CORONAE ABOVE ACCRETION DISCS

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Abstract. A turbulent accretion disc generates acoustic noise which, on propagation upward into the tenuous upper layers, forms shock waves. These give rise to a corona above the disc, thereby providing the possibility to determine observationally the existence (or otherwise) of turbulence in the disc. Models for stationary coronae of this type are calculated on the basis of a simplified thermodynamic cycle in the shocks.

1. Introduction

It is generally assumed that in the accretion discs which occur in some binary stars, angular momentum is transported outward by turbulent viscosity. The origin of the turbulence is somewhat problematic because a Keplerian gaseous disc is axially stable according to the Rayleigh criterion. Also, viscosity is not absolutely necessary for the transport of momentum: it is possible that a standing curved shock in the disc, generated by the flow of gas through the non-circular Roche potential, provides a means for making the matter spiral inward (Icke, 1976). It is therefore important to have observational evidence about the state of motion in the disc. If the turbulence is strong enough to cause considerable viscosity, it must also generate acoustic noise of fairly large amplitude. When these sound waves reach the upper parts of the disc, the low gas density causes them to steepen into shock waves which, by their dissipation, give rise to a corona as an observable consequence of the turbulence below.

The amount of acoustic energy radiated per second by a cubic meter of homogeneous isotropic turbulence can be shown to be

$$E_a = \beta E_t \mathcal{M}^5 \tag{1.1}$$

(Proudman, 1952), where β is a number depending on the velocity correlation, E_t is the amount of energy dissipated by the turbulence, and $\mathcal{M}$ is the average Mach number of the turbulence. In the case of a Heisenberg correlation, one obtains $\beta = 37.7$. The interesting feature of Equation (1.1) is that since E_t is proportional to the disc luminosity, *the ratio of the luminosity of the corona over the luminosity of the disc is a constant* for a given value of $\mathcal{M}$. It is this property which ensures that the coronal radiation is not necessarily obliterated by the disc when the latter is very bright. The steep dependence on the Mach number is a problem, because it makes *a priori* predictions of the coronal luminosity almost impossible. For $\mathcal{M} > 0.2$, $E_a/E_t > 0.01$, but even at such a low fraction the corona should be distinguishable because its radiation will be mostly emitted in a few emission lines of highly ionized heavy elements.

2. A Simplified Acoustic Heat Engine

When a sound wave travels through an atmosphere in the direction of decreasing density, conservation of action causes the wave amplitude to increase. Because the velocity of propagation increases with increasing density, such large-amplitude waves steepen and form shock waves. Shocks will rapidly dissipate, but the liberated energy cannot be easily

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removed by radiation, because the cooling efficiency is insufficient due to the low gas density. Therefore, heat conduction must become an important means of energy transport, and since the conductivity is only sufficient at high temperatures, a corona is formed. This mechanism was extensively studied by Kuperus (1965) for application to the solar corona. I shall presently assume a turbulent accretion disc to be the source of acoustic energy, and largely follow Kuperus' analysis, with the following modifications:

(a) a slightly different, and less efficient, thermodynamic cycle;

(b) integration of the equation of hydrostatic equilibrium instead of using a modified scale height.

Take position at a height z above the plane of the disc. Then a steady shock wave train will pass, so that the pressure P and the density ρ of the ambient gas change in the course of time t somewhat like a sawtooth. Every wave tooth then generates a thermodynamic cycle at its leading edge and immediately behind, thereby heating the gas. Ideally, this cycle consists of (1) adiabatic compression, (2) isochoric compression to a point connected with the initial (P, ρ) by the Hugoniot conditions, (3) linear expansion of P and ρ. In practice, (1) and (2) cannot be expected to occur in that form, and I shall approximate these stages by assuming that the gas, upon passage of a shock front, follows a Hugoniot adiabatic up to a point which is determined by the equilibrium conditions of the atmosphere (Figure 1). The specific energy liberated in this cycle is obtained by path integration. All initial quantities (i.e. those in front of the shock) will have subscript $_0$, final quantities subscript $_1$. Assume the gas to be ideal, with constant ratio γ of specific heats, and mean molecular weight μ. Along the lower branch of the cycle one has

$$I_1 = -\int_{V_0}^{V_1} P \, \mathrm{d} V = \frac{P_0}{\rho_0} \int_{1}^{1/U} P' \rho'^{-2} \, \mathrm{d}\rho', \tag{2.1}$$

$$\left.\begin{aligned} P' &\equiv P/P_0, \\ \rho' &\equiv \rho/\rho_0, \\ U &\equiv \rho_0/\rho_1. \end{aligned}\right\} \tag{2.2}$$

