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A SIMPLE MODEL TO CALCULATE THE COMPACTNESS OF ICE FLOES

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Abstract. This paper describes a simple model for predicting compactness over a given area. The model incorporates equations of continuity and momentum. The momentum equation is based on a balance between air–ice stress, water–ice stress, Coriolis force, and internal ice resistance. Since our interest is in short-range small-scale prediction, as a first step we have neglected sources and sinks terms in the continuity equation and assumed ice to be a film of Newtonian highly-viscous fluid. The solution of a simple equilibrium formulation is used as initial conditions.

Surface winds over the area of interest are obtained from an available model developed on the basis of similarity theory. The compactness model has been applied for a few real cases in summer 1975 over the Beaufort Sea area. The preliminary results of the model are encouraging. The main advantage of this model is that it can run on mini-computers available at most forecasting centres.

Résumé. *Un modèle simple pour calculer la densité de la distribution des glaces flottantes.* Ce papier décrit un modèle simple pour prévoir la densité de distribution des glaces flottantes dans une zone donnée. Le modèle comporte les équations de continuité et de quantité de mouvement. L'équation de quantité de mouvement est basée sur un bilan entre l'effort à l'interface air glace, l'effort à l'interface glace eau, la force de Coriolis et la résistance interne de la glace. Comme notre but est une prévision à court terme et à petite échelle nous avons négligé dans un premier temps la formation et la disposition des glaces flottantes dans l'équation de continuité et assimilé la glace à un film d'un fluide à haute viscosité Newtonienne. On utilise comme condition initiale la solution d'un modèle d'équilibre simple.

Les vents de surface sur la zone étudiée sont obtenus à partir d'un modèle disponible obtenu à partir d'une approche de la théorie de la similitude. Le modèle de densité de distribution a été appliqué dans quelques cas réels de l'été 1975 dans la région de la Mer de Beaufort. Les premiers résultats sont encourageants. Le principal avantage de ce modèle est qu'il peut passer sur de petits calculateurs disponibles dans beaucoup de centres de prévision.

Zusammenfassung. *Ein einfaches Modell zur Berechnung der Kompaktheit von Eisschollen.* Dieser Beitrag beschreibt ein einfaches Modell zur Vorhersage der Kompaktheit in einem begrenzten Gebiet. Das Modell verbindet die Kontinuitätsgleichung mit der Momentengleichung. Die Momentengleichung beruht auf dem Gleichgewicht der Kräfte zwischen Luft und Eis, zwischen Wasser und Eis, der Corioliskraft und der inneren Widerstandskraft des Eises. Da das Interesse auf die kurzfristige Vorhersage in kleinen Bereichen gerichtet ist, wurde in erster Annäherung der Ausdruck für Quellen und Senken in der Kontinuitätsgleichung vernachlässigt und das Eis als Film einer hochviskosen Newtonschen Flüssigkeit betrachtet. Die Lösung eines einfachen Gleichgewichtsmodells dient als Ausgangsbedingung.

Die Oberflächenwinde über dem interessierendem Gebiet werden aus einem verfügbaren Modell gewonnen, das auf der Grundlage der Ähnlichkeitstheorie entwickelt ist. Das Kompaktheitsmodell wurde auf einige wirkliche Fälle im Sommer 1975 über dem Gebiet der Beaufort-See angewandt. Die vorläufigen Ergebnisse sind ermutigend. Der Hauptvorteil dieses Modells liegt darin, dass es mit Minikomputern berechnet werden kann, wie sie auf den meisten Vorhersagezentren verfügbar sind.

1. Introduction

In order to predict the motion of ice floes, a short-range, small-scale dynamical model has been developed (Neralla and others, in press). In this model ice is considered to move under the action of five forces: the air–ice stress, the water–ice stress, the Coriolis force, the pressure-gradient force due to tilting of the sea surface, and the internal ice stress transmitted through the ice pack. The reasonable agreement of this model with satellite-derived ice-floe motions demonstrated the feasibility for adoption into the real-time computerized prediction.

