THE QUALITATIVE EXPLANATION OF OBSERVED PECULIARITIES OF HECUBA AND HILDA ASTEROIDS DISTRIBUTION BY A COMMON INVESTIGATION.

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1. Introduction

Let us consider the planar case of the circular restricted three-body problem. The mass of central body is unit, the radius of the circular orbit of perturbing point P' (of mass $\mu = 1/1047.39$ according to Jupiter's mass) is also unit and the other unit is such that the gravitational constant is equal to 1. The Keplerian elements of the perturbed mass point P (asteroid), with semi-major axis $a < 1$, are given by usual notations; the elements of P' are indicated with a prime (').

It is supposed that the initial mean motions of P and P' satisfy to the condition:

$$[(k+1)n' - kn] \leq \mathcal{O}(\sqrt{\mu}) \tag{1}$$

that is, we have a first-order resonance.

In this case, the use of von Zeipel's method allows us to write the analytical solution in terms of Weierstrass functions [3,4]. In these two articles, the existence of three stationary solutions is shown, two of them, e_1 and e_2, being stable by Lyapunov. The dependence of these solutions with the first integral of our problem:

$$\gamma = a(k + 1 - k\sqrt{1 - e^2})^2, \tag{2}$$

is shown in Fig.1. The investigation of orbits near the stable solutions e_1 and e_2 allows us to obtain some interesting results (in this problem), which explain some regular circumstances of resonant regions in the asteroidal belt.

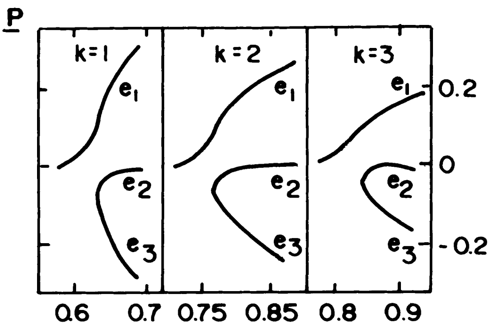

Fig. 1.

S. Ferraz-Mello (ed.), Chaos, Resonance and Collective Dynamical Phenomena in the Solar System, 139–144.

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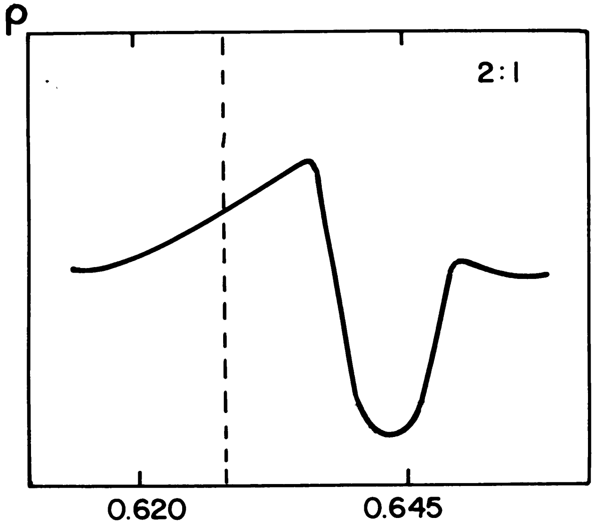
ρ
2:1
0.620
0.645

Fig. 2.

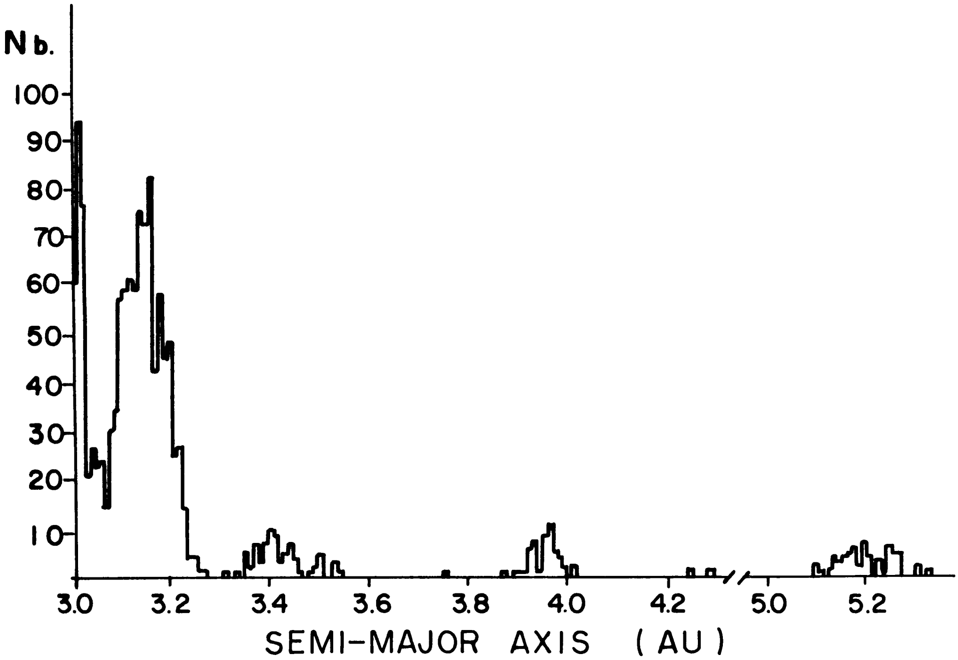
Nb.
100
90
80
70
60
50
40
30
20
10
3.0
3.2
3.4
3.6
3.8
4.0
4.2
5.0
5.2
SEMI-MAJOR AXIS (AU)