Using the Hugoniot conditions across the shock, the Equation (2.1) yields

$$I_1 = \frac{P_0}{\rho_0} \left\{ \frac{U-1}{H} + \frac{H^2-1}{H^2} \log \left(\frac{H-1}{HU-1} \right) \right\}, \tag{2.3}$$

$$H \equiv (\gamma + 1)/(\gamma - 1). \tag{2.4}$$

The upper branch of the cycle is formed by a linear dependence of P and ρ on a formal parameter ζ (i.e. time or vertical distance):

$$\left.\begin{aligned} P &= P_0 + \zeta (P_1 - P_0), \\ \rho &= \rho_0 + \zeta (\rho_1 - \rho_0). \end{aligned}\right\} \tag{2.5}$$

Along this branch, one then obtains

$$I_2 = -\int_{V_1}^{V_0} P \, \mathrm{d} V = \int_{1}^{0} \frac{P_0 + \zeta (P_1 - P_0)}{(\rho_0 + \zeta (\rho_1 - \rho_0))^2} (\rho_1 - \rho_0) \, \mathrm{d}\zeta,$$

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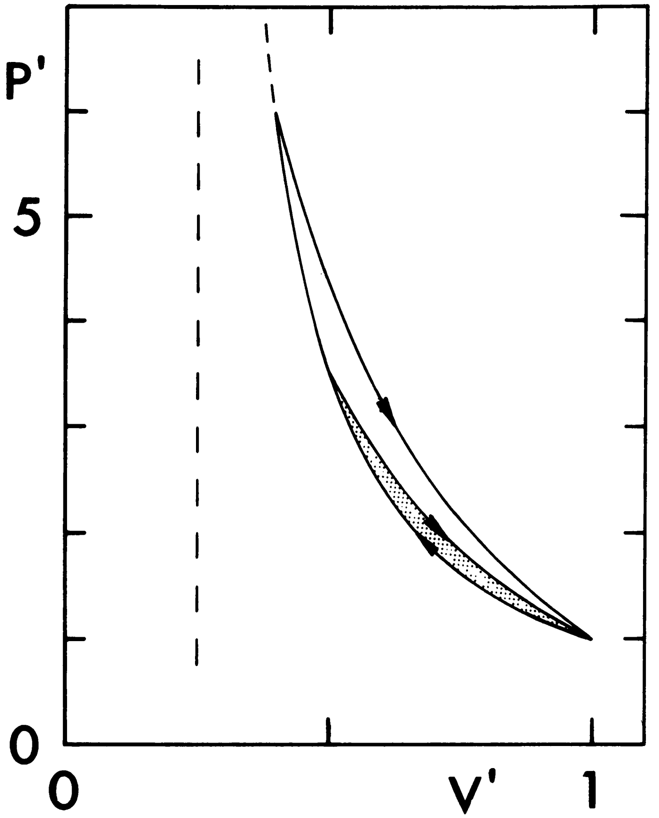

Fig. 1. Thermodynamic cycles traversed on the passage of a shock wave train. Dots: energy yield.

$$I_2 = \frac{P_0}{\rho_0} \frac{1}{HU - 1} \{1 - U^2 + U(H - 1) \log U\}. \tag{2.6}$$

The total energy yield is the sum of I_1 and I_2 plus the energy due to the entropy increase:

$$T_0 \Delta S = \frac{k T_0}{\mu(\gamma - 1)} \log \left(\frac{H - U}{HU - 1} U^\gamma \right) = \frac{P_0}{\rho_0} \frac{H - 1}{2} \log \left(\frac{H - U}{HU - 1} U^\gamma \right) . \tag{2.7}$$

