THEORY OF DUST FORMATION IN R CORONAE BOREALIS STARS

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ABSTRACT. Application of the homogeneous nucleation theory to the problem of R CrB stars shows that the radial distance of the inner boundary of the carbon supersaturation region is at about 12 photospheric radii for a stellar effective temperature T_e = 6000 K. Formation of an optically thick dust shell becomes possible at mass loss rates $\dot{M} \gtrsim 10^{-6}$ $M_\odot$/yr. However, the upper limit of this mass loss rate cannot considerably exceed 10^{-5} $M_\odot$/yr since, at higher $\dot{M}$, the theoretically predicted rate of the visual brightness decline is larger than that derived from observations. Comparison of the theoretically predicted radii of dust grains with those observed in R CrB and RY Sgr shows that the mass loss rate in these stars should be in the range of 1 x 10^{-7} $M_\odot$/yr to 3 x 10^{-6} $M_\odot$/yr.

1. INTRODUCTION

The main observational property of R CrB stars is a sudden drop in their visual light with the amplitude ranging from 1 to 9 mag. It is well known that this is ascribed to the formation of optically thick dust shells. Any theoretical model of grain formation must also be able to explain further properties of R CrB stars, the most important of which are the following:

1. Light minimum occurrence is unpredictable.
2. There is a correlation between the rate of the light decline and the intrinsic colour index at normal state (Pugach, 1977). Typical rates of the light decline are 0.2 mag/day at spectral type F and 0.05 mag/day at spectral type R.
3. While the visual light is decreasing, the ratio $R = \Delta V/\Delta(B-V)$ becomes very large. For instance, according to Tempesti and De Santis (1975), this ratio is $R \approx 25$. However, upon recovery of light the ratio is $R \approx 3.5$, which is typical for the interstellar medium.
4. All R CrB stars also show a cyclic light variation with an amplitude of about a few tenths of mag (Alexander et al., 1972; Fernie, 1982; Kilkenny and Flanagan, 1983), which is due to radial

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pulsations (King, 1980).

5. During the visual light minimum, excesses of infrared radiation can be observed (Feast and Glass, 1973). This is a direct indication of the formation of optically thick dust shells. IR observations by Kilkenny and Whittet (1984) have shown that during the normal state the spectra of R CrB stars can be considered as a superposition of a photospheric and a dust shell contribution. This indicates the existence of permanent dust shells around R CrB stars.
6. Polarimetric observations by Orlov and Rodriguez (1972) have shown that the polarization increases with the decrease of the visual light. After recovery to the normal state, the polarization is different from that before the minimum. This fact strengthens the evidence of optically thin dust shells during the normal state of R CrB stars.
7. The spectral types of R CrB stars are in the range from R to F. It is suspected that also a few stars from spectral type A and B belong to the group of R CrB stars. Thus, any theoretical grain formation model must be able to explain the existence of circumstellar dust around stars with an effective temperature ranging from 2500 K to 6000 K or to even as much as 20000 K.
8. Previous spectral analyses (Searle, 1961; Schönberner, 1975; Orlov and Rodriguez, 1974; Cottrell and Lambert, 1982) revealed a deficiency of hydrogen and an overabundance of helium and carbon. This means that the formation of the circumstellar dust shell is caused by the phase transition of carbon.
9. While the visual light is decreasing, the spectra of all R CrB stars undergo drastic and complex changes. Firstly, a continuum emission at wavelengths shorter than 4000 Å appears (Alexander et al., 1972). In the next phase a great number of sharp emission lines become visible, the intensity of which decays over the time scale of about 20 days (Payne-Gaposhkin, 1963; Alexander et al., 1972). These lines are displaced bluewards by 10 kms^{-1}. They are also assumed to be present during the normal state (Rao, 1981). Finally, broad emission lines of He I, Ca II and Na I appear, indicating a gas outflow with a velocity as great as 200 km/s (Alexander et al., 1972; Rao, 1981).
10. The R CrB stars are supergiants with a bolometric magnitude ranging from -4 mag to -6 mag (Warner, 1967).

