# NECESSITY OF ALTERATION

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In this note we point out a not easily fillable gap in the proof of a major result in Chebyshev approximations on an interval $[\alpha, \beta]$. Tornheim developed a theory for approximation by families unisolvent of degree n (n-parameter families), which was extended by Rice to families unisolvent of variable degree. The principal result is the following:

THEOREM. *Let* F *be unisolvent of degree* n *at* A. *A necessary and sufficient condition that* F *be a best approximation to* f *is that* f-F(A, .) *alternate* n *times*.

The necessity proof of Tornheim and Rice breaks down in a case they did not consider, namely when the error function f-F(A, .) is a non-zero constant. In this case the error curve must be pulled down over the whole interval to get a better approximation, whereas the Tornheim-Rice construction pulls the error curve down only on a portion of the interval.

The author has been unable to find anyone who can complete the proof. It would suffice to show that for any $\varepsilon > 0$, there exist B and C such that

$$F(A, .) - \varepsilon < F(B, .) < F(A, .) < F(C, .) < F(A, .) + \varepsilon,$$

which would be true if the approximating family had a subset unisolvent of degree 1 containing F(A, .). A direct proof of this would likely require a major theoretical advance. The gap does not affect the truth of Tornheim's result, as an alternative proof by Novodvorskii and Pinsker exists. However no other proof is known for Rice's result.

## REFERENCES

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