Fig. 3.

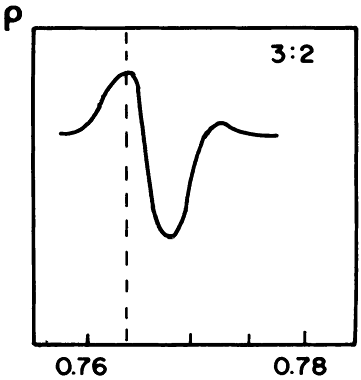

Fig. 4.

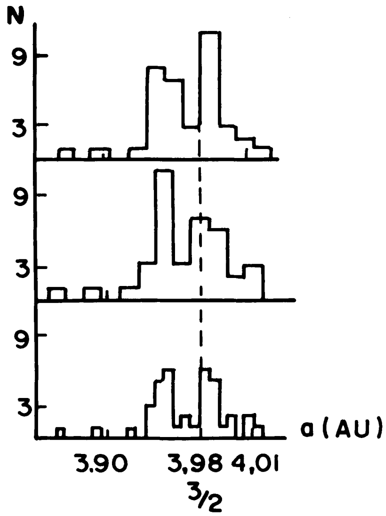

Fig. 5.

2. The evolution of mean motion and formation of gaps

Weierstrass functions are not necessary to define the motions near e_1 and e_2; one may use Jacobi functions [5]. With Jacobi functions the mean motion of P can be written

$$n = n_1 + s \operatorname{cn}(u) \tag{3}$$

where n_1 is the mean motion corresponding to the solutions e_1 and e_2, s a constant depending on the initial conditions and u a linear time function.

We have used equation 3 to model the following problem: one thousand particles with circular orbits distributed by angle variables and with semi-major axis satisfying to eqn. 1 for the resonance 2:1. From the calculations, we can see that, after a time interval of some hundreds revolutions of P', the relative density distribution curve of the particles by semi-major axis will become stable and as shown in Fig.2, where the dashed line gives the value of the exact resonance. If we compare to the real distribution of the asteroid semi-major axes, shown in Fig.3, it can be seen that the real gap has approximately the double of this width. Considering only qualitatively, it is noted that our simple model allows us to define the appearance of gap, with the presence of a larger number of asteroids in the inner side of the gap. In what concerns the depth of the gap, other conditions, as defined in section 3, are necessary.

Now, let us consider the resonance 3:2. Usually, Hilda's group corresponds to a 3:2 commensurability, in opposition to the gap corresponding to Hecuba's group. Let us consider this question in more details. In Fig.4, we show the curve of stationary distribution of density of these asteroids obtained with the method defined earlier. It's simple to see the presence of a gap; however, this gap has a small depth as in the case of 2:1 resonance. Now, let us consider the real distribution of Hilda's group of asteroids. In Fig.5, we show this distribution for different steps of histogram. As we expected, the gap appears really here. This result was obtained by us in 1987 [2]. The absence of other asteroids in this range was explained later [6]. Thus, peculiarities of Hecuba's and Hilda's groups of asteroids are not contradicted and can be explained by our model.

3. The Motion of the Apsidal Line of Resonant Asteroids.

The motion of the apsidal line of asteroids near stable stationary solutions can be represented following [3]:

$$\omega = \omega_0 + \omega_1 u + \omega_2 \frac{\Theta_4'(u)}{\Theta_4(u)} \tag{4}$$

where $\omega_0, \omega_1, \omega_2$ are constants depending on the initial conditions, Θ_4 is the Jacobi Theta-function, thus, the motion consists of a linear time-function and a periodic part.

