REFERENCE SYSTEMS and ASTRONOMICAL STANDARDS

https://doi.org/10.1017/S0252921100046923 Published online by Cambridge University Press

EPHEMERIS ASTRONOMY DEFINITIONS AND CONSTANTS IN GENERAL RELATIVITY

VICTOR A. BRUMBERG[1]
Bureau des Longitudes
Paris, France

Abstract. Currently employed definitions of ephemeris astronomy and the system of astronomical constants are based on Newtonian mechanics with its absolute time and absolute space. To avoid any relativistic ambiguities in applying new IAU (1991) resolutions on reference systems (RS) and time scales one should specify the astronomical constructions and definitions of constants to make them consistent with general relativity (GRT). Such an approach is developed with the aid of the existing relativisting hierarchy of relativistic reference systems and time scales.

1. Introduction

To avoid any relativistic ambiguity the present system of astronomical constants and basic concepts of ephemeris astronomy should be formulated in the GRT framework more thoroughly. This is not a question of changing any numerical value or modifying any traditional concept. The problem is to indicate a corresponding RS (with its time–scale) admitting the unambiguious definition of such constant or conceptual construction. Just this approach based on the hierarchy of the relativistic RSs has been suggested in (Brumberg *et al.*, 1996). Generally speaking, it might be possible to develop ephemeris astronomy concepts, including the system of astronomical constants, not using coordinate systems at all (following the lines of Synge, 1960). But the RS approach enables one to retain the well–elaborated techniques of classical astronomy. One may argue that this problem is not actual with respect to its practical application. Indeed, for most applications one may forget about GRT effects provided that one uses a suitable, well–defined RS. The problem is to indicate such RS and its relationship with the RSs maintained by observations, such as ICRS and ITRS.

[1]On leave from the Institute of Applied Astronomy, St. Petersburg, Russia

I. M. Wytrzyszczak, J. H. Lieske and R. A. Feldman (eds.),
Dynamics and Astrometry of Natural and Artificial Celestial Bodies, 439, 1997.

https://doi.org/10.1017/S0252921100046923 Published online by Cambridge University Press

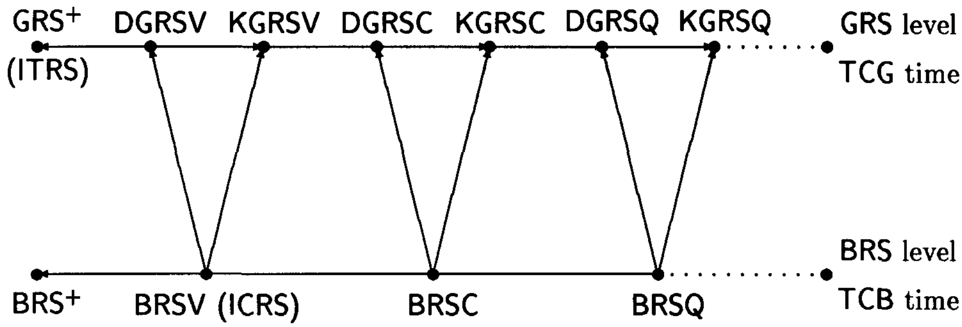

Figure 1. Barycentric and Geocentric Reference Systems (RSs). The following notation is used: B — barycentric, G — geocentric, V — VLBI, C — ecliptical, Q — equatorial, D — dynamical, K — kinematical, $^+$ — rotating, ICRS — IERS Celestial Reference System, ITRS — IERS Terrestrial Reference System.

2. Reference Systems

For the sake of brevity we reproduce here Figure 1 from Brumberg *et al.* (1996) demonstrating the basic RSs at the barycentric and geocentric level. All the designations are self–evident. BRSV is identified here with ICRS as described in Arias *et al.*(1995). GRS$^+$, a geocentric Earth–fixed RS rotating with the Earth, is identified with ITRS (McCarthy, 1992). In doing this, we ignore a small distinction which can be easily taken into account. Indeed, ITRS is often related in practice to TT (TDT in terms of the IAU (1976) recommendations). In accordance with the IAU (1991) recommendations

$$\mathrm{TT} = (1 - L_G)\mathrm{TCG} \tag{2.1}$$

with $L_G = 6.96929 \cdot 10^{-10}$ (Fukushima, 1995). This means that the values of geocentric coordinates $\mathbf{y}$ and mass-factors $G\hat{M}$ in ITRS and GRS$^+$ differ by the same scaling factor

$$(\mathbf{y})_{\mathrm{TT}} = (1 - L_G)(\mathbf{y})_{\mathrm{TCG}}, \qquad (G\hat{M})_{\mathrm{TT}} = (1 - L_G)(G\hat{M})_{\mathrm{TCG}}. \tag{2.2}$$

Just the same relations exist for TDB and TCB quantities. But the corresponding scalar factor $L_B = 1.5505197 \cdot 10^{-8}$ is larger than L_G (Fukushima, 1995). This is the possible explanation why L_G and TT were retained in the IAU (1991) recommendations. One may anticipate that eventually TT will be replaced by TCG.

