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# Abstracts of Australasian PhD theses

## An algebraic formulation of quantum electrodynamics

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In 1967 Strocchi established that the quantisation of the electromagnetic field using a vector potential is impossible within the context of conventional field theory. Although this result is frequently referred to, its significance is largely misunderstood. The fact that the electromagnetic field cannot be described in conventional field theory reflects more upon conventional field theory than theories of the electromagnetic field.

A reappraisal of electromagnetic field theories should therefore be made. It could well be that features of these theories that have been previously regarded as deficiencies are not really deficiencies at all. This thesis is an account of the radiation gauge, Gupta-Bleuler and Fermi methods of quantising the electromagnetic field from that point of view.

The radiation gauge and Gupta-Bleuler methods are well established schemes. Our discussion does not yield any results concerning these methods that cannot be found elsewhere. It does, however, serve to place them in a wider context. The Fermi method is little understood and hence most of this work is concerned with it.

Even though the various formulations of field theory are by no means equivalent, they all eventually reproduce traditional field theory. Thus if we only require that the theory be rigorously formulated for such examples as the neutral scalar field it does not matter which formulation

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we choose.

The differences are, however, important for applications to the quantisation methods of the electromagnetic field. The formulations have to be modified and the point at which such modifications must be made and their nature depends on both the general formulation and the quantisation method.

The formalism that provides the most suitable framework for a rigorous formulation of the Fermi method turns out to be the $C^*$ algebra formulation of Segal. Following Segal, the Weyl algebra of the vector potential is constructed. The Fermi method is then related to a certain representation of the algebra. The representation is specified by a generating functional for a state on the algebra.

Usually, dynamical and kinematical transformations are represented by unitarily implementable automorphisms of the algebra. We prove that this is not always true in the representation given by the Fermi method. The Weyl algebra of the physical field is then constructed as a factor algebra. Difficulties with both the Fermi and Gupta-Bleuler methods can be attributed to the need to use a factor algebra.

The canonical commutation relations $[x_\mu, p_\nu] = -ig_{\mu\nu}$ are formulated as a Weyl algebra. We study the Schrödinger representation of the algebra and find that the Fermi method is just the generalisation of this representation to an infinite number of degrees of freedom. Further analogies are also possible. We can construct factor algebras from the Weyl algebra. The mechanics of such procedures can be studied without the additional complications of an infinite number of degrees of freedom. The Schrödinger representation of the Fermi method is then constructed.

We conclude with a discussion of the results that have been obtained and an indication of ways in which the work might be extended.