## Transformation of Figures.

Similar Figures. According to the ordinary notion of similar figures in plane geometry, such cannot exist on the surface of the sphere, since the area of a figure is determined by its curvature. The following is, however, analogous to the method of transformation by similarity in plane geometry.

If T be a fixed point, P a point on a given curve, Q a point such $\tan\frac{1}{2}TQ/\tan\frac{1}{2}TP$ is constant, the locus of Q is said to be a curve similar to the locus of P.

The curve similar to a circle is another circle.

The tangents at corresponding points on two similar curves are equally inclined to the common radius vector.

The polar of the above method of transformation gives a second method, which is applicable in plane geometry.

Inversion. If in the method of transformation given above the product $\tan\frac{1}{2}TQ\tan\frac{1}{2}TP$ be substituted for the quotient $\tan\frac{1}{2}TQ/\tan\frac{1}{2}TP$, the curve so derived may be called the inverse of the given curve.

In the case of the sphere however the methods of transformation by similarity and by inversion are really not distinct.

To the fact that in transformation by similarity or by inversion tangents at corresponding points are equally inclined to the radius vector, corresponds in the polar methods of transformation the fact that corresponding tangents are equal; and to the fact that the angle of intersection of two curves is unaltered in the one method of transformation, the fact that the distance between the points of contact of common tangents is unaltered in the second method.

[Some of the foregoing results are given in Mulcahy's *Modern Geometry.*]

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# Statical proofs of some Geometrical Theorems.

By John Alison, M.A.

1. Figure 54. If ABC be a triangle, P any point, then the system of forces PA, PB, PC is equivalent to the system PH, PK, PL, where H, K, L are the middle points of BC, CA, AB.

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For the resultant of PA, PB is 2 PL,
of PB, PC is 2 PH,
of PC, PA is 2 PK;
therefore the system PA, PB, PC is equivalent to the system PH, PK, PL.

If H′, K′, L′ be the middle points of the sides of the triangle HKL, then PA, PB, PC is equivalent to PH′, PK′, PL′. If the bisection of the sides of the triangles be continued, the points H, K, L approach one another, and finally coincide at a point G, so that the system is equivalent to 3PG. If PN be the resultant of PA, PB, PC, then P, G, N are collinear, and PN = 3PG.

2. If P be at H, the middle point of BC, then the resultant of HA, HB, HC, since HB, HC are in equilibrium, is HA and is equal to 3HG. Hence the three medians are concurrent at G and are divided at G in the ratio 1 : 2.

Also, since HA, HB, HC is equivalent to HH, HK, HL, therefore HA is the resultant of HK, HL. Hence, HKAL is a parallelogram; the line joining the middle points of two sides of a triangle, is parallel to and equal to half of the third side.

3. When P is at S, the centre of the circumscribing circle ABC, then the system SA, SB, SC = SA, ST, where ST is equal to 2SH, the resultant of SB, SC; and the resultant of SA, ST is SO, which is equal to 3SG. O is on the perpendicular from A on BC, and intercepts a part AO equal to 2SH. Hence the perpendiculars from B and C also pass through O, and BO = 2SK, CO = 2SL.

O, the orthocentre, G, the centroid, and S, the circumscribed centre of any triangle are collinear, and OG = 2GS.

S is the orthocentre of the triangle HKL, and G its centroid; S′, its circumscribed centre is found by cutting off GS′ from GO equal to half of SG, that is, S′, the centre of the nine-point circle of any triangle bisects the line joining its orthocentre and circumscribed centre.

4. Figure 55. If A, B, C, D be the vertices of a tetrahedron, then, as in § 1, it may be shown that the system of forces PA, PB, PC, PD is equivalent to 4PG, where G is the centre of mean position of the four points. Let $G_1$, $G_2$, $G_3$, $G_4$ be the centroids of the triangles BCD,

CDA, DAB, ABC; then if P be at $G_1$, the forces PB, PC, PD are in equilibrium, and the resultant of the system is $G_1A$, and is equal to $4G_1G$. Hence the lines $AG_1$, $BG_2$, $CG_3$, $DG_4$ are concurrent at G, and are divided at G in the ratio 1 : 3. The general proposition is—If each of $n$ points in space be joined to the centre of mean position of the other $n-1$ points, these lines shall be concurrent at the centre of mean position of the $n$ points, and shall be divided in the ratio $1 : n-1$.

5. If P be at H, the middle point of the edge BC of the tetrahedron, then HB, HC are in equilibrium, and the resultant of the system is the resultant of HA, HD, that is, 2HF, where F is the middle point of the opposite edge AD. But the resultant is 4HG, therefore G bisects the line HF. Hence the lines joining the middle points of opposite edges of a tetrahedron are concurrent and bisect each other.

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*Seventh Meeting, May 14th, 1886.*

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DR FERGUSON, F.R.S.E., President, in the Chair.

## Note on Euclid II. 11.

By the Right Hon. HUGH C. E. CHILDERS.

[The following method of dividing a straight line in medial section was communicated to Mr Munn of the Edinburgh High School, by the Right Hon. Hugh C. E. Childers in January last.]

Let AB be the line. (Fig 56).

Draw $AC = \frac{1}{2}AB$, and at right angles to it. Join CB. Bisect the angle ACB with CD, cutting AB at D. Draw DE at right angles to CB. Then the triangles CAD, CED are equal, and $AD = DE$. Draw a circle with DA and DE radii, cutting AB at F.

Then since DEC is a right angle, BC is a tangent to the circle, and triangles BCA, BDE are similar.

But $BA = 2CA$; therefore $BE = 2DE = AF$.

Then $BA \cdot BF = BE^2$,

$$= AF^2,$$

and F is the required point of division.