Mathematical formulations of BRS and GRS transformation, as well as BRS$(t, \mathbf{x})$ $\rightarrow$GRS$(u, \mathbf{w})$, are described in detail in Klioner and Voinov (1993; see also the references therein). There is no need in reproducing them here. The present IAU recommendations give only the initial terms of BRS and GRS metric tensors. The BRS metric tensor with the full post–Newtonian accuracy is used now practically in deriving the BRS post–Newtonian equations of motion. The post–Newtonian BRS→GRS transformation should be served, in future, to specify ICRS→ITRS transformation, as used in IERS.

All RSs shown in Figure 1 may be related to ICRS or ITRS. Indeed, the comparison of the analytical theories of motion of the major planets with observations enables one to construct BRSC with the plane of ecliptic for some definite epoch as the main reference plane. The mutual orientation of the spatial axes of BRSC and BRSV is given by a constant rotation matrix P_C. Under four–dimensional transformation BRS→GRS we get at the G level GRSV and GRSQ for versions D and K. A constant rotation is preserved at the G level. Therefore, GRSC and GRSV are related by means of P_C. K and D versions differ by the geodesic rotation matrix F for V systems and F_C for C systems. The main reference plane of GRSC may be regarded as the ecliptical plane (in K and D versions) at the G level. On the other hand, using an analytical theory of the Earth's rotation in DGRSC, one determines from observations ERP resulting in $\hat{P}(u)$, a rotation matrix relating GRS$^+$ and DGRSC (u=TCG). Matrix $\hat{P}(u)$ may be expressed in terms of three Euler angles φ, θ and ψ . The main reference plane of GRS$^+$ is the equator of date. Having determined its orientation with respect to DGRSC, one may find GRSQ (for versions K and D) with the equator for the chosen epoch as the main reference plane. K and D versions differ here by the geodesic rotation matrix F_Q. Hence, the mutual orientation of the spatial axes of GRSQ and GRSV may be determined with the aid of some constant rotation matrix P_Q. At the B level one may construct then BRSQ, a matrix differing from BRSV by P_Q and resulting to GRSQ under four–dimensional transformation BRS→GRS. The main reference plane of BRSQ may be regarded as the equator for the chosen epoch at the B level. The equivalent of GRS$^+$ at the B level is BRS$^+$, an Earth–fixed RS rotating with the Earth at the B level. BRS$^+$ and DBRSC are related by the rotation matrix $P(t)$ in terms of TCB ($= t$).

This description may be expressed mathematically as follows (using IERS notation):

G level for D version:

$$[\mathrm{GRS}^+] = \hat{P}(u)[\mathrm{DGRSC}] = \hat{P}(u)P_C[\mathrm{DGRSV}] = \hat{P}(u)P_C P_Q^T[\mathrm{DGRSQ}]. \tag{2.3}$$

G level for K version:

$$[\mathrm{GRS}^+] = \hat{P}_K(u)[\mathrm{KGRSC}] = \hat{P}_K(u)P_C[\mathrm{KGRSV}] = \hat{P}_K(u)P_C P_Q^T[\mathrm{KGRSQ}]. \tag{2.4}$$

B level:

$$[\mathrm{BRS}^+] = P(t)[\mathrm{BRSC}] = P(t)P_C[\mathrm{BRSV}] = P(t)P_C P_Q^T[\mathrm{BRSQ}]. \tag{2.5}$$

K and D versions:

$$[\mathrm{KGRSV}] = (E - c^{-2}F)[\mathrm{DGRSV}], \tag{2.6}$$

$$[\mathrm{KGRSC}] = (E - c^{-2}F_C)[\mathrm{DGRSC}], \tag{2.7}$$

$$[\mathrm{KGRSQ}] = (E - c^{-2}F_Q)[\mathrm{DGRSQ}], \tag{2.8}$$

$$\hat{P}(u) = \hat{P}_K(u)(E - c^{-2}F_C) \tag{2.9}$$

with

$$F_C = P_C F P_C^T, \qquad F_Q = P_Q F P_Q^T \tag{2.10}$$

and

$$\hat{P}(u) = D_3(\varphi)D_1(-\theta)D_3(-\psi), \qquad \hat{P}_K(u) = D_3(\varphi_K)D_1(-\theta_K)D_3(-\psi_K), \tag{2.11}$$