In the formulation of ice-dynamics problems, several investigators consider ice as an elastic–plastic material (e.g. Coon and others, 1974) or viscous–plastic continuum (e.g. Hibler, in press). Following Campbell (1965), we treated ice as a film of Newtonian highly viscous fluid. In order to arrive at a realistic result we have emphasized the importance of incorporating the variable compactness (fraction of area covered by ice) in the internal ice-stress formulation (Neralla and others, 1977). The aim of this study is to present a simple model to calculate the compactness over a given area.

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Nikiforov and others (1967) have used the equation of conservation of mass to study the compactness. The most satisfactory theory incorporating the equation of motion and the equation for conservation of mass has been studied by Doronin (1970) for summer ice conditions in the Kara Sea. Except in the treatment of air and water stresses, the approach in this study is similar to Doronin's formulation.

Doronin has also considered sources and sinks terms in the mass equation by taking into account the thermodynamic processes. In our study, we have neglected thermodynamic processes by assuming that mass changes over a period of a few days are small. However, we plan to incorporate them in our future development. In the Arctic Ice Dynamics Joint Experiment (AIDJEX) model (Coon and others, 1974) the mass conservation is obtained by an ice-distribution function.

The study of compactness has several important applications in the Arctic environment. An accurate knowledge of compactness is desirable for more economical and safer navigation and also in the offshore drilling areas, and hence a useful variable in any real-time environmental forecast procedures. In the prediction of an ice-pack front (defined as an edge of a large area of floating ice driven closely together) the internal ice resistance is an important stress which is related to the compactness. A knowledge of compactness is also useful in the vertical heat-exchange computations. The presence of a large concentration of floes inhibits the free movement of icebergs. Hence the study of compactness is important in iceberg grounding problems.

A model for calculating the compactness is discussed in Section 2. This model is applied over the Beaufort Sea area and the results for a few cases are presented in Section 3. Section 4 deals with summary and conclusions of this study.

2. Theory

2.1. *General considerations*

In general the compactness C of ice floes obeys

$$\frac{\partial C}{\partial t} = -\nabla\cdot(C\mathbf{V}_i)+S_i, \tag{1}$$

where $\mathbf{V}_i$ is the ice drift and S_i is the sum of the sources and sinks in the given area (e.g. freezing, melting, or precipitation deposited on the ice), and t is the time. Concentration C_N is related to compactness by

$$C_N = \rho_i h_i C, \tag{2}$$

where ρ_i is the density of ice and h_i is the thickness of ice.

The momentum equation for ice floes (Neralla and others, in press) is given by

$$\rho_i h_i \frac{d\mathbf{V}_i}{dt} = \boldsymbol{\tau}_{ai}+\boldsymbol{\tau}_{wi}+\mathbf{C}_F+\mathbf{P}_G+\mathbf{R}, \tag{3}$$

where $\boldsymbol{\tau}_{ai}$ is the air–ice stress, $\boldsymbol{\tau}_{wi}$ is the water–ice stress, $\mathbf{C}_F$ is the Coriolis force, $\mathbf{P}_G$ is the pressure-gradient force due to tilting of the sea surface, and $\mathbf{R}$ is the internal ice resistance. Since the area under study is small, we have neglected $\mathbf{P}_G$ in Equation (3). Figure 1 illustrates the arrangement of velocities and forces included in the model. For equilibrium, drift equation (3) has been solved by Neralla and others (in press).

The internal ice resistance is expressed as

$$\mathbf{R} = \rho_i h_i \nabla\cdot(K\nabla\mathbf{V}_i), \tag{4}$$

where K is the horizontal kinematic eddy-viscosity coefficient for ice floes. Campbell (1965) assumed K as constant while Doronin (1970) assumed a realistic form, $K = K_H C$, where K_H is a constant value. Doronin's relation for K is used here.

