent evolution will be nonisothermal, with a pressure law $p \propto \rho^{\gamma}$ such that γ is close to 7/5 (appropriate for the adiabatic collapse of molecular hydrogen; see e.g. Winkler and Newman, 1980).

Is further fragmentation of collapsing, rotating clouds possible in the nonisothermal regime? We attempt to answer this question in this paper. While inclusion of radiative transfer effects in a 3D code is necessary to completely answer this question, a provisional answer may be obtained by using the appropriate adiabatic pressure law. Takahara et. al. (1977) have studied the axisymmetric collapse of rotating clouds with an adiabatic exponent γ=5/3, starting from centrally-condensed initial conditions. They found that such clouds form rotating, spheroidal cores, unless the initial model is strongly differentially rotating, in which case a ring forms, if the thermal energy of the cloud is low enough. In a full 3D calculation, a ring is more likely to fragment than a disk, and hence it is important to ascertain under what conditions rings may be formed. In § 2. we show that axisymmetric cloud collapse from uniform rotation and density initial conditions produces rings if the initial ratio of thermal to the absolute value of the gravitational energy (α_i) is sufficiently low, with the critical value of α_i being a function of γ for the cloud. Higher α_i clouds form near-equilibrium spheroids.

Cook and Harlow (1978) calculated the 3D collapse of a rotating adiabatic cloud with γ=5/3, starting with a velocity field which favored ring formation, but containing a small nonaxisymmetric perturbation (NAP). They found that with α_i = .15 the NAP damped and a ring formed, but with α_i = .10, the NAP grew and fragmented the cloud. In § 3. we show that nonaxisymmetric cloud collapse, with an initial cos2ϕ density variation (ϕ is the angle about the rotation axis), results in fragmentation for low enough α_i with γ = 7/5. These results depend slightly on β_i, the initial ratio of rotational to the absolute value of the gravitational energy ($\beta = T/|W|$ in the language of polytropes). Higher α_i clouds form near-equilibrium ellipsoids.

The calculations presented in this paper were performed with computer codes describing self-gravitating gas hydrodynamics in 2D and 3D (Boss, 1980a). Magnetic fields and viscosity were not included in the calculations. All clouds were initially spherical and confined within a fixed spherical boundary throughout the calculation.

2. AXISYMMETRIC COLLAPSE

Twenty models were run with the 2D code, starting from uniform density and rotation initial conditions, with both γ = 7/5 and 5/3 (Boss, 1980c). Each model was 1M$_\odot$ with an initial density of 10^{-12}g cm^{-3}; scaling laws exist to predict the outcome of the collapse of clouds with differing mass and initial density (Boss, 1980c). The initial values of α_i for the clouds ranged between .009 and .34, while β_i ranged from .08 to .32. Because of the absence of pressure gradients in the initial models, the clouds underwent a self-gravitational, dynamic collapse, flattening preferentially down the rotation axis. The models were followed through the dynamic collapse phase until they appeared to be oscillating about an equilibrium state (defined by a drop in the kinetic energy in nonrotational motion); thereafter a dissipative mechanism was employed to reach the final, near-equilibrium state.

Two distinct final states were found: spheroids and rings. The spheroids were differentially rotating polytropes similar to the equilibrium models of Bodenheimer and Ostriker (1973). The rings were differentially rotating, adiabatic analogues of the rings found in isothermal collapse (see e.g. Bodenheimer, 1980). Several models experienced a transitory ring phase, which then evolved into a spheroid. That the final models were close to equilibrium is further indicated by monitoring the final α and β of the cloud; for equilibrium the virial theorem demands $\alpha_f + \beta_f = \frac{1}{2}$. The final models obeyed this relationship to within 10%. The dynamic nature of the calculation assures us that the final near-equilibrium states were physically stable, subject to the axisymmetric constraint.

A $\gamma = 5/3$ cloud collapses to form an equilibrium ring if $\alpha_i \lesssim .01$; a $\gamma = 7/5$ cloud does the same if $\alpha_i \lesssim .04$. For values of $\alpha_i \gtrsim .06$, a $\gamma = 5/3$ cloud forms an equilibrium spheroid; the same occurs for $\alpha_i \gtrsim .08$ for a $\gamma = 7/5$ cloud. For intermediate values of α_i, the clouds collapse to form transitory rings, which thereafter evolve into spheroids. The results appear to be only weakly dependent on the value of β_i.

The equilibrium sequences obtained for both the $\gamma = 5/3$ and 7/5 clouds suggest that equilibrium, rotating, adiabatic clouds will be spheroidal for $\beta_f \lesssim .43$ and ring-like for $\beta_f \gtrsim .43$. This critical value of β is close to the dynamical limit for instability of Maclaurin spheroids to ring-modes at $\beta_d = .457$ (Bardeen, 1971), as well as to the limiting value of $\beta_+ = .437$ above which toroids have a lower energy equilibrium state than spheroids (Marcus, Press, and Teukolsky, 1977). Apparently the analyses for incompressible, uniformly rotating Maclaurin spheroids are approximately correct for differentially rotating, adiabatic, compressible clouds.