Therefore, the total energy e liberated per unit mass by the cycle of Figure 1 is given by

$$e = \frac{P_0}{\rho_0} \left\{ \frac{(H + 1)(1 - U)}{H(HU - 1)} + \frac{H^2 - 1}{H^2} \log \frac{H - 1}{HU - 1} + \frac{H - 1}{2} \log \frac{H - U}{HU - 1} + (H + 1) \left(\frac{U}{HU - 1} + \tfrac{1}{2} \right) \log U \right\} . \tag{2.8}$$

The velocity w behind a shock will be given by a linear relation like Equation (2.5):

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$$w = w_0 + \zeta (w_1 - w_0). \tag{2.9}$$

In the coordinate frame moving with the shock velocity D, this velocity is transformed into

$$u = w - D. \tag{2.10}$$

I assume that there is no net mass transported by the shock wave train. It is possible at this point to incorporate a stellar wind into the model by requiring the mass flux to be constant with z, and positive. I shall not include this extra free parameter, but instead require the average of ρw over a cycle to be zero:

$$\langle \rho w \rangle = \rho_0 U_0 \int_0^1 \left(1 + \zeta\left(\frac{1}{U} - 1\right)\right) \left(1 + \frac{D}{U_0} + \zeta (U - 1)\right) \mathrm{d}\zeta \equiv 0,$$

in which the Hugoniot condition $\rho_0 u_0 = \rho_1 u_1$ has been used to express the velocity ratio as U. Thus one obtains a condition on D:

$$1 + \frac{D}{U_0} = \frac{(2+U)(1-U)}{3(U+1)}. \tag{2.11}$$

Due to the conservation of linear momentum, one has

$$\frac{P_0}{\rho_0} = u_0^2 \frac{HU-1}{H+1}, \tag{2.12}$$

whence Equation (2.11) leads to an expression for the shock velocity:

$$D = \sqrt{\frac{kT_0}{\mu}}\, D(U,H), \tag{2.13}$$

$$D(U,H) \equiv \sqrt{\frac{H+1}{HU-1}}\, \frac{1+4U+U^2}{3(1+U)} \tag{2.14}$$

The structure of the atmosphere will be calculated on the assumption that it is quasistatic. Therefore, it is necessary that the energy e can be liberated within one shock wave length; i.e. the shock thickness λ must be smaller than the distance L between successive shock fronts. The length L is the product of D and the wave period τ (which will be considered below). For a fully ionized gas, the mean free path is

$$\left.\begin{aligned} \lambda &= \alpha T^2 \rho^{-1} && \mathrm{m}, \\ \alpha &= 3.93 \times 10^{-18} && \mathrm{kg\,m^{-2}}\,K^{-2}. \end{aligned}\right\} \tag{2.15}$$

(Allen, 1973, p. 50). Estimating that D is approximately equal to the product of the sound velocity and the shock Mach number $\mathscr{M}$, it follows that $\lambda < L$ leads to the condition

$$\rho > 2.37 \times 10^{-20}\, T^{3/2} \tau^{-1} \mathscr{M}^{-1} \quad \mathrm{kg\,m^{-3}}. \tag{2.16}$$

Therefore, coronal densities of $10^{-14}\,\mathrm{kg\,m^{-3}}$ and temperatures of 10^6 K can be allowed for relatively small Mach numbers. Consequently, the atmosphere can be treated quasi-statically, and the energy e, being generated in the distance λ, is smeared out over the

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distance L, giving an efficiency λ/L to be multiplied with Equation (2.8). Multiplication by ρ_0 and division by τ to obtain the energy yield per unit volume and per unit time then gives

$$E_{\text{shock}} = \frac{\alpha}{\tau^2}\sqrt{\frac{kT_0}{\mu}}\, T_0^2\, E(U,H)/D(U,H) \quad \text{Jm}^{-3}\,\text{s}^{-1}, \tag{2.17}$$

$$E(U,H) \equiv \frac{(H+1)(1-U)}{H(HU-1)} + \frac{H^2-1}{H^2}\log\frac{H-1}{HU-1} + \frac{H-1}{2}\log\frac{H-U}{HU-1} + \\ + (H+1)\left(\frac{U}{HU-1} + \tfrac{1}{2}\right)\log U. \tag{2.18}$$

The working of the heat engine driven by the shock wave train has now been summarized by the cycle average E_{shock} and the propagation velocity D.