2. MODELS OF R CORONAE BOREALIS STARS

The first explanation of the R CrB phenomenon was given by Loreta (1934) and O'Keefe (1939). They presumed that the fading of the visual light is caused by ejection of matter followed by condensation of carbon particles. This hypothesis also qualitatively explains the increase of infrared radiation during the minimum of visual light.

It was preliminary assumed that the ejection of matter occurs in spherically symmetric shells. However, later observations showed that a change in the visual brightness is not always accompanied by a corresponding change of the infrared flux (Forrest and Gillett, 1971,

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1972). These observations favour the idea of local matter ejection (Feast and Glass, 1973). Thus, obscuration of the stellar disc occurs when matter is ejected towards the observer.

It should be noted that the model of local ejection also allows the large values of R during the decline of the visual light to be explained, as well as the dependence of the rate of the light decline on the effective temperature. Following Pugach (1984), let us designate by X the fraction of the stellar disc obscured by the dust cloud. Then, the ratio of the total extinction to the selective extinction can be determined from the following expression:

$$R^{-1} = \frac{\mathrm{Log}\,(1-X+X\exp(-\tau_B))}{\mathrm{Log}\,(1-X+X\exp(-\tau_V))} - 1,$$

where τ_V and τ_B are the optical depths of the dust cloud in the V and the B band, respectively. During the initial phase of light decrease, $X < 1$, $\tau_V > 1$ and $\tau_B > 1$ so that R can reach high values. On the other hand, during the recovery phase, $X = 1$, which means that the increase of the visual brightness is caused by the decrease of τ. For the typical relation $\tau_V = 0.8\,\tau_B$, we obtain $R \approx 4$. Thus, the rate of the visual brightness increase is determined by the dissipation of the dust cloud and, hence, by the decrease of its optical depth. Pugach (1984) has also shown that the theoretical dependence of the rate of the light decline on the intrinsic colour index $(B-V)_o$ is in good agreement with the empirical relation, if $v_t = 0.8\,v_e$ (v_t = tangential expansion velocity of the cloud, v_e = escape velocity).

To explain the time behaviour of the sharp emission line spectrum, Payne-Gaposhkin (1963) proposed that grain formation takes place near the photosphere, whereas the sharp emission lines are formed in the chromosphere. In the frame of this hypothesis, the dust shell blocks the radiative flux and causes the decay of the emission lines. The main difficulty with this model is that there is no way of explaining grain formation close to the photosphere, where the gas temperature can exceed 6000 K.

On the other hand, Fadeyev (1983) proposed that the sharp emission lines are formed in a shock front. This idea is strengthened by the fact that these emission lines are shifted to the blue by about 10 kms^{-1}. Moreover, a simple analysis shows that in pulsating stars with a luminosity to mass ratio of $L/M > 10\,L_\odot/M_\odot$ the frequency of the fundamental mode exceeds the critical frequency for standing waves (Aikawa, 1984). Thus, above the photosphere of such stars, stellar oscillations may occur only in the form of running waves. The decay of the intensity of the emission lines can then be caused by a decrease of the mechanical energy flux in the shock while it propagates through the extended stellar atmosphere.

3. DUST FORMATION IN R CORONAE BOREALIS STARS

So far published models of R CrB stars do not consider grain formation; even the location of the condensation region is still a matter of debate. Simple estimates show that the gas density in a hydrostatic equilibrium atmosphere decreases so rapidly with radius that, in the layers where the gas temperature drops below the evaporation temperature, carbon cannot become supersaturated. The enhancement of the gas density in the outer layers, due to stellar pulsations, seems to be the most probable mechanism for grain formation.

In a first approximation, we assume a spherically symmetric and time-independent outflow of matter, $\dot{M}$. Thus, the gas density ρ at a radial distance R can be estimated from the equation of continuity:

$$\rho = \dot{M}/(4\pi R^2 v) \qquad (1)$$

where v is the velocity of the gas. Let us also assume that the optical thickness of the matter inbetween the photosphere and the considered radius R is negligible. Then the gas temperature T can be estimated from the following relation:

$$T_g = T_e W^{1/4}, \qquad (2)$$

where T_e is the effective temperature and

$$W = 1/2 \quad (1 - (1 - (R_{ph}/R)^2)^{1/2})$$

the dilution factor.

