

CLUSTER CORRELATIONS FOR SCALE-FREE SPECTRA

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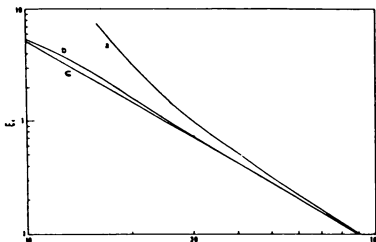
The cluster autocorrelation function ξ_c and the galaxy-cluster cross correlation ξ_{gc} are used to test the biased structure formation for scale-free spectra $P(k) \propto k^n$. Following Kaiser (1984), we assume that rich clusters form only at high density regions with the matter distribution represented by a Gaussian random field. Then, the correlation ξ of two regions with characteristic scales R_1 and R_2 lying above the thresholds ν_1 and ν_2 ($\bar{\delta} \equiv \nu\sigma$), is given by the expression for the bivariate Gaussian

$$\xi(r, R_1, \nu_1, R_2, \nu_2) = -1 + \frac{2}{\pi^{1/2}} \left[\operatorname{erfc}\left(\frac{\nu_1}{2^{1/2}}\right) \operatorname{erfc}\left(\frac{\nu_2}{2^{1/2}}\right) \right]^{-1} \int_{-\infty}^{\infty} dx e^{-x^2} \operatorname{erfc}\left[\frac{2^{-1/2} \nu_2 - z x}{(1-z^2)^{1/2}} \right],$$

where $z(r)$ is the covariance function, i.e. $z \equiv \xi_c(r)/\xi_c(0)$ and $\xi_c(r)$ is the correlation of the matter distribution Gaussian filtered on the comoving scales R_1 and R_2 . From the previous equations, one can obtain for $P(k) \propto k^n$, $z \ll 1$ and large r a power-law form either for ξ_c or ξ_{gc} . Moreover, the amplification factor $A_{c-gc} \equiv \xi_c/\xi_{gc}$ is

$$A_{c-gc} \approx \left[\frac{G(\nu_1)}{G(\nu_2)} \right]^{1/2} 2^{n+1} \left(\frac{R_2}{R_1} \right)^{\frac{n+3}{2}}, \quad G(\nu) \equiv e^{-\nu^2} \left[\operatorname{erfc}\left(\frac{\nu}{2^{1/2}}\right) \right]^{-2}.$$

If we want to reproduce the observed autocorrelation $\xi_c = (r_0/r)^{1.8}$, $r_0 \approx 25 h^{-1} \text{ Mpc}$, we should then take $n = -1.2$, $R_c = 4.5 h^{-1} \text{ Mpc}$ (which implies $\nu_c = 2.9$ for the observed number density $n_c \approx 6 \times 10^{-6} h^3 \text{ Mpc}^{-3}$). Let us consider that the same index $n = -1.2$ applies to galaxies, then one obtain in the best case for the values of R_g - allowed by the observed masses of galaxies - a decrement $A_{c-gc} \approx 0.2$. This is in clear contrast with the observations, where it is found an amplification of ≈ 4 . The previous result do not changes for different R_c . Therefore, our conclusion is that a scale-free spectrum can reproduce ξ_c (see Fig.) but is in contradiction with ξ_{gc} .



Cluster autocorrelation function $\xi_c(r)$ for $R_c = 4.5 h^{-1} \text{ Mpc}$.
 a) Exact expression.
 b) Approximated $\xi_c \approx G(\nu)z$.
 c) Observed $\xi_c = (25/rh)^{1.8}$.