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BOOK REVIEWS

PADBERG, M. *Linear optimization and extensions* (Algorithms and Combinatorics, Vol. 12, Springer-Verlag, Berlin–Heidelberg–New York–London–Paris–Tokyo–Hong Kong 1995), xix + 449 pp., 3 540 58734 9, DM 148.

The publisher's announcement tells us that this book offers a comprehensive treatment of linear programming as well as of the optimization of linear functions over polyhedra in finite-dimensional Euclidean vector spaces. An introduction surveying fifty years of linear optimization is given.

The main topics treated in the book include: simplex algorithms and their derivatives including the duality theory of linear programming, polyhedral theory, projective algorithms, Newtonian barrier methods, ellipsoid algorithms in perfect and in finite precision arithmetic.

My overriding impression is that this is a very solid book, having 449 pages, mostly very densely packed with lots of in-line equations and rather few figures (only five in the first six chapters). One might imagine that it would be difficult to say so much about linear programming, especially as combinatorial optimization problems only make a late appearance in the last chapter (Chapter 10). The reason is that large parts of the text are devoted to the development of the underlying theory such as matrix algebra, vector spaces, polyhedral descriptions and projective geometry. On the one hand this can be very useful in that the reader does not need to rely on results from other sources, but on the other hand it makes the technical parts of the book rather daunting.

I had the good fortune to meet Manfred Padberg at the Mathematical Institute at Oberwolfach. Those who know him better than I recognize him as a charming fellow and excellent company, who, amongst his many other qualities, is an accomplished jazz pianist. This lively personality shows through in the more descriptive parts of the book, which are enhanced by anecdotes from the formative years of linear programming. Professor Padberg also shows a remarkable knowledge of classical Greek history and intertwines the mathematical development with many episodes taken from this period. I was interested to learn for example that *devexus* is the Latin for *downward sloping*, which explains why the DEVEX code of Paula Harris is so called. There are many similar illuminating remarks which make the descriptive text informative and entertaining for the reader at any level.

I would however be uncomfortable in teaching linear programming from this book. The notation a_j^i instead of the common a_{ij} is strange to me and the use of A_I and A_{N-I} for basic and nonbasic variables is also unusual. Less tangibly I feel that the intuitive understanding of linear programming is somewhat neglected in favour of a rather arid presentation of the properties of basic feasible solutions and the fundamental theorem of linear programming.

The author makes it a feature that he does not use a tableau representation to teach the Simplex Method, influenced by bad experiences from his formative years. This same problem (whether to present a tableau format) also confronted me when writing my own text on optimization. At that time I also felt with Padberg that the matrix based approach is more intuitive and is closer to actual practice in real-life codes. However, I did include a brief description of the tableau format and have since come to realise its value in developing methods for the resolution of degeneracy in linear programming. Also I now see that the tableau format of the Linear Complementarity Problem readily provides a way of understanding and unifying many methods for quadratic programming (which by the way is an extension of LP not covered in the book).

I was pleased to see that the author does not perpetuate the myth that cycling in LP can be ignored. Indeed he is quite clear in pointing out that degeneracy is a common feature in many practical LP models. Unfortunately the cure that he describes and justifies, that is, the least index rule, is certainly not a practical method in floating point. As far as I could see, there is no description of techniques such as DEVEX and EXPAND which are used in practical codes. In general, although the author gives some feeling for the attention to detail needed to write a good LP code, I do not think that this book is a good source for finding out about this detail.

The book contains a description in Chapter 8 of various interior point methods, although it comes after a heavy dose of projective geometry. This follows a very detailed chapter on the properties of polyhedra. The presentation hereabouts in the book will appeal to those of a pure mathematical bent. There is probably quite a bit of material here that cannot easily be found in other texts and, for example, I found the description of the exact arithmetic (division free) Gaussian elimination algorithm to be of particular interest.

I was surprised that combinatorial optimization occupies a relatively small part of the book, particularly in view of the author's research interests in this area. The reader will find the discussion of the *branch and cut* method particularly interesting. The book ends with three appendices, each describing a large scale application of LP. Added to some other non-trivial examples in the introduction, these form a very useful feature.

R. FLETCHER

ARNOLD, V. I. *Topological invariants of plane curves and caustics* (University Lecture Series, Vol. 5, American Mathematical Society, Providence, RI, 1995), 60pp., paperback, 0 8218 0308 5, £17.50.

The book contains lectures on a range of topics from the author's most recent investigations. It is devoted to Vassiliev type invariants of plane curves and problems of low-dimensional symplectic and contact geometry.

The singularity theory approach to topological classification of objects of any nature follows a standard strategy going back to Poincaré. One considers the infinite-dimensional space of objects of interest, including both generic and degenerate objects. In this space degenerate objects form a discriminant hypersurface. An invariant is a locally constant function on its complement. The difference of its values on two neighbouring connected components can be related to the degeneracy corresponding to the piece of the discriminant which separates them.

This general approach was used with great success by Vassiliev in his work on knot invariants. In this book Arnold is taking the first step in constructing a similar theory for immersed plane curves and plane fronts (cooriented curves with cusps). Arnold represents three basic first order invariants which are dual to the three bifurcations: triple points on regular curves and two types of self-tangencies (with coinciding and opposite coorientations of the branches). He studies their properties and gives a series of geometrical interpretations. For example, the coinciding self-tangency invariant is basically the Bennequin-Tabachnikov number of Legendrian knots in the solid torus with the standard contact structure of spherisation of cotangent bundle of the 2-plane.

As a whole the theory of invariants of plane curves promises to be a sort of non-commutative version of knot theory.

Arnold's introduction of Vassiliev's technique to the study of plane curves was a by-product of investigations on a different topic (the story of Vassiliev invariants repeats!). The major aim (not yet achieved in the maximal desired generality) of Arnold's programme was to prove what might be called the Last Geometrical Theorem of Jacobi. Jacobi observed that the caustic of a point (that is, the set of intersection points of infinitesimally close geodesics starting at this point) on a closed convex surface should have a cusp. In fact, a point generates an infinite sequence of caustics. Jacobi's *Vorlesungen über Dynamik*, which was published after his death, contains an unfinished manuscript in which he announces (giving no proof) that for the surface of an ellipsoid the number of cusps on a caustic is equal to four.