3. NONAXISYMMETRIC COLLAPSE

Ten models were run with the 3D code, starting from uniform rotation, with an initial density variation $\rho_i = \rho_o\ (1 + .5 \cos 2\phi)$, for clouds with $\gamma = 7/5$ only (Boss, 1980d). The initial values of α_i for the clouds ranged between .05 and .4, while β_i ranged from .05 to .3. These models also underwent a dynamic collapse phase characterized by flattening about the equatorial plane. No dissipation was employed in the 3D calculations, however.

Two distinct final states were obtained: ellipsoids and binary systems. The initial density variation produced an intermediate bar configuration shortly after one free-fall time into the collapse. For the initially high-thermal-energy clouds, the bar was stable with respect to fragmentation, and the NAP slowly died out, producing a near-equilibrium ellipsoid similar to those obtained with the 2D code (§2.). For the initially low-thermal-energy clouds, the bar was unstable and broke up into two distinct fragments, which orbited each other as a binary system. The behavior was somewhat dependent on the initial value of β_i. For $\alpha_i \gtrsim .06 + .3\ \beta_i$, ellipsoids resulted, while binary systems obtained for lower values of α_i. These results would presumably change somewhat for collapses with different initial density variations.

The near-equilibrium ellipsoids were monitored for 3 or more local free fall times or ¼ to 1 local rotation periods, without showing any tendency to fission; rather, the ellipsoids appear to be evolving into axisymmetric spheroids. One model ($\alpha_i = .1$, $\beta_i = .05$) experienced a transitory phase of trailing spiral arm growth from the ends of the intermediate bar, but in this case the NAP died away over 1 rotation period of the bar. The ellipsoids so obtained had $\beta_f \lesssim .27$, which agrees favorably with the dynamical limit for the instability of Maclaurin spheroids to nonaxisymmetric modes at $\beta = .274$ (Bardeen, 1971).

The binary fragments formed are similar to those encountered in isothermal collapse calculations, in that they have masses and spin specific-angular momenta roughly 0.1 that of the original cloud. The adiabatic fragments, however, have sufficiently high thermal energies ($\alpha_f \sim .3$ to 1.) that subsequent dynamic collapse and further fragmentation is unlikely. The reduction of spin angular momentum is achieved by preferentially assimilating low-angular momentum material at the center of the cloud and by storing angular momentum in the orbital motion of the binary system.

4. IMPLICATIONS FOR STAR FORMATION

The calculations show that a suitably perturbed low-thermal-energy cloud ($\alpha_i \lesssim .06 + .3\ \beta_i$) with $\gamma = 7/5$ may dynamically fragment into a binary protostar system. The fragments obtained from 3D isothermal collapse calculations typically have $\alpha_i \sim .05$ and $\beta_i \sim .05$ to .3 (Boss, 1980b); such fragments will undergo collapse and enter the nonisothermal regime. Provided that such fragments may be adequately modelled by a $\gamma = 7/5$ cloud, we see that these fragments may be expected to undergo a subsequent dynamic fragmentation. The subfragments so produced, however, should be sufficiently stable to resist further fragmentation, though they may fission on a secular time scale. This fragmentation mechanism allows stellar masses with reduced spin angular momentum to form out of massive parent clouds, while also producing copious binary star systems. These conclusions, however, are contingent upon the results of calculations with a 3D code with a proper radiative transfer treatment.

We have also seen that the dynamical instability limits for Maclaurin spheroids are quite close to the numerically derived limits for differentially rotating, adiabatic clouds, implying that the classical analyses may have wider applicability than was previously supposed.

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DISCUSSION

JONES: Do you have any feeling how much diffusion you get in your numerical scheme?

BOSS: We've done quite a bit of testing of the codes, primarily in 2D. We have some indications that they are correct because we ran the same or similar collapses with different codes with different amounts of diffusion in them and we get essentially the same results.

JONES: You may have to wait until you use higher resolution.

BOSS: We have done some calculations with 50% more points. With my code things don't change too terribly much, but we are still making more high resolution runs.

TSCHARNUTER: Fragmentation of the blobs or clouds depend very much on radiative transfer. The radiation temperature will differ from the kinetic temperature.

BOSS: As some of these calculations get high enough in temperature, one should take into account hydrogen dissociation which can prompt another collapse. This is another important effect as well as radiation transfer. These calculations were done to see if it was possible to get fragmentation at all so that we can decide whether to write an explicit or implicate 3D code.