GEORGE DAVID BIRKHOFF.

It is impossible in the small space here available to comment fully on the work of one who has occupied for many years the leading position among American mathematicians and whose contributions to the development of modern mathematics have been of such width and depth as those of G. D. Birkhoff. He published about 130 papers, the first in 1904 at the age of 20, and several posthumously since his sudden death on Nov. 12th, 1944. Full accounts of his researches have appeared in other journals . Only a few of his achievements will, therefore, be noted here; they are typical of his work, which was always characterised by his habitual effort to push his investigations to the limits of generality, and by his delight in harmonious unification. They exemplify also the extent to which he was a disciple of Poincaré, taking over both his problems and his techniques, and like Poincaré dividing his interests between pure and applied mathematics.

Birkhoff's work as an analyst included many studies on linear differential, difference and q-difference equations and systems of such equations. A typical result (1913) was his extension to such systems of the Riemann problem of constructing a linear differential equation with assigned singularities; a by product of this work was a theorem on matrices of analytic functions, from which he was led (1916) to the generalisation for such matrices of the classical theory of the representation of an analytic function as an infinite product.

Throughout his life, Birkhoff was deeply interested in dynamics. The great aim which inspired much of his work in this field was his desire to obtain a reduction of the most general dynamical system to a normal form, from which a complete qualitative characterisation of the system could be inferred. Towards this he found many theorems of great value and generality. In the case of systems with two degrees of freedom he came near to achieving the aim, embodying the results in a memoir (1917) which was awarded the American Mathematical Society's newly established Bôcher Memorial Prize (1923), and on which Birkhoff is said to have remarked that it was as good a piece of work as he was ever likely to do. Two other celebrated achievements were his proof (1913) of Poincaré's topological "last geometrical theorem," with corollaries in the theory of orbits, and his proof (1931) of the ergodic theorem. The greater part of his work on dynamical theory was expounded in a book *Dynamical Systems* (1927).

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Birkhoff made many contributions to the discussion of the theory of relativity, publishing two characteristically original books (*Relativity and Modern Physics*, 1923; *The Origin, Nature and Influence of Relativity*, 1925). His point of view depended less than the usual on physical intuition and involved a maximum appeal to mathematical symmetry and simplicity. He was engaged at the time of his death on the new theory of matter, electricity and gravitation which he put forward in 1943. Unlike Einstein's theory, it was based on flat space-time and involved a gravitational tensor potential governed by a linear differential equation.

Birkhoff was intrigued by the problem of analysing the essentials of artistic and musical form, and his treatise, *Aesthetic Measure* (1933), set out to do this mathematically. The basic idea was that the value or *aesthetic measure* of a work of art in its formal aspects is directly proportional to its *order* (depending on the harmonious interrelation of its parts), and inversely proportional to its *complexity*. As members of the St Andrews Mathematical Colloquium of 1938 will remember, he was able to expound and illustrate his ideas on this subject with great fascination even for those who remained sceptical of their validity.

The " four colour " problem may be mentioned as one of Birkhoff's sidelines. A paper on it was communicated to this Society and published in the *Proceedings* (1930). He introduced " chromatic polynomials " $P(x)$ equal to the number of ways in which a given map can be coloured in x colours. Although the main objective of showing that $P(4) \geqq 1$ was not achieved, many properties of $P(x)$ were obtained.

He lectured to the Edinburgh Mathematical Society at its St Andrews Colloquia of 1926 and 1938 on " The significance of dynamics for scientific thought," " Analytic deformations and autoequivalent functions," and " The mathematical theory of art." Many members will recall his commanding figure and courteous friendliness. He was elected an Honorary Member of the Society (1926) and an Honorary Fellow of the Royal Society of Edinburgh (1943), received the honorary degree of LL.D. at St Andrews (1938), and was similarly honoured by many other societies and academies throughout the world. His death at the height of his creative power was a great and sudden loss. His son, Garrett Birkhoff, is already well known as a versatile and accomplished mathematician.

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