The main quantity determining the phase transition is the supersaturation ratio $S = P_c/P_s(T)$, where P_c is the partial pressure of carbon, and

$$P_s(T) = -A/T + B \qquad (3)$$

is the equilibrium vapour pressure. The quantities A and B are constants for a given material and are determined in laboratory measurements. If the supersaturation ratio exceeds unity, vapour condenses to grains whereas, for $S < 1$, the inverse process, namely evaporation, takes place.

The problem, however, is complicated due to the fact that the phase transition takes place in the stellar radiation field. It is hence necessary to take the energy equilibrium into account, i.e. the radiation absorbed and re-emitted by the dust grains. If we neglect the energy exchange between grains and molecules, the expression for the energy equilibrium can be written in the following form:

$$4\pi r^2 \sigma T_e^4 WQ(T_e, r) = 4\pi r^2 \sigma T_d^4 Q(T_d, r) \qquad (4)$$

where r and T_d are the radius and temperature of a dust particle and Q(T,r) the Planck-averaged absorption coefficient.

According to Lefevre (1979), the supersaturation ratio at the surface of the dust particle, with the temperature T_d, is determined by the relation

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$$S = \frac{P_c}{P_s(T)} \left(\frac{T_d}{T_g}\right)^{1/2} \qquad (5)$$

where the equilibrium vapour pressure P_s corresponds to $T = T_d$.

Firstly, it is necessary to find the radial distance of the layer where the supersaturation ratio is $S = 1$. This layer is the inner boundary of the condensation region since, at smaller radii, the supersaturation ratio is $S < 1$. Unfortunately, a direct estimate of this radial distance is impossible since the temperature of the dust particles is determined by their mean radius which is an unknown quantity. The results given below were obtained for the grain radius $r = 10$ Å which is typical for the condensation region.

The radial distance of the saturation level increases rapidly with effective temperature (Fig. 1). This is caused by the stellar radiation which heats the dust particles. The radial distance of the saturation layer also increases when the rate of mass loss decreases since, at the resulting smaller gas densities, saturation of carbon can be reached only when the temperatures of gas and dust particles are also lower. As can be seen from Figure 2, an increased radial distance of the saturation layer leads, furthermore, to an increase of the temperature differences between gas and dust.

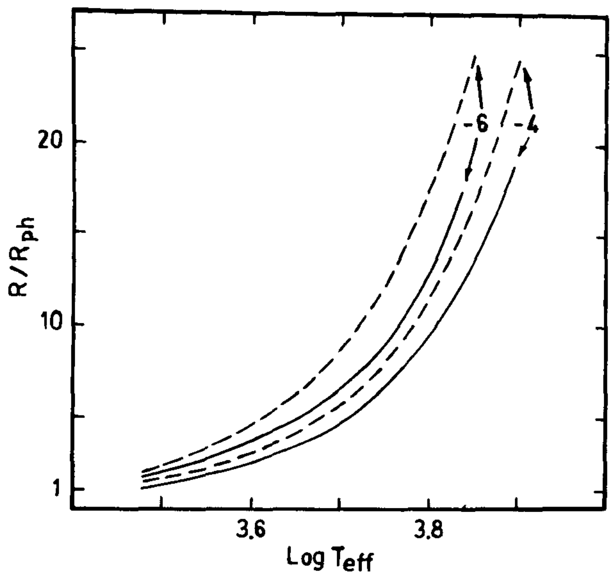

Fig. 1: Radial distances of the saturation layer (fully drawn) and the layer of maximum nucleation (dashed) as a function of the effective temperature for $\log(\dot{M}/M_\odot/yr) = -6, -4$.

