PROPER ELEMENTS OF THE ASTEROIDS

A SEMI - ANALYTICAL METHOD

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Abstract. We propose here the first results of the semi analytical method of Henrard(1990) applied to the calculation of proper elements for the asteroids.

Key words: Proper Elements-Asteroids-Semi analytical Method

1. Introduction

The asteroidal belt is still today a natural laboratory where small planets are moving on orbits not really easy to calculate with the usual tools of the perturbation theory. They could be fragments of larger bodies after several violent collisions , but the motion of those parent bodies is very uncertain and not well defined. A way of answering these questions is the reconstitution of asteroids families i.e. asteroids presenting the same dynamical or chemical properties, which can be all "children" of the same initial body. A way of tackling the problem is to find invariants of motion for each planet and to regroup the "close enough" candidates in a same family.

The interesting point for us is the calculation of these invariants of motion for each asteroid, also called Proper Elements of the planet. A complete calculation of the proper elements is quite impossible because of the resonance problems that we encounter. A partial set of proper elements can be calculated for the main families; the interest is not only the history of the Solar System but also the understanding of the motions of asteroids in some particuliar regions. even if we cannot give a precise proper eccentricity or a proper inclination for a specific asteroid the behavior is much better known after the dynamical study performed in those research fields.

2. Principles

We would like to give here the main ideas concerning the calculation of proper elements. We start with the restricted three body problem Sun - Main Planet - Asteroid. The hamiltonian function, which we call H, is then dependent on the semi major axis, a, the eccentricity, e, the inclination , i, the longitude of the pericenter, ϖ, the longitude of the ascending node, Ω and the mean longitude λ and on the corresponding primed elements for the

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perturbing main planet. The first step consists in averaging the hamiltonian function over the short periodic terms i.e. terms with periods of a few years, corresponding to the periods of rotation of the planets around the Sun. It is obvious that this step should be performed in the regions of the phase space not too close to a mean motion resonance. The elliptic elements after averaging are called the mean elements. The mean hamiltonian function $\bar{H}$ is, after that, independent of the mean mean longitude and the mean semi major axis is then a constant. The mean elements which are still varying are then $\bar{e}$, $\bar{i}$, $\bar{\varpi}$ and $\bar{\Omega}$. The second step is the averaging over the long periods i.e. periods of thousands of years of ϖ and Ω with the problem of the secular resonances. The resulting elements after the two averages are called the Proper Elements and the doubly averaged Hamiltonian is only function of $\bar{\bar{e}}$ and $\bar{\bar{i}}$. So we define the classical proper elements : $\bar{a}$, $\bar{\bar{e}}$ and $\bar{\bar{i}}$. The basic frequencies of the problem i.e. the derivatives with respect to time of $\bar{\bar{\varpi}}$ and of $\bar{\bar{\Omega}}$ are given by the first derivatives of the doubly averaged Hamiltonian.

This scheme is very general and different approaches exist. Their differences come either from the choice of the small parameter to perform the second averaging by a first order perturbation theory , or from the way, numerical or analytical, the averaging steps are calculated.

3. Main contributions

Two important contributions are to be mentioned : this of Williams in 1969 and the combined one of Yuasa (1973), Knezevic (1988 - 1989) and Milani and Knezevic (1990). The second one is purely analytical, based on a power series in eccentricity and inclination of the Hamiltonian up to the degree 4. Let us remind that each averaging step needs to choose a small parameter ; in the short periodic case, we start with :

$$H = H_0 + \epsilon H_1 \tag{1}$$

where ϵ is the mass of the disturbing planet. After the averaging process we end up with :

$$\bar{H} = \bar{H}_0 + \epsilon \bar{H}_1 + \epsilon^2 \bar{H}_2 \tag{2}$$

which we divide again into two parts with a second small parameter, μ :

$$\bar{H} = K_0 + \mu K_1 \tag{3}$$

where K_0 is integrable. In the analytical theory of Yuasa, Knezevic and Milani, this second parameter is smaller than e^2 or $\sin^2 i$ or ee' (which they keep in the H_0 term) i.e. that they keep in K_0 all the terms in eccentricities and inclinations up to degree 2; so, there is no difference, in order of magnitude, between the values of the eccentricities of the main planets and of the asteroids. After a time dependent translation, they get a K_0 independent of the angles and then, integrable.

The elimination of the short periodic terms is performed by a second order perturbation theory up to degree 4 in the asteroids and planets elements and the averaging over the long periodic terms by a first order perturbation method. Yuasa uses Hori's formalism and works in the invariant plane; Knezevic rewrites the same results in the heliocentric coordinates (adding the indirect part of the disturbing function for the calculation of the mean elements). Milani and Knezevic (1990) use the Lie formalism and add an iterative process to evaluate with more precision the two basic frequencies of the problem.

