Correspondence

When is a hypothesis null?

DEAR SIR,

At a recent meeting of the Midland Branch, the topic of the *null hypothesis* arose in discussion. It was clear on that occasion that the concept causes trouble in teaching statistics in schools, and examiners' reports confirm this from time to time.

Since the meeting I have looked at a number of books on statistics and it appears to me that one of the causes of confusion amongst teachers is the vagueness of textbooks (my own included) and, indeed, differences in the interpretation of the term. Has, I wonder, the usage of the phrase changed since its introduction by Fisher? For instance, Siegel writes in *Nonparametric statistics:* "The *null hypothesis* is a hypothesis of no differences. It is usually formulated for the express purpose of being rejected." This would appear to exclude, for instance, a hypothesis about the ratio of (i) two heads, (ii) a head and a tail, (iii) two tails, on spinning two coins a number of times.

My own experience suggests that it is partly the unfamiliarity of the concept that causes trouble, but also the word *null*—which does not convey to the modern person the essence of what is meant. If I may suggest another word (for the classroom, rather than the examination question), it would be *skittle*: a skittle hypothesis is specially set up in order to be knocked down. But, seriously, would any statistician be prepared to say what is the modern interpretation of the phrase null *hypothesis*—in words, please, that the average sixth-former can understand?

Yours faithfully,

J. K. BACKHOUSE

Department of Educational Studies, 15 Norham Gardens, Oxford OX2 6PY

Discontinuity

DEAR SIR,

The article on *The Transition from School to University Mathematics* in the October 1972 *Gazette* concluded "The present evidence suggests that in many instances the sixth-form teacher has not made himself aware of the work required for a first-year honours course, nor has the university lecturer familiarised himself with school conditions". What conditions? Small classes and the asking of questions in class? Or the approach used in school mathematics? What is "work required for first-year honours courses" that the teacher should make himself aware of? Analysis? The subject matter of school calculus? What is the fundamental difference between the subject matter of these two courses? Why do the majority of sixth-formers think of calculus as the "topic most enjoyed" and the majority of first-year undergraduates report analysis the "most difficult subject"? What is the first-year analysis course other than the subject-matter of sixth-form calculus and convergence taken from the point of view of a mathematician? Why should differentiation, integration and functions defined by series be topics that are powerful and appealing to sixth-formers, and the same topics a year later be difficult, baffling and often nauseating to mathematics undergraduates?

We all know the answer to this question—it is the Onset of Rigour (or *rigor*?). This search for rigour is a necessary part of the process of turning the 'look for a pattern' child of 11 into the sophisticated mathematician of 21. At 18, we drop him, having blessed him with good grades in A level double-subject mathematics, from the frying-pan of never having seen why he should not differentiate $\sin^{-1}(1 + x^2)$ or any other expression he can write down, into the fire of proving that if $f(a) < 0$ and $f(b) > 0$, then $f(x) = 0$ at some point x between a and b, provided that f satisfies an obvious condition that every reasonable function which he has ever heard of satisfies anyway. Is it any wonder that he regards

https://doi.org/10.2307/3615358 Published online by Cambridge University Press

analysis as baffling *or* trivial *or* unreasonable *or* a great song and dance about things that everyone knows are true anyway?

Is it not time that we stopped seeing new undergraduates as people who have a great regard for logical deduction from (possibly rather strange) sets of axioms, and started viewing students in a more realistic light, recent sixth-formers who have learnt to enjoy mathematics as a powerful tool for solving problems? Certainly they need to understand the intricacies of the real number system, and its consequences. But they do not know in the first week in October that they need to understand the completeness axiom—indeed, they think they know about the real numbers. Is it beyond the wit of man to devise a first-year undergraduate analysis course which helps students to see why analysis must be like it is, and which does not present a vast edifice of well-known results balanced precariously on an unexplained and unlikely set of axioms?

Yours in anger at the wastefulness of this method,

HILARY SHUARD

Homerton College, Cambridge

Reviews

Perspectives in mathematics, by David E. Penney. Pp xiv, 349. £4·65. 1972 (Benjamin/Addison-Wesley)

This book was planned primarily as a text for a one-year course in mathematics for American undergraduates not intending to take calculus. Each of the ten chapters is a self-contained study of a topic "chosen from one of the major branches of modern mathematics". Chapter 1 is called "The Bolyai–Gerwin Theorem"—given two polygons of equal area one can be dissected into pieces which will fit to form the other. Chapter 2, "Brunnian Links", considers interlinking loops of string and provides an algebra by which linkages can be symbolised and problems of disentanglement solved. Chapter 3, "The Well-tempered Clavichord"—natural harmonics and equal temperament—and Chapter 8, "Animal Populations"—competitive and predator–prey situations—cover familiar ground, the former using logarithms and continued fractions and the latter avoiding technical calculus by using admirable diagrams. Chapter 5, "Polyhedra", deals with Euler's formula and a converse—given V, E, F such that $V-E+F=2$ a polyhedron exists provided $V+4 \leqslant 2F \leqslant 4V-8$, and finishes with some map-colouring. Chapter 9, "The Art-Gallery Theorem", shows the value of set-notation in an investigation into convex sets, hulls and kernels. The theorem, attributed to Krasnoselskii, says that if we are given points on the boundary of a closed (re-entrant) polygon and every three of them can be 'seen' from some point within the polygon, then there is a point within the polygon from which all of them can be 'seen'. ['Seeing' X from P means that PX does not cut the boundary.] Of the remaining chapters 4, "Group Theory", 6, "Infinite Sets", and 7, "Number Theory" give introductions. In Chapter 10, "The Real Number System", the general policy of using exercises to clarify ideas and lead up to proofs is seen to good advantage. A real number is eventually defined as "the collection of all equivalent sequences of nested intervals of rational numbers", and $\sqrt{2}$ is shown to exist (in R).

It will be seen that the book provides applications to fields (unlike, say, electromagnetism) where little specialised knowledge is needed and also theory of some logical and conceptual difficulty. An intelligent layman with O level mathematics should be able to follow most of the text. The writing is clear and the argument well-motivated and presented. The author provides answers and hints for the exercises in an appendix and also a few references for further reading. This is a useful source-book for teachers and young mathematicians should find much of it fascinating. It should be in the libraries of all schools and colleges where mathematics is taken seriously. It gives a true flavour of what pure mathematics is about.

Singleton Lodge, Blackpool R. C. LYNESS