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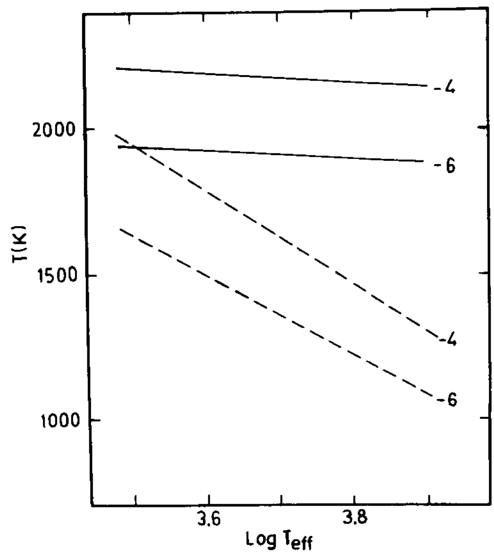

Fig. 2: Temperature of dust particles (fully drawn) and temperature of gas (dashed) at the saturation layer as a function of effective temperature for log $(\dot{M}/M_{\odot}/yr) = -6, -4$.

S = 1 for condensation is only a necessary condition. In reality, vapour has to be supersaturated before a perceptible part of carbon can be transformed into a solids. In supersaturated vapour, the formation of cluster becomes possible due to the agglomeration of monomer molecules. The equilibrium distribution of the resulting clusters of n molecules is determined by the following relation:

$$N_n = N_1 \exp(-\Delta G_n/KT), \quad (6)$$

where N_1 is the concentration of monomers and ΔG_n the free energy from the cluster formation.

Of decisive importance is the formation of critical-sized clusters. In these sufficiently large clusters or nuclei, the further accretion of monomers is more probable than evaporation. According to Draine and Salpeter (1977), the number of monomers contained in such a nucleus is governed by the following relation:

$$n_* = 1 + (\theta / T_d \ \ln S)^3 \quad (7)$$

where

$$\theta = 2 (4 \pi/3)^{1/3} \ \Omega^{2/3} \sigma / K \quad (8)$$

is the bulk volume of the molecule in its solid form and σ the surface tension. Using relations (7) and (8), one can easily obtain the expression for the free energy of nucleus formation:

$$\Delta G_*/KT = 1/2 \, (\theta/T_d)^3 \, / \, (\ln S)^2 \qquad (9)$$

It should be noted that the theory of homogeneous nucleation can be used only for rather large values of n since, for very small clusters, the conception of the surface tension energy becomes meaningless.

The rate of nucleation

$$J = Z \, \omega N_1 \, \exp(-\Delta G_*/KT) \qquad (10)$$

gives the number of nuclei formed per unit time and per unit volume. Here, Z is the Zeldovich factor, taking into account the fact that the real cluster distribution differs from the equilibrium distribution. According to Draine (1981),

$$Z = (n_* - 1)^{-2/3} \, ((1/6\,\pi)\,(\theta/T_d))^{1/2} \qquad (11)$$

The second factor in (10) gives the accretion rate of monomers colliding with the nucleus:

$$\omega = \alpha_s \, (4\,\pi r_*^2) \, P_c \, / \, (2\,\pi\, mKT_g)^{1/2} \qquad (12)$$

where α_s is the sticking coefficient, r_* the radius of the nucleus and m the mass of the monomer.

The rate of nucleation increases almost exponentially with increasing supersaturation. Simultaneously, the growth of the particles reduces the density of the vapour molecules. Further condensation, hence, is controlled by the growth of the already condensed particles and by the mean frequency of monomer collisions. The time-dependence of the main quantities describing the condensation process in a R CrB star model with T_e = 6000 K, M_b = -5 mag and $\dot{M} = 10^{-6}$ $M_\odot$/yr is shown in Figure 3.

The growth of the mean size of the particles after the nucleation rate maximum has been reached is accompanied by the growth of the optical depth of the dust shell and by the increase of the radiative pressure acting on the dust. As a result, dust grains and gas molecules are accelerated, the latter due to momentum transfer from the dust grains so that, within the time interval of about one month, the velocity of the gas exceeds 100 kms^{-1}. The drift velocity of the dust grains through the gas is in the range 10 kms^{-1} to 20 kms^{-1}.

The deviation of the phase transition from the dynamical equilibrium increases both with increasing effective temperature and decreasing mass loss rate (see also Fig. 2). Figure 4 shows the radial dependences of the supersaturation ratio S and the nucleation rate J, calculated for the mass loss rates $\dot{M} = 10^{-6}$, 3.16 x 10^{-6} and 10^{-7} $M_\odot$/yr. As we have seen previously, decrease of M corresponds to a decrease of the gas density. This leads, hence, to a decrease of the collision frequency. In this case, the exponential growth of the nucleation rate is confined to higher supersaturation ratios which lead not only to an increase of the number density of dust grains but also to a decrease of their finite mean radius.