On the upper part of Fig.6, we show the period of revolution of the apsidal line (in Jupiter's revolution units) for the resonance 2:1, with $e_1 = 0.1$, as a function of the difference of mean motions $\delta n = n - n_1$ (in seconds of arc; for Jupiter $n' = 300''$). As shown by a more complete analysis, near the solution e_1, the apsidal line evolves in opposite directions, then the speed of this rotation reduces and becomes zero, after what it changes to a positive value [4]. In the lower part of Fig.6 we show the amplitude of the periodic term of formula 4 (in radians).

One may pay attention to the small period of revolution of the apsidal line for orbits near e_1. In Fig.7, we show the relation of frequencies of resonant and non-resonant orbits corresponding to values of the eccentricities for the solution e_1 of the resonances 2:1 (curve 1), 3:2 (curve 2) and 4:3 (curve 3). From these dependences we can conclude that resonant orbits near stationary solutions have rotational speed of the apsidal line one or two orders larger and, since the possibility of collision of this asteroid with another is proportional to this speed, the time of existence of such asteroids is one or two orders smaller than the mean values. Therefore, this fact explains the absence of librational asteroids with eccentricities $e < 0,35$

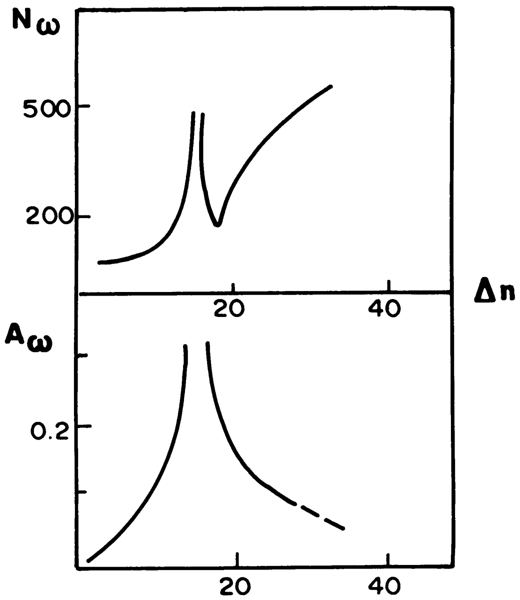

Fig. 6.

(just three asteroids of this group, Nos. 1921, 1922 and 1362, are librational; their eccentricities and inclinations with respect to Jupiter's plane are shown in Table 1). This circumstance is the cause of increasing the 2:1 gap width. The passage to the elliptic case, as we have shown earlier [1], do not change our results.

TABLE I
Librational asteroids of Hecuba's group.

	e	i(degrees)
1921	0.41	20
1922	0.36	23
1362	0.48	36

4. Conclusion.

It is possible to explain all peculiarities of the distribution of asteroids with commensurabilities 2:1 and 3:2 by a common investigation.

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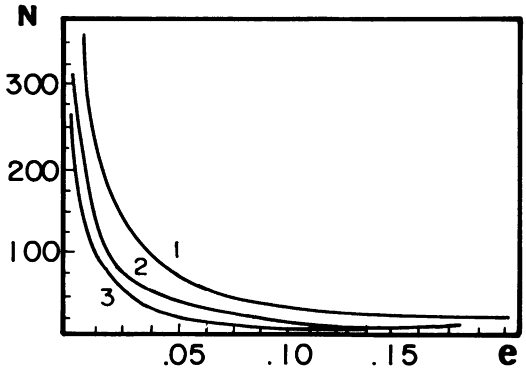

Fig. 7.

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Discussion

S.Ferraz-Mello – (1) I have also shown the actual existence of a statistical gap inside the Hilda's group (*Astronomical Journal* **96**, 400, 1988). However, the gap of the resonance 2:1 is not a statistical gap but one related with chaotic motions (the corresponding gap, in the 3:2 resonance, would be situated at the right of the Hilda's group and not inside it). (2) There is at least one librating asteroid in the resonance 2:1 with an eccentricity smaller than 0.35. It is (3789) Zhongguo (formerly known as 1928 UF) for which $e = 0.19$."

E.V. Alfimova (1) At first, the existence of the gap in Hilda's group has been shown by us in [2], in 1987. The gap in Hecuba's group can be explained by adding the stochastic effect and the disruption of librating asteroids from fast motion of the apsidal line. (2) By circular problem, we have found the librating asteroids 1362, 1921, 1922, 3688, 3789 (and apocentric librating asteroids 401 and 1121).

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