

ADAMS, M. and GUILLEMIN, V., *Measure theory and probability* (Wadsworth Mathematics Series, Wadsworth, 1986), \$32.40.

This book was written for the purpose of teaching measure theory to undergraduates, using probability to supply motivation and examples. This stands in contrast to Kingman and Taylor (1966), who lay a thorough foundation in measure theory in order to discuss probability and stochastic processes; to Billingsley (1979), who provides a graduate introduction; and to Grimmett and Stirzaker (1982), who emphasize the probability theory and give only a sketch of requisite measure theory.

The purpose is reflected in the choice of topics. The book is organized into three chapters, respectively Measure Theory, Integration, and Fourier Analysis. The discussion of measure theory employs coin sequences, and later random walks, to motivate and develop a discussion of an abstract probability measure space. A good touch is that the authors introduce the discrete Dirichlet Problem at an early stage, which should encourage those students who might otherwise rebel at the impending dose of technicalities about rings of sets, set functions, and so forth.

Measure theory is used to develop the Lebesgue integral and associated theory. This is applied to describe the expectation of a random variable. I think a probabilist might wish to reverse this order, discussing first the expectation as a linear operator (hence preparing the ground for functional analysis), and then exhibiting Lebesgue measure on $[0, 1]$ as a special example.

The book closes with a discussion of Fourier analysis, introducing the Lebesgue spaces $\mathcal{L}^1(X, \mu)$ and $\mathcal{L}^2(X, \mu)$ (and $\mathcal{L}^p(X, \mu)$ in exercises), Fourier series and integrals, and applying the results to study random walks, Toeplitz forms, and the Central Limit Theorem. These applications seem the most attractive part of the book: the authors give careful and clean discussions. Note however that the distinction between a function and its "almost-everywhere" equivalence class, while stated, is rather underplayed.

There are two appendices: one on metric spaces and the other on \mathcal{L}^p -spaces.

The authors have wisely set themselves a limited objective, to use probability to explain measure theory, and have in my view succeeded. However, it is interesting to consider whether their objective could have been attained in any other ways. For example in the light of recent advances in expert systems it would be appropriate to use Bayesian statistics as well as coin-tossing to motivate measure theory. This would inevitably require more material on conditional probability and expectation: no bad thing since the idea of conditioning is pervasive through all probability theory. Following from this it would be natural to adopt the linear operator approach to expectation, which would provide a good jumping-off point for further courses on analysis.

In conclusion, this would make a good text from which to teach a first course in measure theory. A course in probability theory would require a more probabilistic bias, though the book would make a useful supplementary text.

REFERENCES

- P. BILLINGSLEY, *Probability and Measure* (Wiley, New York, 1979).
 G. R. GRIMMETT and D. R. STIRZAKER, *Probability and Random Processes* (OUP, Oxford, 1982).
 J. F. C. KINGMAN and S. J. TAYLOR, *Introduction to Measure and Probability* (CUP, Cambridge, 1966).

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CHAO, J.-A. and WOYCZYŃSKI, W. A. (eds.) *Probability theory and harmonic analysis* (Pure and Applied Mathematics: a series of monographs and textbooks, Vol. 98, Marcel Dekker, 1986), 312 pp., \$71.50.

This is composed of fifteen papers on various aspects of probability and harmonic analysis, arising from a conference in Cleveland in 1983 and from various seminars. As mentioned in the preface, topics range from martingales, stochastic integrals, diffusion processes on manifolds, through random walks and harmonic functions on graphs, random Fourier series, to invariant differential and degenerate elliptic operators, and singular integral transforms. I shall mention just