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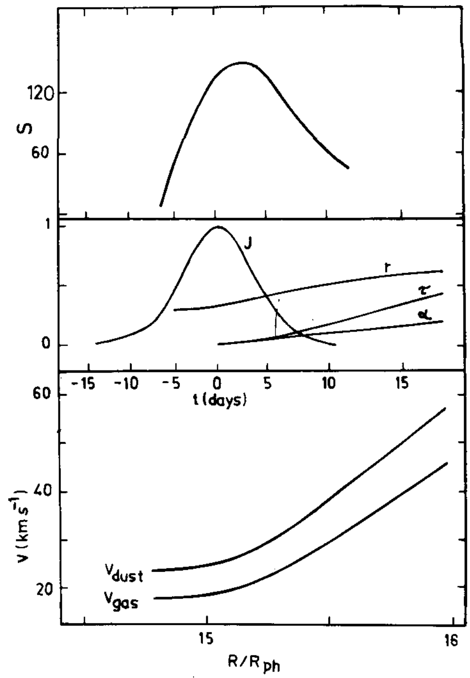

Fig. 3: Supersaturation ratio S, nucleation rate J, mean grain radius r, optical thickness , condensation degree , dust velocity v_{dust} and gas velocity v_{gas} as a function of time in the model with T_e = 6000 K and $\dot{M} = 10^{-6}$ $M_\odot$/yr.

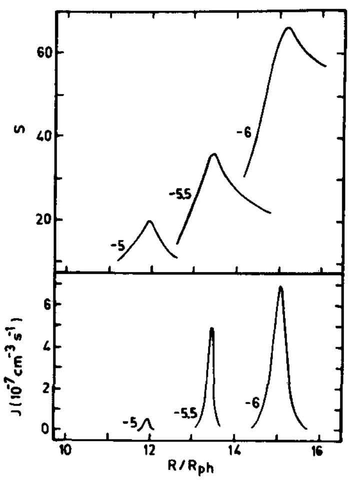

Fig. 4: Supersaturation ratio S and nucleation rate J as a function of radial distance in the model with T_e = 6000 K. The mass loss rates are as follows: log ($\dot{M}/M_\odot$/yr) = -5, -5.5, -6.

In the limit of very low mass loss, the mean collision time becomes comparable with the expansion time of the outer layers. In this case, carbon does not condense, even if $S > 1$. Unfortunately, the current theory of homogeneous nucleation does not allow this process to be studied in detail, since the number of molecules contained in the critical-sized cluster reduces to the order of unity.

At the effective temperature $T_e \approx 6000$ K, the final mean radius of the dust grains r_∞, depends on the mass loss rate as follows:

$$\mathrm{Log}\ r_\infty = -0.67 + 0.81\ \mathrm{Log}\ \dot{M}, \quad (13)$$

where r_∞ and $\dot{M}$ are expressed in units of cm and $M_\odot$/yr, respectively. This relation can be used to estimate the mass loss rate, provided that the radii of the dust grains are known from observations. For instance, recent IUE observations of R CrB and RY Sgr (Holm et al., 1982; Hecht et al., 1984) have shown the existence of carbon particles with radii ranging from 50 A to 600 A. These radii then correspond to mass loss rates from 1.2×10^{-7} $M_\odot$/yr to 2.5×10^{-6} $M_\odot$/yr.

