

## THE J vs. M RELATION FOR BINARY STARS

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ABSTRACT. For a given mass  $M$  and environmental temperature  $T$ , there is a well-defined angular momentum  $J_{\max}$  above which physical systems cannot exist as self-gravitating entities. The quantity  $J_{\max} \propto M^2 T^{-1/2}$ . Observations of  $J$  and  $M$  in young binary systems should put useful constraints on the temperature of the medium from which they formed.

Brosche (1963) was the first to publicize the idea that a plot of  $\log J$  vs.  $\log M$  for a wide range of astronomical systems shows a strong correlation fit nicely by a line of slope +2, i.e.,  $d \log J / d \log M = 2$ . In his initial analysis, Brosche identified this relationship in a plot that contained systems ranging in mass from planet-satellite systems orbiting the Sun to the local supercluster of galaxies--a range covering more than 20 decades in mass and more than 40 decades in  $J$ . (If this plot is extended in angular momentum over an additional 60 decades--i.e., covering a total of more than 100 decades in  $J$ --the point defined by the Planck mass and  $H$  falls very nearly on the same line than runs through the points in Brosche's diagram!) It behooves us to understand the physical origin of this universal  $d \log J / d \log M$  relation.

A number of discussions of the  $d \log J / d \log M$  relation can be found scattered through the literature over the past twenty-five years. Many (Carrasco *et al.* 1982, and references cited therein; Trimble 1984) have focused on systems having a limited mass or period range--such as normal binary star systems--for which data correlations often show  $d \log J / d \log M$  closer to +5/3 than to +2. Our discussion will be confined to Brosche's more universal slope.

Because very slowly rotating objects or, ideally, zero angular momentum systems fall well below the data shown in Brosche's Fig. 2, it is unreasonable to adopt the line of slope +2 as an absolute correlation obeyed by all physical systems. Instead, it is preferable to identify from Brosche's paper an upper envelope that Nature sets on  $J(M)$  for all systems. With this in mind, we should not only be concerned with explaining the slope but also the absolute location of the line (i.e., its  $y$ -intercept) in the  $\log J$ - $\log M$  plane.

It appears as though both the position and slope of this upper

envelope can be explained in terms of the characteristic sound speed--or temperature--of the medium from which astronomical systems form under the influence of gravity. For a given total mass and sound speed  $c$ , there is an angular momentum  $J_{\max}$  above which a system cannot become, or cannot exist as, a self-gravitating entity. The limit is set by (Tohline and Christodoulou 1988)

$$J_{\max} = f \frac{G}{c} M^2, \quad (1)$$

where  $G$  is the gravitational constant and  $f \approx 0.1$  is a dimensionless coefficient. (Alternatively, for a given  $M$  and  $c$ , the maximum allowed orbital period  $P_{\max} = 2\pi f^3 GM/c^3$ .) This limit on  $J$  can also be ascertained from Chandrasekhar's (1961) dispersion relation for gravitational instability in a uniformly rotating, infinite homogeneous medium. Clearly, for fixed  $c$ , this relation demands an upper envelope that is a line of slope +2 in a  $\log J - \log M$  diagram. Furthermore, the value of  $J_{\max}$  is reasonable. All of Brosche's data lies below the line defined by a sound speed of  $0.3 \text{ km s}^{-1}$ , indicating that the angular momentum for all these systems has been limited by environments warmer than 10 K.

Using relation (1), it is interesting to examine data sets having a restricted range of masses or orbital periods, such as the spectroscopic binaries discussed by Trimble (1984). Virtually all binaries shown in her Fig. 8 having  $P > 1000$  days lie below the envelope set by a temperature  $T = 1000 \text{ K}$  ( $c = 3 \text{ km s}^{-1}$ ). This strongly suggests that these "long period" binary systems formed in an environment having  $T \gtrsim 1000 \text{ K}$ . The short period ( $P < 2$  days) systems shown in Trimble's Fig. 8 are confined below a line defined by relation (1) and a temperature  $\sim 10^6 \text{ K}$ , reflecting the much warmer environment in which the stellar components of these systems reside.

Models that have been developed to explain the dynamical process of star formation have generally included standard physical processes which are believed to be important in dictating what the gas temperature  $T(\rho)$  is as a function of the gas density  $\rho$  during cloud collapse. Direct checks of this  $T(\rho)$  relation are precluded by the large optical depth in protostellar clouds and/or the spatial resolving power of our present day instrumentation. An indirect test of the environmental temperature at which binary systems form may be available, however, through relation (1). Careful observational documentation of the  $J$  vs.  $M$  relationship among the youngest binary star systems can be used to put useful constraints on physical models of star formation.

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