3. The Structure of a Stationary Corona

The quantities P_0, ρ_0, T_0, u_0 and U describe the structure of the corona. These must be linked by five equations. The first two of these have already been introduced, namely the equation of state for a perfect gas, and the Hugoniot momentum equation:

$$P_0 = \rho_0\, k\, T_0/\mu, \tag{3.1}$$

$$\frac{P_0}{\rho_0} = u_0^2\,\frac{HU-1}{H+1}\,. \tag{3.2}$$

The third equation is the one describing the transfer of energy. Since there is no net mass flux, no matter is carried up against the gravitational potential, whence one obtains simply

$$\frac{\text{d}}{\text{d}z}\left(\rho w h + \tfrac{1}{2}\rho w^3 - \kappa\frac{\text{d}T}{\text{d}z}\right) = E_{\text{rad}} - E_{\text{shock}}. \tag{3.3}$$

The radiant energy loss E_{rad} is to be considered below. Geometrical dilution effects have been ignored for simplicity but could, if necessary, be taken into account. However, this refinement would also necessitate a detailed consideration of the sideways propagation of the sound waves. In the above, h is the specific enthalpy, and the thermal conductivity κ is given by

$$\kappa = 10^{-11}\, T^{5/2} \quad \text{Jm}^{-1}\,\text{s}^{-1}\,\text{K}^{-1}, \tag{3.4}$$

(Allen, 1973, p.50). The wave period τ is independent of z: since the corona is driven from below, τ is a boundary condition at $z = 0$. The assumption of a single period is necessary for the calculation of a *stationary* corona. The error thus introduced is probably not large: any shock wave train is expected to be dominated by the period of its strongest shocks, because the velocity of propagation increases with amplitude so that weaker shocks are overtaken and obliterated. I assume that the dominant period of the vertical component of the turbulence in the disc corresponds to the period of oscillation around the $z = 0$ plane in the gravitational field of the accreting star:

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$$\tau = 2\pi \sqrt{\frac{R^3}{MG}}, \tag{3.5}$$

assuming the oscillation amplitude to be small with respect to the distance R of the accreting star with mass M. The period is connected with the distance L between shocks and the shock velocity D by

$$\tau = L/D. \tag{3.6}$$

The fourth equation for the corona is the momentum equation

$$\frac{1}{\rho}\frac{dP}{dz} + w\frac{dw}{dz} = -\frac{d\Phi}{dz}, \tag{3.7}$$

where Φ is the gravitational potential in the disc:

$$\Phi = -\frac{MG}{\sqrt{R^2 + z^2}}. \tag{3.8}$$

Finally, the increase in entropy S caused by the shocks must be taken into account. From the Hugoniot conditions, the jump ΔS can be calculated, and with Equation (3.6) one obtains

$$\frac{dS}{dz} \approx \frac{\Delta S}{L} = \frac{c_v}{\tau D} \log\left(\frac{H-U}{HU-1}\, U^{\gamma}\right).$$

Because $S = c_v \log P\rho^{-\gamma}$, this can be written (using Equations (3.1, 2) and (2.13))

$$\tau\sqrt{\frac{kT_0}{\mu}}\left(\frac{d\log T}{dz} - \frac{2}{H-1}\frac{d\log\rho}{dz}\right) = \frac{1}{D(U,H)}\log\left(\frac{H-U}{HU-1}\, U^{\gamma}\right). \tag{3.9}$$

All the necessary equations are now at hand. It remains to determine the radiative energy loss E_{rad}. I have used the cooling function for an optically thin plasma calculated by Pottasch (1965), approximating the numerical values by the analytical form

$$E_{rad} = \begin{cases} 3.79 \times 10^9\, \rho^2 T^2 \quad \mathrm{Jm^{-3}\,s^{-1}} \quad \text{for} \quad T < 9.12 \times 10^4\,\mathrm{K}, \\ 2.88 \times 10^{24}\, \rho^2 T^{-1} \quad \mathrm{Jm^{-3}\,s^{-1}} \quad \text{for} \quad T > 9.12 \times 10^4\,\mathrm{K}, \end{cases} \tag{3.10}$$

