INTEGRAL MEANS OF UNIVALENT FUNCTIONS ON AN ANNULUS

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We study pointwise and integral analogues of the Schwarz lemma for holomorphic functions on an annulus.

Counterexamples reveal that a pointwise version of the Schwarz lemma for holomorphic functions on an annulus would not be possible. If holomorphic functions on an annulus are either not univalent or not normalised by having zero constant coefficient in the Laurent series expansion then the pointwise analogue of the Schwarz lemma fails in a more dramatic fashion.

We also examine an integral means version of the Schwarz lemma for univalent holomorphic functions on an annulus and its relation with normalisation and univalence of the functions under consideration. It turns out that an integral means version of the Schwarz lemma also fails for an annulus even when the functions are univalent and normalised. If either the univalence condition or the normalisation condition is dropped then the integral means version of the Schwarz lemma fails in a more dramatic way. Finally, we determine a partial analogue of the integral means version of the Schwarz lemma for univalent holomorphic functions on an annulus.

We also obtain sharp estimates for the integral means of certain classes of univalent functions that need not be bounded on an annulus and estimates for the integral means of derivatives of such functions.

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