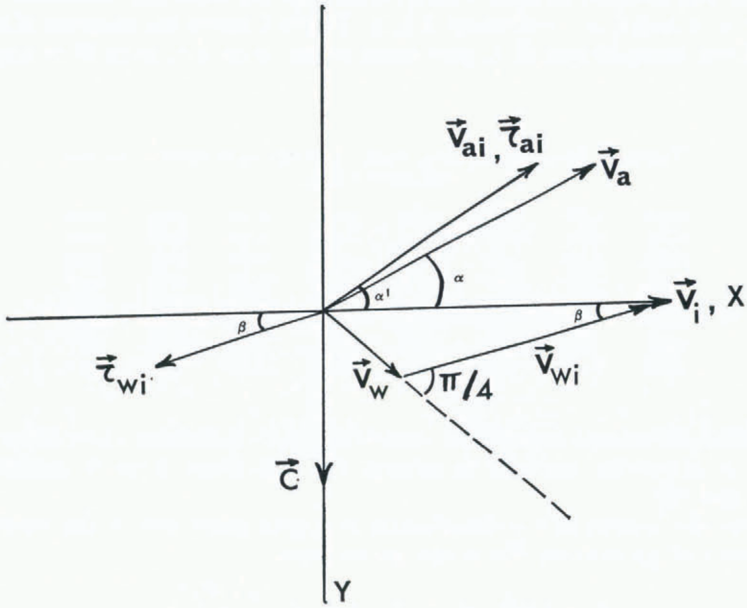

Fig. 1. Diagram of force and velocity vectors.

For a single ice floe the internal ice resistance is zero and the momentum equation for equilibrium drift becomes

$$\boldsymbol{\tau}_{\mathrm{ai}}+\boldsymbol{\tau}_{\mathrm{wi}}+\mathbf{C}_{\mathrm{F}} = 0, \tag{5}$$

which is the equation studied by Shuleykin (1938) and Reed and Campbell (1962). Shuleykin obtained water currents from the empirical expression of Ekman (1905). Reed and Campbell obtained water currents from the requirements of continuity of mixing lengths and the eddy viscosity at the interface between the boundary and the spiral layers. The solution of Equation (5) is obtained as discussed in detail in Neralla and others (1977). The difference between our method and Reed and Campbell's method is that the latter assumed $\mathbf{V}_{\mathrm{ai}} = \mathbf{V}_{\mathrm{a}}$.

2.2. *Compactness of ice floes*

The equations for compactness and momentum in the component form become

$$\frac{\partial C}{\partial t} = -\left(\frac{\partial}{\partial x}\,(CV_{\mathrm{i}}{}^{x})+\frac{\partial}{\partial y}\,(CV_{\mathrm{i}}{}^{y})\right), \tag{6}$$

$$\frac{\partial}{\partial x}\left(C\,\frac{\partial V_{\mathrm{i}}{}^{x}}{\partial x}\right)+\frac{\partial}{\partial y}\left(C\,\frac{\partial V_{\mathrm{i}}{}^{x}}{\partial y}\right) = -\frac{1}{\rho_{\mathrm{i}}h_{\mathrm{i}}K_{\mathrm{H}}}\,(\tau_{\mathrm{ai}}{}^{x}+\tau_{\mathrm{wi}}{}^{x}+C_{\mathrm{F}}{}^{x}), \tag{7}$$

$$\frac{\partial}{\partial x}\left(C\,\frac{\partial V_{\mathrm{i}}{}^{y}}{\partial x}\right)+\frac{\partial}{\partial y}\left(C\,\frac{\partial V_{\mathrm{i}}{}^{y}}{\partial y}\right) = -\frac{1}{\rho_{\mathrm{i}}h_{\mathrm{i}}K_{\mathrm{H}}}\,(\tau_{\mathrm{ai}}{}^{y}+\tau_{\mathrm{wi}}{}^{y}+C_{\mathrm{F}}{}^{y}), \tag{8}$$

where the superscripts x and y denote the x- and y-components.

The solutions of Reed and Campbell's model (see Equation (5), Section 2.1) as modified by Neralla and others (1977) are used as initial conditions. No slip boundary condition for $\mathbf{V}_{\mathrm{i}}$ and no diffusive boundary condition for C are used.

If S denotes the fraction of land area of a grid point, then the maximum value of C is $C = 1$ when $S = 0$ and $C = 1-S$ when $S > 0$. Table I shows the fraction of land area of a grid point. If the compactness at a grid point is less than 0.1, then $\mathbf{R}$ is neglected in the computations.