which is sufficiently precise for the present purpose (see Figure 2). Because the shock thickness λ is much less than a shock wave length L, the velocity w is but a small perturbation when averaged over L; therefore, I shall assume a quasistatic state with $w \approx 0$. Similarly, all thermodynamic quantities are on the average equal to those describing the situation in front of each shock, whence all zero indices will be suppressed from now on. Assembling the structure equations for the corona one finds after some manipulation

$$\frac{d\rho}{dz} = -\left(\frac{\mu}{k}\frac{d\Phi}{dz} + \frac{dT}{dz}\right)\frac{\rho}{T}, \tag{3.11}$$

$$\frac{dT}{dz} = \frac{T}{\gamma\tau}\sqrt{\frac{\mu}{kT}}\frac{1}{D(U,H)}\log\left(\frac{H-U}{HU-1}U^{\gamma}\right) - \frac{2}{H+1}\frac{\mu}{k}\frac{d\Phi}{dz}, \tag{3.12}$$

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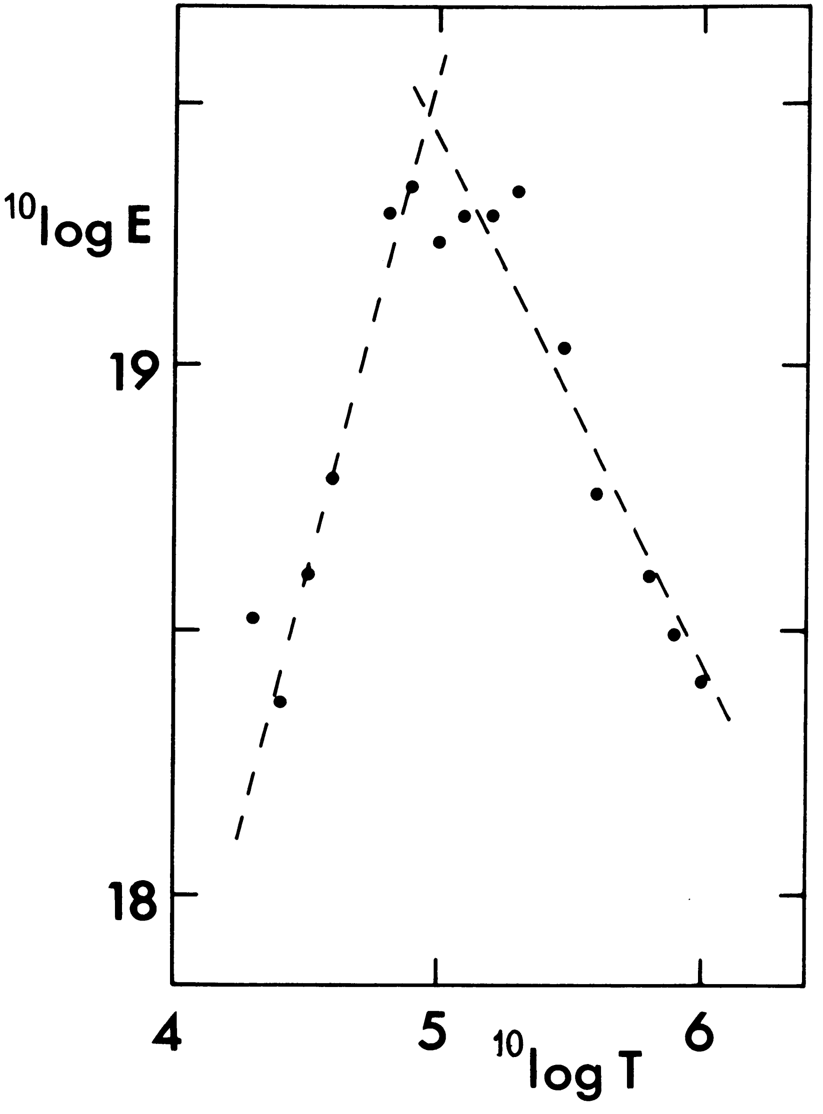

Fig. 2. Dots: cooling function calculated by Pottasch (1965). Dashes: approximation adopted for the integrations.

