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The observations of MHD turbulence in the solar wind indicate that this is in a state characterized, to a good degree by the absence of non linear interactions. It is argued that this is a general property of incompressible MHD turbulence in a magnetized plasma.

A well known result of the measurements of hydromagnetic turbulence in the solar wind (Belcher and Davis, 1971; Burlaga and Turner, 1976) is the correlation between velocity ( $\delta \underline{v}$ ) and magnetic field ( $\delta \underline{B}$ ) fluctuations, which satisfy to a good degree (especially in the regions of the streams' trailing edges) the relation

$$\delta \underline{v} = \pm \delta \underline{B} / \sqrt{4\pi\rho} \quad (1)$$

with a sign implying that, of the two possible modes, only those propagating away from the Sun are present.

This property can be shown (Dobrowolny et al., 1979) to imply that the solar wind MHD turbulence is in a very peculiar state characterized by the absence of non linear interactions. This follows from the equations of incompressible magnetohydrodynamics written in the form

$$\frac{\partial}{\partial t} \delta \underline{Z}^{\pm} + (\underline{C}_A \cdot \nabla) \delta \underline{Z}^{\pm} + (\delta \underline{Z}^{\pm} \cdot \nabla) \delta \underline{Z}^{\pm} = -\nabla(p + B^2/8\pi) \quad (2)$$

$$\Delta^2(p + B^2/8\pi) = -\nabla \cdot [(\delta \underline{Z}^{\pm} \cdot \nabla) \delta \underline{Z}^{\pm}] \quad (3)$$

where

$$\delta \underline{Z}^{\pm} = \delta \underline{v} \pm \delta \underline{B} / \sqrt{4\pi\rho} \quad (4)$$

and  $\underline{C}_A = \langle \underline{B} \rangle / \sqrt{4\pi\rho}$  is the Alfvénic speed in the average field  $\langle \underline{B} \rangle$ . Note that, in the case of infinitesimal fluctuations,  $\delta \underline{Z}^{\pm}$  represent the two possible Alfvénic waves propagating away and toward the Sun. The property (1) is then equivalent to having, either  $\delta \underline{Z}^{\pm} = 0$  and

$\delta Z^- \neq 0$  or viceversa. As the non linear terms in eq. (2) are of the type  $(\delta Z^{\mp} \cdot \nabla) \delta Z^{\pm}$ , it follows that, in fact, the magnetohydrodynamic turbulence in the solar wind is, to a good approximation, in a state without non linear interactions.

We argue now that this peculiar property is not a particular one of the turbulence in the solar wind, but must be rather considered a general property of developed incompressible MHD turbulence in the presence of a background magnetic field. The point is the following: waves generated at the Sun must be necessarily outwardly propagating, if observed at 1 AU (Belcher and Davis, 1971). However the maximum power in Alfvén waves generated from motions in the convection zone and getting out in the solar wind, is found to be mainly in periods of the order of one to few hours, which correspond also to typical scales of supergranulation (Hollwegg, 1978). Much less power can get out at higher frequencies which, on the other hand, may suffer severe photospheric damping (Osterbrock, 1961). In contrast to this, the power spectra measurements (extending to periods of  $\sim 1$  s), indicate that there is appreciable power in the shorter periods also (from 1 s to 1 h). Adding to this the fact that there are mechanisms of local wave generation, at the higher frequencies, for example from velocity shear or microscopic instabilities, we conclude it is unlikely that the higher frequencies observed in the wind are of solar origin. The argument of solar origin, as an explanation for having only outwardly propagating waves does not therefore apply to such higher frequencies which, however, consist still of outwardly propagating waves.

Thus the property of being in a state characterized by the absence of non linear interactions, seems to be a general outcome of the development of incompressible anisotropic MHD turbulence. We will now show that physical arguments of the same dimensional type than those used to derive the  $k^{-3/2}$  law for the spectrum of isotropic turbulence (Kraichnan, 1965), indicate in fact an evolution of developed anisotropic MHD turbulence toward a state where one of the possible modes has disappeared.

Consider the interaction between vortices of the same scale  $l$ . The elementary interaction time, determined by the velocity  $C$  in the background magnetic field is  $\tau_{int} \sim l/C_A$ . From eq. (2) we can derive, in order of magnitude, the variation  $dZ^{\pm}$  in amplitude of a given vortex ( $\delta Z^{\pm}$ ), in one interaction time, due to its interaction with a vortex of the opposite type ( $\delta Z^{\mp}$ ). This will be

$$dZ^\pm \sim \tau_{int} (\delta Z^\pm \delta Z^\mp) / \ell$$

In  $N$  such stochastic interactions the amplitude variation will be  $\Delta Z^\pm = \sqrt{N} dZ^\pm$ . Hence, the number of interactions  $N^\pm$  it takes to obtain a variation, for a given vortex  $\delta Z^\pm$ , equal to its initial amplitude (i.e.  $\Delta Z^\pm \sim \delta Z^\pm$ ) will be given by

$$N^\pm \sim \ell^2 / (\delta Z^\mp)^2 \tau_{int}^2$$

The corresponding time  $T^\pm$  needed to obtain such a variation is

$$T^\pm \sim C_A \ell / (\delta Z^\mp)^2 \tag{5}$$

and should be considered as a typical time for a significant local energy transfer across the spectrum for a given type of mode.

Using (5), we could easily reproduce the power law  $-3/2$  for the spectrum of the symmetric case ( $\delta Z^+ \sim \delta Z^-$ ). As this is a known result (Kraichnan, 1965), we refer rather to the asymmetric case  $\delta Z^+ \neq \delta Z^-$ , i.e. a case where there is, at some time, an unbalance between the energy in the  $\pm$  modes. For example, suppose  $\delta Z^+ > \delta Z^-$ . Then, from (5)

$$T^+ / T^- \sim (\delta Z^+ / \delta Z^-)^2 > 1 \tag{6}$$

which means that the mode  $\delta Z^-$  transfers energy across the spectrum faster than the mode  $\delta Z^+$  (and then dissipate at the short wavelenghts). As the energy per unit time pumped at the source is constant, the energy in the modes  $\delta Z^-$  will further decrease. We thus see that, starting from an initial unbalance  $|\delta Z^+| > |\delta Z^-|$ , we have necessarily a cascade towards a situation where all the energy available remains in the mode  $\delta Z^+$ . This is precisely the state indicated by the observations of MHD turbulence in the interplanetary plasma and the above physical reasoning derives it quite generally as a consequence of the nature of the non linear terms in the equations of incompressible MHD.

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*DISCUSSION*

*Heinemann:* Belcher and Davis' original argument depended, essentially, on the solar wind flow speed; that is, that the flow speed eventually became greater than the Alfvén speed. I believe you neglected the flow speed in your equations: all that appeared was the Alfvén speed. Would your results change in a qualitative way if you retained the flow velocity?

*Dobrowolny:* My arguments refer to the case of an average uniform velocity of the fluid as you can locally suppose in the interplanetary medium on the scale length of Alfvén waves. On the other hand, the argument of solar origin, to explain the presence of only outwardly propagating waves does not probably apply at least to the highest frequencies observed (which may be locally generated) and which are also found to be outwardly propagating.