TABLE I. FRACTION OF LAND AREA AT EVERY GRID POINT OVER THE BEAUFORT SEA

0.20	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.20	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.60	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.70	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.70	0.00	0.00	0.00	0.00	0.00	0.25	0.00
1.00	0.70	0.20	0.00	0.00	0.70	1.00	0.40
1.00	1.00	1.00	0.30	0.10	1.00	1.00	0.60
1.00	1.00	1.00	0.80	0.20	0.10	1.00	1.00

The modified Liebmann successive over-relaxation technique (Carnahan and others, [c1969]) with a projection method to control the non-linear form is applied for solving Equations (7) and (8).

Let (l, m) be the indices of x, y coordinates at a grid point and N the iteration number. The x-component of ice velocity $V_i{}^x$ is calculated from

$$V_i{}^{x,\,N+1}(l, m) = V_i{}^{x,\,N}(l, m) - \frac{\omega}{4}\frac{R_x{}^N}{C(l, m)}, \tag{9}$$

where

$$R_x{}^N = \alpha_1 V_i{}^{x,\,N}(l-1, m) + \alpha_2 V_i{}^{x,\,N}(l+1, m) + \alpha_3 V_i{}^{x,\,N}(l, m-1) + \\ + \alpha_4 V_i{}^{x,\,N}(l, m+1) - 4C(l, m) V_i{}^{x,\,N}(l, m) + Q_x{}^{N+\frac{1}{2}}(l, m), \tag{10}$$

$$\alpha_1 = C(l, m) - \frac{C(l+1, m) - C(l-1, m)}{2}, \tag{11}$$

$$\alpha_2 = C(l, m) + \frac{C(l+1, m) - C(l-1, m)}{2}, \tag{12}$$

$$\alpha_3 = C(l, m) + \frac{C(l, m+1) - C(l, m-1)}{2}, \tag{13}$$

$$\alpha_4 = C(l, m) - \frac{C(l, m+1) - C(l, m-1)}{2}, \tag{14}$$

$$Q_x{}^{N+\frac{1}{2}}(l, m) = -\frac{\Delta x^2}{\rho_i h_i K_H}(\tau_{ai}{}^{x,\,N+\frac{1}{2}} + \tau_{wi}{}^{x,\,N+\frac{1}{2}} + C_F{}^{x,\,N+\frac{1}{2}}), \tag{15}$$

$$\Delta x = \Delta y,$$

ω is the relaxation factor and $\tau_{ai}{}^{x,\,N+\frac{1}{2}}$, $\tau_{wi}{}^{x,\,N+\frac{1}{2}}$, and $C_F{}^{x,\,N+\frac{1}{2}}$ are evaluated by using $V_i{}^{x,\,N+\frac{1}{2}}$ as

$$V_i{}^{x,\,N+\frac{1}{2}} = \tfrac{1}{2}(V_i{}^{x,\,N+1} + V_i{}^{x,\,N}). \tag{16}$$

The y-component of ice velocity, $V_i{}^y$ is calculated in a similar way.

3. RESULTS

Figure 2 shows the variation of ice drift and deviation angles with wind speed for two sets of air–ice drag coefficients and ice thicknesses representing smooth and thin ice ($C_d{}^a = 1.4\times10^{-3}$ and $h_i = 2$ m) and rough and thick ice ($C_d{}^a = 2.6\times10^{-3}$ and $h_i = 5$ m) (Banke and Smith, 1971). The following values are used for other constants: $C_d{}^w = 3.8\times10^{-3}$,

$\omega = 7.29 \times 10^{-5}\ s^{-1}$, $\rho_i = 0.9\ g\ cm^{-3}$, $\rho_a = 1.29 \times 10^{-3}\ g\ cm^{-3}$, $\rho_w = 1.03\ g\ cm^{-3}$. The ice drifts are in reasonable agreement with other studies (e.g. Shuleykin, 1938; Reed and Campbell, 1962).

The present model is applied for five different cases over the Beaufort Sea area (Fig. 3). It is to be noted here that ice floes are considered to be the rough and thick category (i.e. $C_d^a = 2.6 \times 10^{-3}$, and $h_i = 5$ m). The value used for K_H in this study is $3.3 \times 10^{13}\ g\ s^{-1}$.

