

ADAMS, M. and GUILLEMIN, V., *Measure theory and probability* (Wadsworth Mathematics Series, Wadsworth, 1986), \$32.40.

This book was written for the purpose of teaching measure theory to undergraduates, using probability to supply motivation and examples. This stands in contrast to Kingman and Taylor (1966), who lay a thorough foundation in measure theory in order to discuss probability and stochastic processes; to Billingsley (1979), who provides a graduate introduction; and to Grimmett and Stirzaker (1982), who emphasize the probability theory and give only a sketch of requisite measure theory.

The purpose is reflected in the choice of topics. The book is organized into three chapters, respectively Measure Theory, Integration, and Fourier Analysis. The discussion of measure theory employs coin sequences, and later random walks, to motivate and develop a discussion of an abstract probability measure space. A good touch is that the authors introduce the discrete Dirichlet Problem at an early stage, which should encourage those students who might otherwise rebel at the impending dose of technicalities about rings of sets, set functions, and so forth.

Measure theory is used to develop the Lebesgue integral and associated theory. This is applied to describe the expectation of a random variable. I think a probabilist might wish to reverse this order, discussing first the expectation as a linear operator (hence preparing the ground for functional analysis), and then exhibiting Lebesgue measure on $[0, 1]$ as a special example.

The book closes with a discussion of Fourier analysis, introducing the Lebesgue spaces $\mathcal{L}^1(X, \mu)$ and $\mathcal{L}^2(X, \mu)$ (and $\mathcal{L}^p(X, \mu)$ in exercises), Fourier series and integrals, and applying the results to study random walks, Toeplitz forms, and the Central Limit Theorem. These applications seem the most attractive part of the book: the authors give careful and clean discussions. Note however that the distinction between a function and its "almost-everywhere" equivalence class, while stated, is rather underplayed.

There are two appendices: one on metric spaces and the other on \mathcal{L}^p -spaces.

The authors have wisely set themselves a limited objective, to use probability to explain measure theory, and have in my view succeeded. However, it is interesting to consider whether their objective could have been attained in any other ways. For example in the light of recent advances in expert systems it would be appropriate to use Bayesian statistics as well as coin-tossing to motivate measure theory. This would inevitably require more material on conditional probability and expectation: no bad thing since the idea of conditioning is pervasive through all probability theory. Following from this it would be natural to adopt the linear operator approach to expectation, which would provide a good jumping-off point for further courses on analysis.

In conclusion, this would make a good text from which to teach a first course in measure theory. A course in probability theory would require a more probabilistic bias, though the book would make a useful supplementary text.

REFERENCES

- P. BILLINGSLEY, *Probability and Measure* (Wiley, New York, 1979).
 G. R. GRIMMETT and D. R. STIRZAKER, *Probability and Random Processes* (OUP, Oxford, 1982).
 J. F. C. KINGMAN and S. J. TAYLOR, *Introduction to Measure and Probability* (CUP, Cambridge, 1966).

W. S. KENDALL

CHAO, J.-A. and WOYCZYŃSKI, W. A. (eds.) *Probability theory and harmonic analysis* (Pure and Applied Mathematics: a series of monographs and textbooks, Vol. 98, Marcel Dekker, 1986), 312 pp., \$71.50.

This is composed of fifteen papers on various aspects of probability and harmonic analysis, arising from a conference in Cleveland in 1983 and from various seminars. As mentioned in the preface, topics range from martingales, stochastic integrals, diffusion processes on manifolds, through random walks and harmonic functions on graphs, random Fourier series, to invariant differential and degenerate elliptic operators, and singular integral transforms. I shall mention just

three articles corresponding to my own interests. Chavel and Feldman discuss the Wiener sausage (the tube swept out by a ball undergoing Brownian motion) in the novel context of a Riemannian manifold. Durrett expounds aspects of the theory of reversible diffusion processes, including Brownian motion on manifolds and reversible diffusions in random media. Gray, Karp, and Pinsky discuss the average time at which Brownian Motion escapes from a tube in a Riemannian manifold. In particular they provide a stochastic characterization of minimal embeddings in the 3-sphere.

There is something in this collection for almost every aficionado of probability or harmonic analysis: certainly a useful book to recommend for library purchase.

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FISHER, S. D. *Complex variables* (Wadsworth Mathematics Series, Wadsworth, 1986), xii + 403 pp., \$39.90.

Quot homines, tot sententiae! The old Latin adage seems applicable to the teaching of complex analysis to judge by the number of books on the market. Various potential audiences have to be catered for and seeking a book which is just right for a particular group of students can readily lead to a non-existence theorem. For some tastes, the present book may contain too little on certain topics or too much on others. However, there should be enough to keep everybody reasonably happy.

A distinctive feature of the book is the amount of space allocated to applications. There are five long chapters of which the last two could be regarded almost entirely as applications of the theory. Earlier chapters also contain some cultural digressions but these are clearly marked and can be omitted if desired. A prominent role is taken by the topic of conformal mappings and even in the first chapter the seeds are being sown in preparation for a bumper harvest. At the end of the book, in Appendix 2, there is a useful list (an atlas, possibly?) of standard conformal mappings with diagrams for easy reference.

The first three chapters contain most of the standard material which is found in any first course on complex analysis. Indeed the needs of applied mathematicians, engineers *et al.* will probably be completely met. In the case of pure mathematicians, however, one or two ingredients could be missing. A glance at the index reveals the absence of such items as uniform convergence, the Weierstrass M -test and the principle of analytic continuation. To be fair, some of the arguments for which the M -test is often used are developed in the exercises. One of the standard examples to which analytic continuation relates is the gamma function. Surprisingly perhaps, this does not appear until page 332 and then merely in the exercises, where one instance of analytic continuation can be found nestling.

Chapter 4 deals with applications of analytic and harmonic functions. Laplace's equation, the Poisson integral representation and Green's functions for boundary-value problems are discussed. There is also a section showing how Laplace's equation is connected with problems in electrostatics, elasticity, steady-state heat flow and flow of an ideal fluid. The latter develops ideas contained in optional sections of earlier chapters. Conformal mappings including the Schwarz-Christoffel transformation are extensively employed.

Chapter 5 deals with transform methods. The Fourier and Laplace transforms are discussed and there is a mention of the Fast Fourier Transform. There is also a section on the Z -transform with applications to control theory and, in particular, the stability of a discrete linear system. (Stability of solutions of a system of linear differential equations is treated in one of the optional sections of Chapter 3.)

For the most part, the book is well written. Periodically, things go slightly wonky, as on pages 308–9 where the heat kernel has the wrong constant and a number of 2's are missing during the first part of Example 13. Again, a purist may carp at statements that a set has no interior or boundary (pages 25 and 28). Over 220 worked examples are sprinkled liberally throughout the