At the opposite the work of Williams (1969) is mainly numerical. He does not use any series development in e and $\sin i$ but he does not take into account perturbation terms smaller than e' or $\sin i'$. The parameter μ of the long periodic averaging process is chosen as e' or $\sin i'$. So there is an obvious difference of order of magnitude between the main planets and the asteroids elements. So his work is of degree ∞ in the asteroids elements and of degree 1 in the main planets elements and this is a first order pertubation theory because there is no correction in m'^2 after the first averaging. There is no iterative process to correct the frequencies.

4. The semi numerical perturbation method

Our contribution is in between the other two precited ones. We are not interested in the first averaging process and we start with the $\bar{H}$ as given by Yuasa (1973) and corrected by Knezevic (1988 - 1989) but we use the same parameter μ as Williams i.e. that we let in K_1 all the terms proportional to e' or $\sin i'$. The idea under this subdivision is obviously to calculate proper elements especially for asteroids with large eccentricities or inclinations. We start with :

$$\bar{H} = K_0(e,i,\omega) + \mu K_1(e,i,\omega,\Omega) \tag{4}$$

It means that, see Kozai(1985), thanks to that division, the first part of the hamiltonian function is integrable. The only angle is ω, the argument of the pericenter.

Using then canonical variables defined as :

$$q = \omega \tag{5}$$

$$p = -\omega - \Omega \tag{6}$$

$$Q = G - H \tag{7}$$

$$P = L - H \tag{8}$$

where L, G and H are Delaunay classical momenta, we end up with :

$$\bar{H} = K_0(P,Q,q) + \mu K_1(P,Q,p,q) \tag{9}$$

which is exactly the required form to start the semi numerical first order perturbation method of Henrard (1990).

We calculate the proper elements as the mean elements plus the corrections expressed in the Lie formalism as the first derivatives of the generator of the transformation. All the corrections are evaluated by numerical integrations along the corresponding trajectories defined by K_0. We repeat the operation a few times to get the correct proper elements i.e. to integrate on the correct torus (defined by the two basic frequencies of the problem) which is comparable with the iterative process introduced by Milani and Knezevic.

5. Results

We have first to mention here the difficulties present in this kind of problem to compare different sets of proper elements calculated by different methods. First of all, the models are not the same and in some particular regions of the phase space this is a very important choice. Secondly, Williams 's process is not iterative so we have to compare our first iteration with his final result. Thirdly when we end up the calculation, we have action angle proper elements and we have to convert these results in proper eccentricity and proper inclination. The way of performing this transformation is very different in the three theories : Williams chooses the minimal value of the eccentricity (i.e. the maximal value of the inclination), Milani and Knezevic give a kind of averaged proper eccentricity and averaged proper inclination, and we would like to introduce a third calculation based on the topology induced by K_0, the maximal eccentricity and the minimal inclination. With this third definition, all the orbits can be treated in the same way, because this calculation coincides with the intersection of the orbits with the axis y where $y = \sqrt(2Q) \sin q$. All the orbits, inside or outside the critical curves of the K_0 Hamiltonian, cross the y axis while the resonant ones, inside the critical curve, do not cross the x axis where $x = \sqrt(2Q) \cos q$.

In the good regions (not too close to a mean motion resonance, not too close to a secular resonance, characterized by small values of the eccentricities and the inclinations) the three theories gather and give similar results for $\bar{\bar{e}}$ and $\bar{\bar{i}}$ as well as for the two basic frequencies. Elsewhere, there is no possible comparison.

All the results of our method are going to be summarized in a paper (Lemaitre - Morbidelli (1992)) actually in preparation. A second part of the work will be present in the same paper, using the same semi numerical perturbation method of Henrard (1990) but applied to a K_0 and a K_1 in a close form in eccentricity and inclination of the asteroid, obtained by interpolation on a grid numerically calculated. The set of proper elements calculated by this last version (including the m'^2 terms and the iterative process) should be the most fiable for asteroids with large inclinations.

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Discussion

H.Kinoshita – Do you have a plan to reclassify asteroidal family with use of the proper elements that are derived from your new methods?

A.Lemaître – No.

Y.Kozai – Yuasa made a mistake in formulating his theory. Namely, he did not include indirect part of the disturbing function. To compute secular motions according to his theory, this must be taken into account and this can de done very easily.

A.Lemaître – Zoran Knežević checked Yuasa's calculation, corrected a few mistakes (misprints) and added the indirect part.