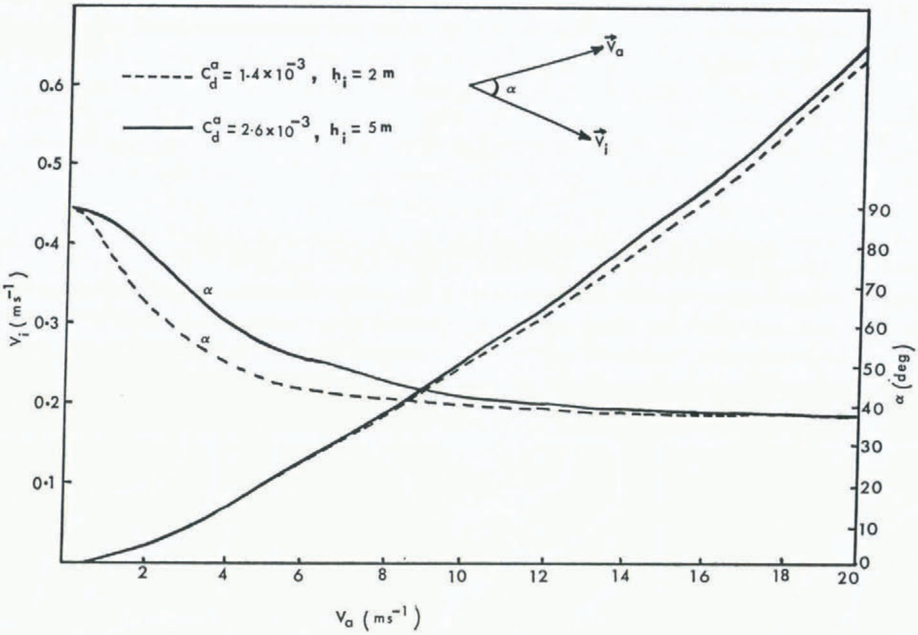

Fig. 2. Variation of ice speed V_i and deviation α with wind speed for two sets of air–ice drag coefficients and ice thicknesses.

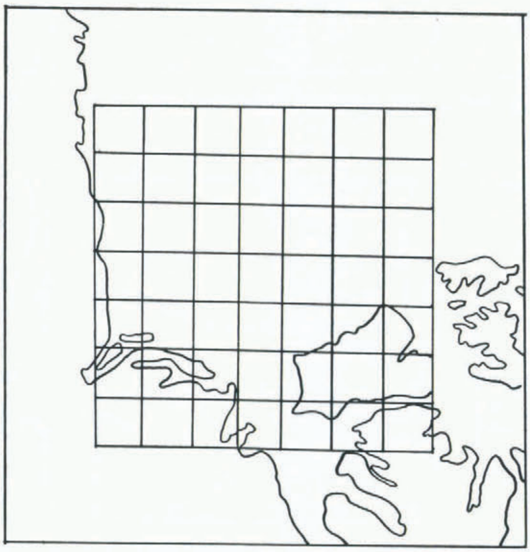

Fig. 3. Diagram showing location of 8 × 8 grid array in the Beaufort Sea.

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Based on the similarity theory approach, Agnew (1977) developed a model to diagnose surface winds over a grid (Fig. 3). The grid distance in our study is 127 km. We have used this model to obtain surface winds at every 24 h for the period of our interest.

The observed information on ice compactness was obtained from the subjectively prepared daily ice charts at the Ice Forecasting Central, Ottawa. The required grid-point data are hand-abstracted from these charts.

Five cases have been selected during the period of July and August 1975. After every time step (24 h) new winds are read in. The integrations have been carried out for a period of 5 d. The predicted compactness is verified with observed hand-abstracted data.

Figures 4 to 8 show, for each case, the initial, predicted, and observed values of compactness. With a steady large-scale flow over the area of interest, the agreement of model predictions with observations for cases in late July (Figs 4 and 5) is reasonable. The predicted distribution of compactness (in particular, 0.5 isopleth) in Figures 4 to 7 agrees well with the observed distribution. The model performance for the period 24–29 August 1975 (Fig. 8) is only marginal, presumably due to abrupt changes in winds caused by a fast-moving intense weather system.

Although the predictions agree reasonably well with observations, it would be more realistic to incorporate the sources and sinks term in the continuity equation for compactness. However, the present results from the simple formulation demonstrate the applicability of the technique for short-term forecasting of compactness over a small area.

