CORRECTION TO "AN EXTENSION OF MEYER'S THEOREM ON INDEFINITE TERNARY QUADRATIC FORMS"

B. W. JONES

Mr. G. L. Watson of University College, London showed, by a counter-example, the falsity of Theorem 6 in my paper entitled "An extension of Meyer's Theorem on Indefinite Ternary Quadratic Forms" which appeared in this Journal, vol. 4 (1952), pp. 120–128. The error in Theorem 6 stemmed from an error in Theorem 2 which should read as follows:

THEOREM 2. *If the form f above is properly primitive, if $(\Omega, \Delta) = p$, $\Omega \not\equiv 0 \not\equiv \Delta \pmod 4$ and if p^3 does not divide $|g|$, and if there is an integer q prime to p and satisfying the following conditions:*

(i) *q is an odd prime or double an odd prime,*

(ii) *$-\Omega q$ is represented by the reciprocal form of g,*

(iii) *every solution of the congruence*

$$x^2 - qy^2 \equiv 1 \pmod p \tag{3}$$

is congruent (mod p) *to a solution of the Pell equation*

$$x^2 - qy^2 = 1, \tag{4}$$

then the form f is in a genus of one class.

The condition that f be properly primitive is probably not essential. The essential correction is in condition (ii) for which $(\Omega, \Delta) = p$ is necessary.

First we deal with Theorem 2 as altered and then indicate the other necessary changes in the paper. The purpose of condition (ii) is to show that G, the matrix of g, can be written in the form

$$\begin{bmatrix} \Omega B & \Omega b_1 \\ \Omega b_1^T & b \end{bmatrix}, \text{ replacing } \begin{bmatrix} pB & pb_1 \\ pb_1^T & b \end{bmatrix}$$

of page 122.

To show that the new condition (ii) assures this, notice first that the invariants Ω and Δ/p of G are relatively prime by the conditions of the theorem, and f properly primitive implies [**2**, Theorem 41] that we may consider

$$g \equiv \Omega a x^2 + \Omega b y^2 + (\Delta/p) c z^2 \pmod{8\Omega^4\Delta^2p^k}.$$

Hence the reciprocal form g_0 of g is congruent to

$$bc(\Delta/p)x^2 + ac(\Delta/p)y^2 + \Omega ab z^2.$$

Thus if g represents a binary form $\Omega\phi$ where $|\phi| = -q$, its reciprocal form represents $-q\Omega$ and conversely.

https://doi.org/10.4153/CJM-1953-030-6 Published online by Cambridge University Press

Lemmas 1, 2, 3 are proved without any but trivial alterations, which establish Theorem 2 as stated above since Theorem 1 shows that g is in a genus of one class. To Corollary 1 should be added the condition:

$$f \text{ properly primitive and } (\Omega, \Delta) = p.$$

To make Theorem 4 apply to the new situation we must add the condition that the reciprocal form of g represents $-\Omega q$ in all $R(r)$ for prime divisors, r, of Ω. It is not hard to see [**2**, Theorem 34 and the remark] that this condition is, in terms of the form g given above,

$$c_r(g_0) = (-|g_0|, \Omega q)_r, \text{ whenever } \Omega q\, |g_0| \text{ is a square in } R(r)$$

[**2**, Corollary 14], c_r being the Hasse symbol. Since $|g_0| = \Omega(\Delta/p)^2$, the condition reduces to $c_r(g_0) = (q|r)$ whenever r occurs in Ω to an odd power and q is a square in $R(r)$, that is,

(iv) $c_r(g_0) = 1$ if $(q|r) = 1$ and r occurs in Ω to an odd power.

Then Theorem 4, to apply to our situation, should read as follows:

THEOREM 4. *Let p be a fixed odd prime and f a properly primitive quadratic form for which $(\Omega, \Delta) = p$, neither Ω nor Δ being divisible by 4 or p^2, and g its p-related form. Then the reciprocal form of g represents $-\Omega q$ if and only if it represents it in $R(r)$ for all prime divisors of $2\Delta/p$ and* (iv) *holds for each r occurring to an odd power in Ω.*

Condition (iv) then must be added to Theorem 5.

Since, in Theorem 6, $p = 3$, $(q|3) = 1$, we must add the condition:

$$c_r(g_0) = 1$$

or its equivalent above for all r occurring in Ω to an odd power.

In the two examples given in my paper, this condition holds and the result stated is correct. The example given by Mr. Watson,

$$f = 2x^2 + 12y^2 + 6yz + 12z^2,$$

is, in the first place, improperly primitive and hence barred by the above discussion, but it has been indicated that this restriction is probably not essential. However, the form g_0 for this form is

$$5x^2 + 8y^2 - 4yz + 8z^2$$

and here $c_3(g_0) = -1$. Hence the essential condition is not met.

University of Colorado

https://doi.org/10.4153/CJM-1953-030-6 Published online by Cambridge University Press