of M is the sum of the maxima numbers for the M_r. In particular, if M has n independent eigenvectors, every M_r has n_r.

Now, suppose that the matrices A, B, C, ... satisfy our two conditions. If A has k distinct latent roots $\lambda_1, \lambda_2, \ldots$, with n_r eigenvectors for the root λ_r, we may suppose the matrices to have been subjected to a T-transformation, with the n eigenvectors of A taken in order as the column-vectors of T. A will then be diagonal, and its eigenvectors for the root λ_r are the vectors v_r of the rth sub-space. Again, $ABv_r = BAv_r = \lambda_r Bv_r$, so that Bv_r is an eigenvector of A for the root λ_r, and hence belongs also to the rth sub-space. It follows as above that $B = \text{Diag}\ (B_r)$, and so also for C, D, etc.

Since B has n independent eigenvectors it follows (as in the case of M above) that B_r has n_r, which are eigenvectors of B, and also of A for the root λ_r. Taking $r = 1, 2, 3, \ldots$, we see that A and B have a common set of n independent eigenvectors; and the proposition is true for two matrices.

Again, $BC = CB$, *i.e.* $\text{Diag}\ (B_rC_r) = \text{Diag}\ (C_rB_r)$, hence B_r and C_r commute; and the same holds for $D_r, E_r, \ldots$.

Now, suppose that if $m-1$ matrices commute and have each a complete set of independent eigenvectors, they have a common set; and consider m matrices $A, B, C, \ldots, M$, satisfying the conditions. We have seen that the $m-1$ matrices $B_r, C_r, \ldots, M_r$ commute and have each a complete set of n_r independent eigenvectors; they have therefore a common set, and these are all eigenvectors of A for the root λ_r. Taking $r = 1, 2, \ldots, k$, we find a common set for the m matrices $A, B, \ldots, M$. But we have seen that the proposition is true for two matrices, hence it is true universally.

To reduce the matrices to diagonal form, we have only to take the common set of eigenvectors as the columns of T.

M. F. E. & R. E. I.

CORRESPONDENCE.

AN ENIGMA

To the Editor of the *Mathematical Gazette.*

SIR.—Several correspondents have kindly explained the capital letters in the statement (Note 2333)

" $2x^2 - x - 1 = (2x+1)(x-1)$ by F.M.O.L.".

They form a mnemonic for multiplying together two binomials and stand for

Firsts, Middles, Outers, Lasts.

Another version is F.O.I.L., where I = Inners. I fear the suggestion that the candidate meant to write " by F.M.O.F.", meaning " by fair means or foul ", must be rejected.

Yours, etc., C. O. TUCKEY.

A TRIANGLE FORMULA

To the Editor of the *Mathematical Gazette.*

SIR,—It is not often that the *Gazette* takes one back in memory to schooldays of more than seventy years ago.

The proof of the formula for $\tan \frac{1}{2}(B-C)$ on p. 50 of the current number (February, 1953) can be found on pp. 75–6 of *Trigonometry for beginners* by I. Todhunter, published in 1871; but it was not included in his larger work, *Trigonometry for the Use of Colleges and Schools.*

Yours, etc., A. S. RAMSEY.

https://doi.org/10.2307/3608936 Published online by Cambridge University Press