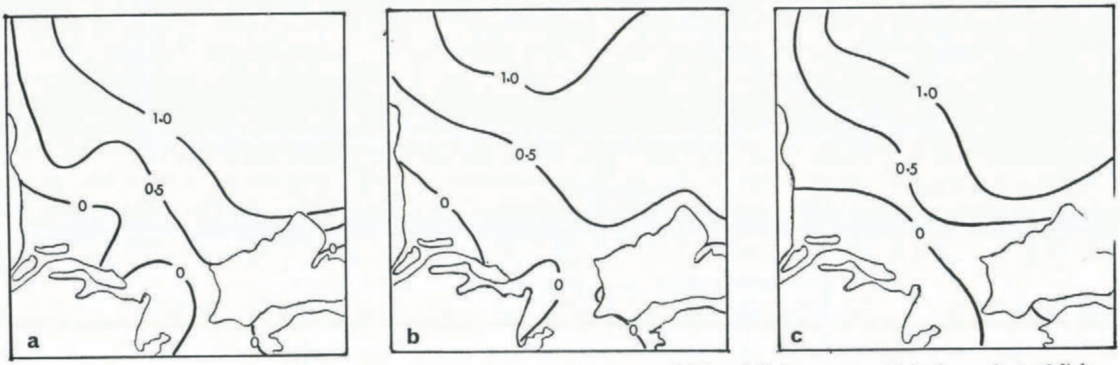

Fig. 4. Compactness (a) at initial time, 21 July 1975, (b) 5 d forecast valid for 26 July 1975, and (c) observed at 26 July 1975.

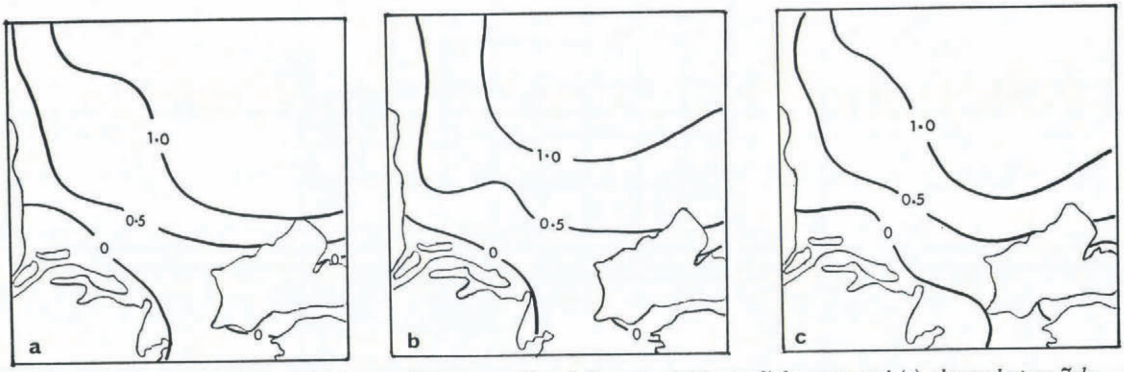

Fig. 5. Compactness (a) at initial time, 24 July 1975, (b) 5 d forecast valid for 29 July 1975, and (c) observed at 29 July 1975.

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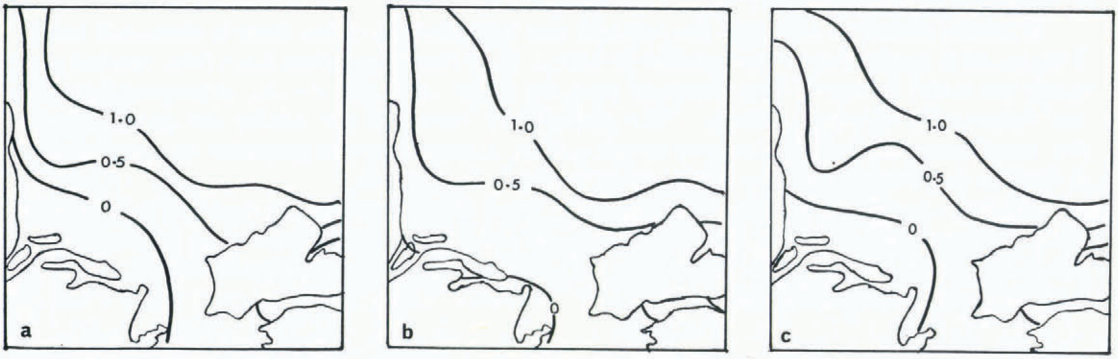

Fig. 6. Compactness (a) at initial time, 15 August 1975, (b) 5 d forecast valid for 20 August 1975, and (c) observed at 20 August 1975.