D_i being elementary rotation matrices. The matrix components F_C^{ij} may be expressed in terms of the vectorial components F_C^k $(i, j, k = 1, 2, 3)$

$$F_C^{ij} = \varepsilon_{ijk}F_C^k, \qquad \varepsilon_{ijk} = \frac{1}{2}(i-j)(j-k)(k-i). \tag{2.12}$$

F_C^k are evaluated in (Brumberg *et al.*, 1992). The initial terms of these expressions are reproduced here:

$$c^{-2}F_C^1 = 0''\!.955 \cdot 10^{-8}t - 1''\!.303 \cdot 10^{-6}\cos(\lambda_3 + D - F) + \ldots, \tag{2.13}$$

$$c^{-2}F_C^2 = 0''\!.119 \cdot 10^{-8}t - 1''\!.303 \cdot 10^{-6}\sin(\lambda_3 + D - F) + \ldots, \tag{2.14}$$

$$c^{-2}F_C^3 = 0''\!.019198830t + 153''\!.111 \cdot 10^{-6}\sin(\lambda_3 - 1.796598) + \\ + 1''\!.919 \cdot 10^{-6}\sin(2\lambda_3 - 3.5932) + \ldots \tag{2.15}$$

with the mean longitude λ_3 of the Earth and the Delaunay arguments D and F for the Moon. In these expressions t is measured in Julian years from J2000.0.

The angles φ_K, θ_K, ψ_K may be considered as given by observations while φ, θ, ψ are given by a theory of the Earth's rotation. From (2.9) and (2.11) one gets easily

$$\varphi - \varphi_K = -\frac{c^{-2}}{\sin\theta}\left(F_C^1 \sin\psi + F_C^2 \cos\psi\right), \tag{2.16}$$

$$\theta - \theta_K = c^{-2}\left(F_C^1 \cos\psi - F_C^2 \sin\psi\right), \tag{2.17}$$

$$\psi - \psi_K = c^{-2}\left[F_C^3 - \frac{\cos\theta}{\sin\theta}\left(F_C^1 \sin\psi + F_C^2 \cos\psi\right)\right]. \tag{2.18}$$

In taking into account only the geodesic precession and nutation in narrow sense, one has $F_C^1 = F_C^2 = 0$ and, hence, $\varphi = \varphi_K$, $\theta = \theta_K$. Expressions

(2.19)–(2.21) together with (2.16)–(2.18) show the difference between K and D systems at the G level.

Practically, BRSQ differs very little from BRSV and this difference may be often neglected. For estimation purposes one may put

$$P_Q = E, \qquad P_C = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\varepsilon & \sin\varepsilon \\ 0 & -\sin\varepsilon & \cos\varepsilon \end{pmatrix}, \tag{2.19}$$

ε being the mean obliquity.

It remains to indicate (Brumberg, 1996) that the Earth's rotation matrices at the B and G levels are related by

$$\hat{P}_K(u) = P(t^*) \tag{2.20}$$

where u and t^* satisfy the 'time ephemeris' equation (Fukushima, 1995)

$$u = t^* - c^{-2}A(t^*), \qquad \dot{A}(t) = \frac{1}{2}\mathbf{v}_E^2 + \overline{U}\left(t, \mathbf{x}_E(t)\right), \tag{2.21}$$

$\mathbf{v}_E$ is the BRS Earth's velocity and $\overline{U}(t, \mathbf{x})$ is the BRS potential of all solar system bodies excluding the Earth (external potential).

3. GRS Earth's Rotation Equations

To derive the GRS Earth's rotation equations with the main GRT terms, it is not necessary to specify here V, C or Q versions of GRS. The Newtonian potential in the GRS metric may be written as

$$\hat{U}(u, \mathbf{w}) = \hat{U}_E(u, \mathbf{w}) + Q_i(u)w^i + \hat{U}_T(u, \mathbf{w}) \tag{3.1}$$

with geopotential $\hat{U}_E(u, \mathbf{w})$, non–geodesic acceleration of the geocentre Q_i and tidal potential (Klioner and Voinov, 1993)

$$\hat{U}_T(u, \mathbf{w}) = \overline{U}\left(t^*, \mathbf{x}_E(t^*) + \mathbf{w}\right) - \overline{U}\left(t^*, \mathbf{x}_E(t^*)\right) - \overline{U}_{,k}\left(t^*, \mathbf{x}_E(t^*)\right) w^k \tag{3.2}$$

with comma denoting the derivative with respect to the indicated variable.