$$-\frac{\mathrm{d}}{\mathrm{d}z}\left(\kappa \frac{\mathrm{d}T}{\mathrm{d}z}\right) = E_{\mathrm{rad}} - E_{\mathrm{shock}}. \tag{3.13}$$

With (2.17), (3.8, 10) these form a closed set for the variables ρ, T and U as a function of z. In order to make the equations manageable for numerical integration, I shall express temperature in units of 10^4 K, length in units of R which is taken to be 10^8 m, and density in 10^{-14} kg m^{-3}. The mass M of the accreting star is one solar mass (1.989×10^{30} kg) and the average molecular weight is 8.36×10^{-28} kg. The length scaling

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means, that a *decrease* in the radius R of the basis point in the disc entails a corresponding *increase* in z.

The Equations (3.11–13) were solved numerically by a predictor-corrector method. Integration was performed from high z towards low z rather than the other way around, for computational stability. Several densities and temperatures at various heights were tried as initial conditions, and a good set of results (satisfying condition (2.16)) is shown in Figures 3 and 4. The acoustic energy flux density E_a is calculated from Equations (1.1), (2.17). At the lowest z point in Figures 3 and 4, the values of E_a are 4.3, 3.8 and 3.4 $Jm^{-3}\,s^{-1}$ for the cases 1, 2 and 3, respectively. At these points the integration was stopped because of the close approach of U to the singular value $1/H$. Multiplication of these E_a by a volume $\frac{4}{3}\pi R^3\,m^3$ gives energy fluxes of about 1.8×10^{25}, 1.6×10^{25} and 1.4×10^{25} $J\,s^{-1}$ in the three examples. This could be compared with the disc luminosities calculated by Bath *et al.* (1974), ranging from 5×10^{23} $J\,s^{-1}$ for a disc with temperature 10^4 K to 10^{27} $J\,s^{-1}$ for a 4×10^5 K disc.