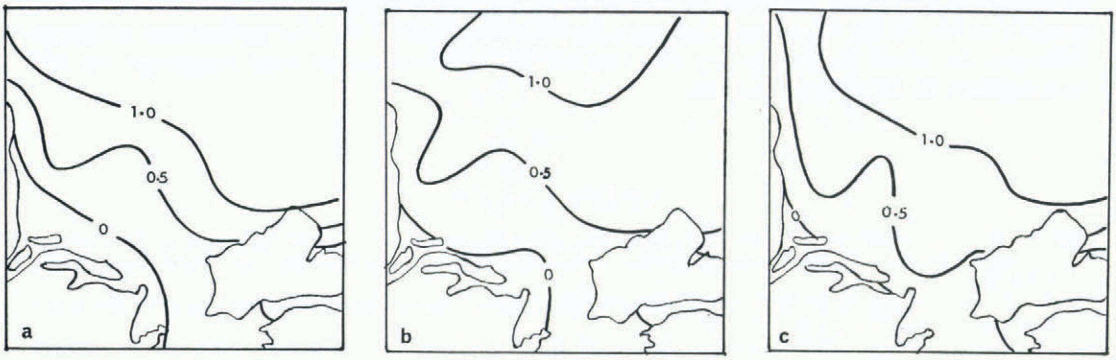

Fig. 7. Compactness (a) at initial time, 19 August 1975, (b) 5 d forecast valid for 24 August 1975, and (c) observed at 24 August 1975.

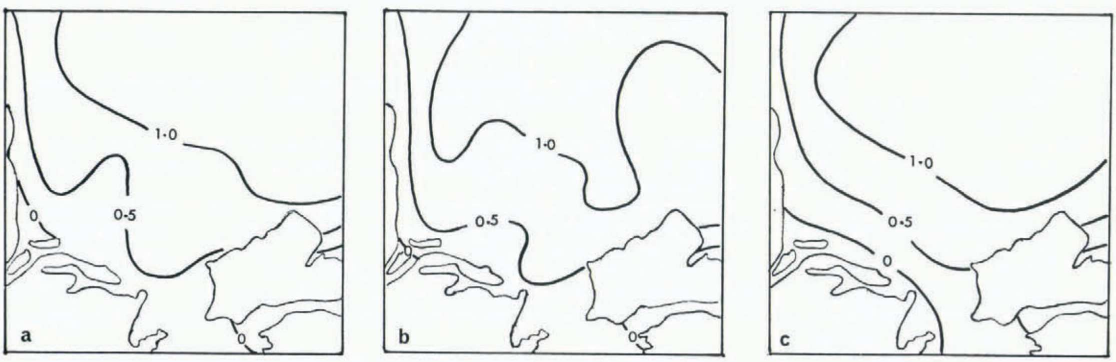

Fig. 8. Compactness (a) at initial time, 24 August 1975, (b) 5 d forecast valid for 29 August 1975, and (c) observed at 29 August 1975.

4. Summary and conclusions

A simple model to calculate the ice drift and compactness of ice floes is presented here. This model is applied over the Beaufort Sea area. Five-day integrations have been carried out using the initial ice-cover data obtained from the daily composite ice charts prepared at the Ice Forecasting Central, Ottawa. The preliminary results of the model are encouraging. For computing compactness over a longer period, thermodynamic processes are important. A further development incorporating these processes is planned.

Since the model is wind driven, reasonably good wind forecasts are required to improve the model performance. The larger grid distance used here may obscure some of the phenomena such as cracks and leads. Reducing the grid size may improve forecasts and also help to explain cracks or narrow leads.

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