Of great importance is the comparison of the observed light curves of R CrB stars with predictions. Let us assume that the dust grains completely re-radiate the stellar energy. Then the decline in visual

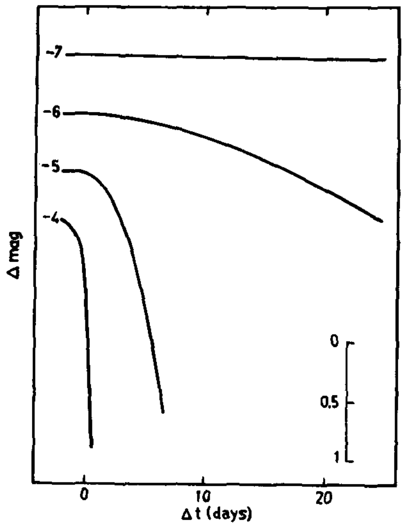

Fig. 5: Temporal behaviour of the visual brightness for the model with T_e = 6000 K. The mass loss rates are log $(\dot{M}/M_\odot/\mathrm{yr})$ = -7, -6, -4.

brightness Δm is related to the optical thicknes of the dust cloud as $\Delta m = 1.086\tau$. In Figure 5, for models with T_e = 6000 K and mass loss rates of $\dot{M} = 10^{-7}$, 10^{-6}, 10^{-5} and 10^{-4} $M_\odot$/yr, the variation of Δm with time is shown. Accordingly, for $\dot{M} > 10^{-5}$ $M_\odot$/yr, the rate of brightness decline is faster than observed, and we conclude that dust formation in R CrB stars occurs in outflowing matter with rates $\dot{M} < 10^{-5}$ $M_\odot$/yr. Figure 6 shows the predicted rate of the visual brightness decline,

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dm/dt, as a function of the stellar effective temperature. We see that the spherically symmetric model of dust shell formation predicts a decrease of dm/dt with increasing T_e, for a given mass loss rate $\dot{M}$. This behaviour comes about because at higher effective temperatures the condensation process takes place at lower gas densities which results in a decrease of the growth rate of dust particles with T_e. In Figure 6 also the observed values of T_e and dm/dt for R CrB and RY Sgr, respectively, are reproduced (error bars). From this diagram follows that the mass loss rate in these stars is of the order of 10^{-5} $M_\odot$/yr to 10^{-6} $M_\odot$/yr.

If we insert in Figure 6, the typical rate brightness decline for the (cool) R type R CrB stars, namely dm/dt = 0.05 mag/day, then we read off a mass loss rate of only $M \approx 10^{-7}$ $M_\odot$/yr, which is below what has been observed and far below the rate obtained for the hotter stars, R CrB and RY Sgr. If M varies with T_e (at constant luminosity), then one should expect a decrease with T_e rather than an increase. Hence, we conclude that the spherically symmetric model is not adequate to describe dust around R CrB stars (see also the review given by Feast, these proceedings).

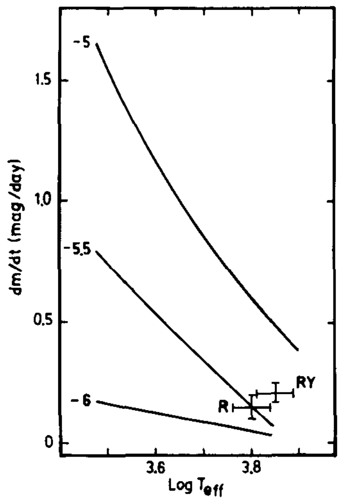

Fig. 6: The rate of the visual brightness decrease as a function of effective temperature. the mass loss rates are log ($\dot{M}/M_\odot$/yr) = -6, -5.5, -5. The observational estimates of T_e and dm/dt for R CrB and RY Sgr are shown as R and RY, respectively.

4. CONCLUSIONS

Application of the homogeneous nucleation theory to the problem of R CrB stars allows the conclusion that the grain formation process can occur only at larger distances from the stellar surface. At the effective temperature T_e = 6000 K, the radius of the inner boundary of the dust shell is in the range of 10 to 15 photospheric radii. In order to supersaturate carbon vapour one has to postulate another mechanism for the origin mass loss. At present we only know of one mechanism, namely the radial stellar oscillations with accompanying shock waves.

If the matter were leaving the star in a spherically symmetric way, the formation of an optically thick dust shell would happen for $\dot{M} \gtrsim 10^{-6}$ $M_{\odot}$/yr at T_e = 6000 K. However, observations indicate that the geometry of the dust clouds formed in R CrB stars is non-spherical. Thus, the above estimates of M are an upper limit. The actual mass loss rate depends on what fraction of the surface area is covered by the dust cloud.

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