The Newtonian equations of the Earth's rotation involve the second derivatives of the tidal potential. Proceeding as in Klioner and Voinov (1993) and in Brumberg (1996) one gets

$$\hat{U}_{T,rs}(u, \mathbf{w}) = \overline{U}_{,rs}\left(t^*, \mathbf{x}_E(t^*) + \mathbf{w}\right). \tag{3.3}$$

Considering that

$$t - t^* = c^{-2}\mathbf{v}_E(t)\mathbf{w} \tag{3.4}$$

one may rewrite the previous expression as

$$\hat{U}_{T,rs}(u,\mathbf{w}) = \overline{U}_{,rs}\,(t, \mathbf{x}_E(t) + \mathbf{w}) - c^{-2}\overline{U}_{,rst}\,(t, \mathbf{x}_E(t) + \mathbf{w})\,\mathbf{v}_E(t)\mathbf{w}\,. \quad (3.5)$$

Both expressions are convenient to transform the BRS luni-solar and planetary tidal action on the Earth into the GRS quantities. By means of the technique by Fock (1955) the GRS equations of the Earth's rotation may be presented in the form

$$\frac{dS^i}{du} = Q^i + 2c^{-2}(1-q)\hat{I}\left(\varepsilon_{ijk}\dot{F}^j\hat{\omega}^k + \ddot{F}^i\right)\,. \quad (3.6)$$

Here S^i are the GRS components of the Earth's spin

$$S^i = \hat{\omega}^i\hat{I}_{kk} - \hat{\omega}^k\hat{I}_{ik} \quad (3.7)$$

expressed in terms of the GRS components $\hat{\omega}^i$ of the Earth's angular velocity and the GRS Earth's inertia moments

$$\hat{I}_{ij} = \int_{(E)} \rho w^i w^j d^3w\,, \quad (3.8)$$

ρ being the Earth's mass density. Q^i are the GRS components of the torque moment acting on the Earth

$$Q^i = \varepsilon_{ijk}\int_{(E)} \rho w^j \hat{U}_{T,k} d^3w = \varepsilon_{ijk}\hat{I}_{jm}\hat{U}_{T,km}(u,0) + \ldots\,. \quad (3.9)$$

For simplicity, we retain here only the quadrupole terms. Non–geodesic acceleration Q_i occurring in (3.1) has no influence since

$$\int_{(E)} \rho w^i d^3w = 0 \quad (3.10)$$

by the definition of GRS. The main relativistic terms indicated in the right–hand side of (3.6) result from the explicit GRT corrections to S^i and Q^i (Brumberg, 1995). In deriving them, the Earth was regarded as a spherical body with the constant value of $\hat{I} = \hat{I}_{kk}/3$. Thus, the direct GRT terms ignored in (3.6) are proportional to the Earth's non–sphericity. The GRT terms indicated explicitly in (3.6) are actually present in KGRS ($q = 0$) and absent in DGRS ($q = 1$).

4. GRS$^+$ Earth's Rotation Equations

Equations of the Earth's rotation are considered usually in GRS$^+(u, \mathbf{y})$. Let GRS$^+$ components of the Earth's angular velocity, its spin, its inertia and torque moments be designated by $\hat{\Omega}^i$, T^i, $\hat{J}_{ij}$ and M_i, respectively. Then together with

$$y^i = \hat{P}_{ik} w^k \tag{4.1}$$

one has

$$\hat{\Omega}^i = \hat{P}_{ik}\hat{\omega}^k, \quad T^i = \hat{P}_{ik} S^k, \quad \hat{J}_{ij} = \hat{P}_{ir}\hat{P}_{js}\hat{I}_{rs}, \quad M_i = \hat{P}_{ik} Q^k. \tag{4.2}$$

