

Hierarchical stellar clusters in molecular clouds

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Abstract. Observations show that massive stars always form in clusters or associations, and that the most massive stars form in the dense cores of large clusters. This suggests that accretion processes in cluster cores may be responsible for the formation of stars. In addition, young stellar clusters have been found to contain subclusters, so that star formation can be seen to be a hierarchical process that involves clustering on a range of scales. In this paper, we propose a fractal model of the parental molecular cloud, namely that of the Julia set given by $f(z) = z^2 + c$, where z and c are complex numbers and $c = -0.745430 + 0.113008i$, to explain this phenomenon and the associated complex structures seen in star-forming regions.

Keywords. stars: luminosity function, mass function, ISM: clouds, open clusters and associations: general

1. Introduction

Observations show that massive molecular clouds are sources of recent or ongoing star formation, and that the most massive stars tend to form in the dense cores of large clusters. Lower-mass clouds contain sites of low-mass stellar systems, as in Taurus and Ophiuchus (Williams & McKee 1997). Many young stellar clusters have been found to contain subclusters and, on the smallest scales, most stars are in multiple or binary systems. Star formation can, therefore, be seen to be a hierarchical process that involves clustering on a range of scales. A very good example of this type of clustering can be seen in the Orion Nebula, where one such cluster is embedded in the Kleinmann–Low infrared nebula and the other is in the Trapezium cluster.

Stars form in a hierarchy of clusters and subclusters in a molecular cloud, depending on the amount of material available for star formation. This is borne out by the fact that the mass of the most massive star present increases systematically with the mass of the associated molecular cloud (Larson 1995). In addition, the mass of the most massive star in a cluster is proportional to the total mass of stars and inversely proportional to the number of stars in that cluster. Each level in a stellar hierarchy corresponds to a fixed logarithmic interval in mass, with the number of stars in that hierarchy inversely proportional to their mass.

Giant star–gas complexes (SGC), with sizes ranging from 170 to 700 pc, contain 90% of star clusters with ages from 2 to 3×10^7 years, as well as molecular clouds with masses from 10^5 to $10^6 M_{\odot}$. The larger ones contain 10 or more stellar groupings, with the largest ones still having H I cloud remnants. OB associations in SGCs are found to be from 0.3 to 1.2×10^7 years old, with the older ones located towards the outer edges. In the inner regions of the SGCs are found OB associations with ages of less than 6.5×10^6

years, HII regions, cloud clumps, clusters of early spectral types and young water masers. An example is SCG 2, located in the Sagittarius arm (Sitnik 1990).

2. The initial mass function

In 1955, Salpeter showed that the number of stars, N , per unit logarithmic mass interval in the solar neighbourhood followed the relation

$$\frac{dN}{d \log M} \propto M^{-1.35}, \quad (2.1)$$

where M is the stellar mass (Salpeter 1955). This stellar initial mass function (IMF) was later modified by Silk (1995), to include molecular clouds, into

$$\frac{dN(M)}{dM} = AM^{-(1+\gamma)}, \quad (2.2)$$

where $dN(M)$ is the number of stars or cloud cores in the mass interval dM , $\gamma = 1.5 \pm 0.3$, a range which includes the Salpeter IMF and A is a constant. Observational support for the Salpeter IMF and its connection with its parental cloud has come from *HST* data (Maíz Apellániz 2001) and submillimetre observations (Clark *et al.* 2007).

3. The IMF and fractal molecular clouds

Old models of the IMF arising from molecular-cloud fragmentation, ambipolar diffusion, stellar winds and collisions had to be abandoned because of insufficient evidence (Chabrier 2003), leaving turbulence in the parental molecular cloud (Elmegreen & Scalo 2004) to be the only plausible explanation. In addition, a hierarchical stellar structure is possible only from a fractal parental molecular cloud created from chaotic dynamics. Evidence for the fractal structure of molecular clouds has been found by various authors (e.g., Larson 1995; Elmegreen & Falgarone 1996; Kramer *et al.* 1998). It has been further shown (Datta 2001, 2003, 2007) that fractal molecular-cloud structure is in the shape of a Julia set given by

$$f(z) = z^2 + c, \quad (3.1)$$

where $c = -0.745430 + 0.113008i$, of average box dimension 1.67.

4. Relation between the IMF and fractal dimension

We now propose that there exists a connection between the box dimension of the molecular cloud and the IMF (Equation 2.2) for molecular clouds. To make that connection, we look at the later stages of molecular-cloud evolution in the presence of strong gravitational forces. At this stage, the cloud IMF is related to the core-mass function (CMF; Goodwin *et al.* 2008) and our assumption of cloud cores to be spheres is reasonable. In a large collection of such cores, as in a fractal, we can use the mean mass, M_c , to represent core masses in the range $M_c \pm dM_c$ (law of large numbers). Replacing the frequency by the number of cores, $N(M_c)$, in the numerator of Equation (2.2) and taking logarithms, we get

$$\log N(M_c) / \log M_c = \log A / \log M_c - \gamma. \quad (4.1)$$

We also know that the mass of the core is a function of the core radius, R_c , so substituting this we get

$$\frac{\log N(R_c)}{\log 4/3\pi\rho R_c^3} = \frac{\log A}{\log 4/3\pi\rho R_c^3} - \gamma. \quad (4.2)$$

Taking limits as R_c tends to zero and since $4/3\pi\rho$ is a constant, we get

$$\lim_{R_c \rightarrow 0} \frac{N(R_c)}{\log 1/R_c} = 3\gamma. \quad (4.3)$$

From the definition of Kolmogorov (or Minkowski) dimension (Falconer 1997) given by

$$D = \limsup_{\epsilon \rightarrow 0} \frac{\log N(\epsilon)}{\log 1/\epsilon}, \quad (4.4)$$

where $N(\epsilon)$ is the minimum number of open balls of radius ϵ centred on points of a nonempty compact subset of a metric space X covering the set and ‘sup’ is the supremum (Datta 2003). Equation (4.4) is equivalent to the box dimension of the set or physical object, as in our case. Comparing Equations (4.3) and (4.4), we get

$$D \equiv 3\gamma. \quad (4.5)$$

Hence, the result.

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