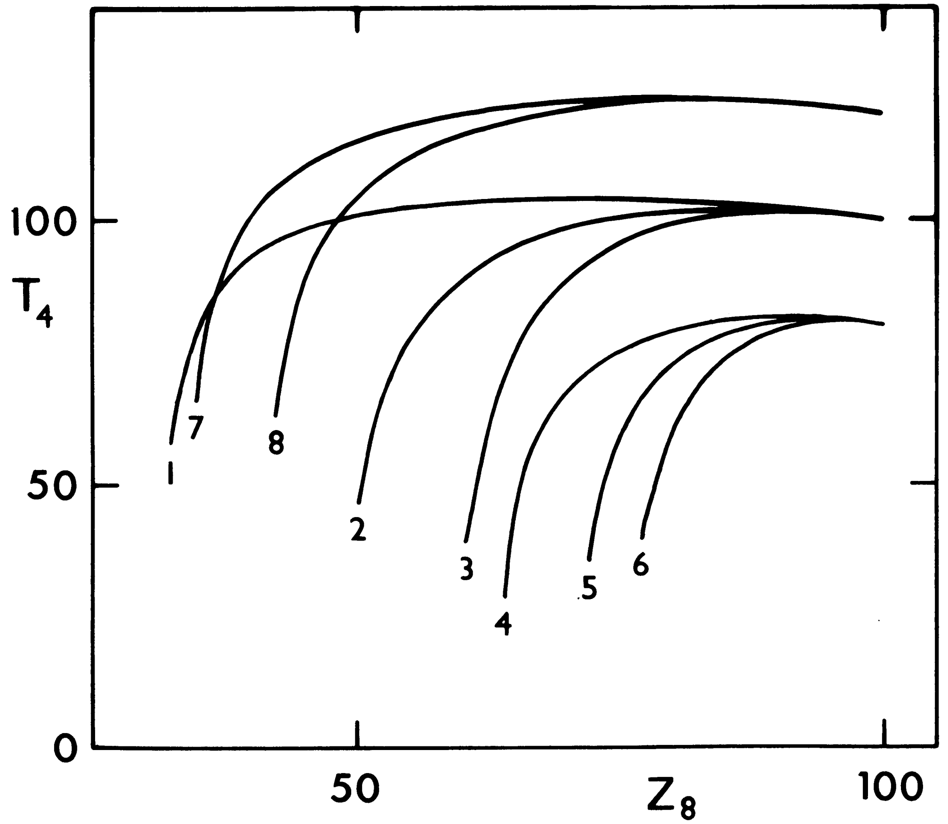

Fig. 3. Temperature in the corona, in units of 10^4 K, as a function of height, in units of 10^8 m. Scaling of z upwards with a factor of f means moving to a radius R in the disc scaled down by the same factor (e.g. $z = 1$ corresponds to $R = 10^8$, $z = 10$ to $R = 10^7$ etc.).

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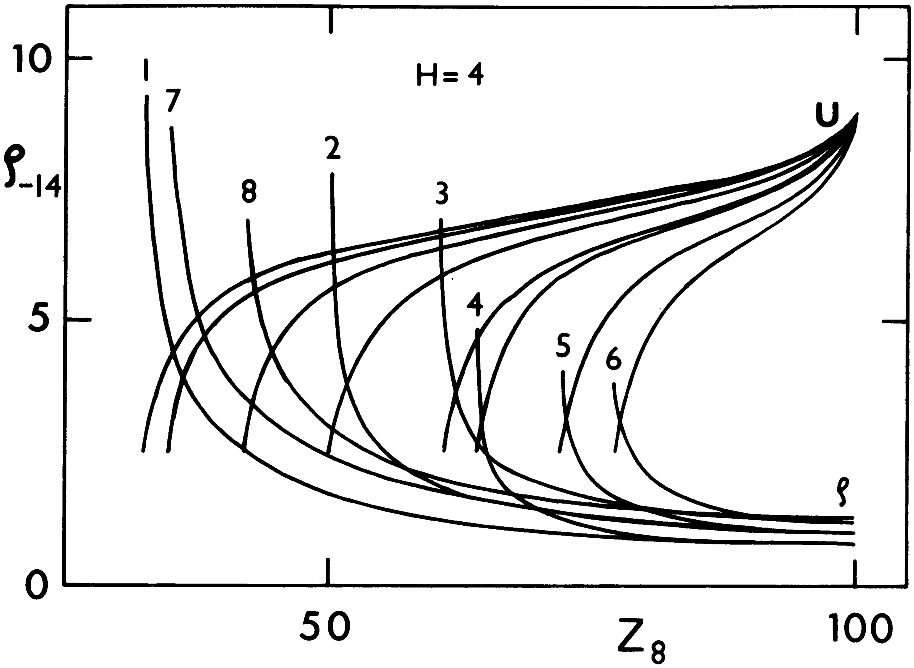

Fig. 4. Density in units of 10^{-14} kg m^{-3}, and shock strength parameter U, for the cases shown in Figure 3.

4. Conclusions

Turbulent accretion discs generate coronae, and it should be possible to obtain observational evidence as to whether turbulence, and hence turbulent viscosity, plays a role in the angular momentum transport necessary for accretion. One should expect to see (under favourable circumstances, such as eclipse by a sufficiently dark companion) inner coronae in the ultraviolet, and emission lines from outer coronae with excitation temperatures of possibly millions kelvin. The detailed spectrum will be difficult to predict because of complications with collisional de-excitation and dielectronic recombination.

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DISCUSSION

Jones: The idea of strong turbulence in the disc has been brought up several times during this session and I would like to make a cautionary remark about this. We have seen from Dr Flannery's results that the disc is quite cold and so we are led to think in terms of what is often referred to as 'supersonic turbulence'; by this one presumably means random motions with supersonic velocities. By its very nature 'supersonic turbulence' is highly dissipative (though not via an eddy cascade process) and I have doubts as to whether such a state of fluid motion could be set up and maintained. For one thing, the principal result of trying to establish such a state would be to heat up the disc and thereby reduce the effective flow Mach number to a value less than unity. It is necessary to exercise care, therefore, in constructing models of the disc where turbulence may be present.

Icke: I agree wholeheartedly.

Hall: Let me draw your attention to KU Cygni, a binary which seems to show evidence of a corona such as you have proposed.

Icke: I apologize for not being familiar with the observations, but I will certainly look it up.

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