Matrix $||\hat{P}_{ik}||$ coincides with one of the rotation matrices occurred in (2.3) or (2.4). The inertia moments

$$\hat{J}_{ij} = \int_{(E)} \rho y^i y^j d^3y \tag{4.3}$$

are constant. If the spatial axes of GRS$^+$ coincide with the principal inertia axes of the Earth, then $\hat{J}_{rs} = 0$ for $r \neq s$ and

$$\hat{J}_{11} = \frac{1}{2}(B + C - A) \tag{4.4}$$

with the similar expressions for two other moments. A, B, C (don't mix with A in the time ephemeris equation) are the principal Earth's inertia moments. Then, equations (3.6) take the form

$$\frac{dT^i}{du} + \varepsilon_{ijk}\hat{\Omega}^j T^k = M_i + 2c^{-2}(1-q)\hat{J}\frac{d}{du}(\hat{P}_{ik}\dot{F}^k) \tag{4.5}$$

with $\hat{J} = \hat{J}_{kk}/3$,

$$T^1 = A\hat{\Omega}^1, \qquad T^2 = B\hat{\Omega}^2, \qquad T^3 = C\hat{\Omega}^3 \tag{4.6}$$

and

$$M_i = \varepsilon_{ijk}\hat{J}_{jm} V_{,km}(u, 0). \tag{4.7}$$

Here

$$V(u, \mathbf{y}) = \hat{U}_T(u, \mathbf{w}) \tag{4.8}$$

is the tidal potential in the GRS$^+$ coordinates with

$$V_{,km}(u, \mathbf{y}) = \hat{P}_{kr}\hat{P}_{ms}\hat{U}_{T,rs}(u, \mathbf{w}). \tag{4.9}$$

For $i = 1$ the first equation in (4.5) takes the form

$$A\frac{d\hat{\Omega}^1}{du} + (C-B)\hat{\Omega}^2\hat{\Omega}^3 = (C-B)V_{,23}(u,0) + 2c^{-2}(1-q)\hat{J}\frac{d}{du}(\hat{P}_{1k}\dot{F}^k). \tag{4.10}$$

Two other equations are similar.

The new IAU time–scales satisfy the relation

$$TCG = (1 - L_C)TCB - c^{-2}(A_p + \mathbf{v}_E\mathbf{w}) \tag{4.11}$$

where the solution of the time ephemeris equation (2.21) is presented in the form

$$c^{-2}A = L_C t + c^{-2}A_p\,, \qquad L_C = L_B - L_G. \tag{4.12}$$

In accordance with (3.3) the right–hand members of (4.5) should be calculated for the moment

$$TDB^* = (1 - L_G)TCG + c^{-2}A_p\,, \tag{4.13}$$

if originally expressed in TDB, or for the moment

$$TCB^* = (1 + L_C)TCG + c^{-2}A_p\,, \tag{4.14}$$

if originally expressed in TCB.

5. System of Astronomical Constants

The hierarchy of RSs of Section 2 and the equations of the Earth's rotation of Section 4 enable one to give the GRT-compatible interpretation of any astronomical constant and ephemeris astronomy concept. That was done in Brumberg *et al.*, (1996) and in Brumberg (1996). The estimations (2.13)–(2.15) and (2.16)–(2.19) together with the solution of the time ephemeris equation show the order of magnitude of necessary changes in operating with different GRSs or BRSs.

References

Arias, E.F., Charlot, P., Feissel, M., and Lestrade, J.-F.: 1995, "The extragalactic reference system of the International Earth Rotation Service (ICRS)", *Astron. Astrophys.* **303**, 604.

Brumberg, V.A.: 1995, "General relativistic description of the Earth's rotation in different reference systems", *J. Geodyn.* **20**, 181.

Brumberg, V.A.: 1996, "Reference systems and astronomical constants in general relativity", *Earth Rotation, Reference Systems in Geodynamics and Solar System, Journées 1995* (N. Capitaine, B. Kolaczek, S. Debarbat, eds), Warsaw, 29.

Brumberg, V.A., Bretagnon, P., and Francou, G.: 1992, "Analytical algorithms of relativistic reduction of astronomical observations", *Systèmes de Référence Spatio-Temporelles, Journées 1991* (N. Capitaine, ed.), Paris, 141.

Brumberg, V.A., Bretagnon, P., and Guinot, B.: 1996, "Astronomical units and constants in the general relativity framework", *Celest. Mech. & Dyn. Astron.*, in press.

Fock, V.A.: 1955, *The Theory of Space, Time and Gravitation*, State Tech. Publ., Moscow (in Russian).

Fukushima, T.: 1995, "Time ephemeris", *Astron. Astrophys.* **294**, 895.

Klioner, S.A. and Voinov, A.V.: 1993, "Relativistic theory of astronomical reference systems in closed form", *Phys. Rev. D* **48**, 1451.

McCarthy, D.D.: 1992, "IERS Standards (1992)" *IERS Tech. Note 13*, Obs. de Paris.

Synge, J.L.: 1960, *Relativity: The General Theory*, North–Holland, Amsterdam.