

# CONFORMAL METHODS IN GENERAL RELATIVITY

This book is a systematic exposition of conformal methods and how they can be used to study the global properties of solutions to the equations of Einstein's theory of gravity. It shows that combining these ideas with techniques of the theory of partial differential equations can elucidate the stability of the basic solutions of the theory. Introducing the differential geometric, spinorial and PDE background required to gain a deep understanding of conformal methods, this text provides an accessible account of key results in mathematical relativity over the last 30 years, including the stability of de Sitter and Minkowski spacetimes.

For graduate students and researchers, this self-contained account includes useful visual models to help the reader grasp abstract concepts and a list of further reading, making this the perfect reference companion on the topic.

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JUAN A. VALIENTE KROON is a Reader in Applied Mathematics at Queen Mary University of London. He was a Lise Meitner fellow of the Austrian Science Fund (FWF), an Engineering and Physical Sciences (EPSRC) Advanced Research fellow and he specialises in various aspects of mathematical general relativity.

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JUAN A. VALIENTE KROON

*Queen Mary University of London*





Shaftesbury Road, Cambridge CB2 8EA, United Kingdom  
One Liberty Plaza, 20th Floor, New York, NY 10006, USA  
477 Williamstown Road, Port Melbourne, VIC 3207, Australia  
314–321, 3rd Floor, Plot 3, Splendor Forum, Jasola District Centre, New Delhi – 110025, India  
103 Penang Road, #05–06/07, Visioncrest Commercial, Singapore 238467

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