

THE  
JOURNAL  
OF  
SYMBOLIC LOGIC

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VOLUME 14

1949

PUBLISHED QUARTERLY BY THE ASSOCIATION FOR SYMBOLIC LOGIC, INC.

WITH THE AID OF SUBVENTIONS FROM

HARVARD UNIVERSITY

PRINCETON UNIVERSITY

INSTITUTE FOR ADVANCED STUDY

SMITH COLLEGE

UNIVERSITY OF PENNSYLVANIA









The four numbers of Volume 14 were issued at the following dates:

Number 1, pages 1–80, May 16, 1949.

Number 2, pages 81–144, June 23, 1949.

Number 3, pages 145–208, October 6, 1949.

Number 4, pages 209–283, January, 1950.

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## ERRATA

### VOLUME 3

Page 134, line 32. For the second occurrence of « (IV) », read « (V) ».

### VOLUME 6

Page 19, line 31. For « to to », read « to ».

Page 93. In each of the last five lines on this page, replace the last occurrence of = by  $\neq$ .

Page 188, second column, line 19 from the bottom and line 6 from the bottom. For 31, read 30.

### VOLUME 7

Page 103, line 7 from the bottom. For « Kuratowski », read « Kuratowskiego ».

Pages 111, 112. See the author's footnote on page 9 of volume 14.

### VOLUME 9

Page 104, line 25. For « Tarksi's », read « Tarski's ».

### VOLUME 10

Page 12, line 7. For « moduc », read « modus ».

### VOLUME 11

Page 115. In the second line of XXVI, for the last  $\neg$ , read  $\supset$ .

Page 116, line 23. For  $(\alpha_1)(\alpha_2)B \dots (\alpha_m)$ , read  $(\alpha_1)(\alpha_2) \dots (\alpha_m)B$ .

Page 118, footnote 7, line 3. For  $A_n$ , read  $A_i$ .

### VOLUME 13

Page 133. In †741, change  $\cup$  to  $\cap$ , and insert a bar over  $\cup$ .

Page 168, lines 16–17. For « Adjukiewicz », read « Ajdukiewicz ».

Page 176, line 7. For « dan », read « dans ».

Page 225, line 23. For « mathematiche », read « matematiche ».

Page 225, line 34. For « does », read « goes ».

Page 233, line 6 from the bottom. Delete the first comma in the line.

### VOLUME 14

Page 56. Supply the page number, « 56 », in the upper left corner.

Page 63, line 21. For « SMIELEW », read « SZMIELEW ».

Page 71, line 12 from the bottom. For « sciences », read « science ».

Page 70, line 36. For « esprimibilità », read « esprimibilità ».

Page 114. In [8], for « Congrès », read « Congrès ».

Page 116, line 5 from the bottom. For  $PTQ_1$  read  $PRQ_1$ .

Page 118, line 2. For  $B_1U_2^*$ , read  $B_1^*U_2^*$ .

Page 118, line 4. For  $p + 1 \geq i$ , read  $i \geq p + 1$ .

Page 118, line 3 from the bottom. For  $T baab$ , read  $R baab$ .

Page 139, line 13. For « basés », read « basées ».

Page 140, line 16. For « paraissantes », read « paraissant ».

Page 143, line 2 from the bottom; page 144, lines 2, 4, 7. For XIV (62), XIV (63), XIV (64),

Page 163, line 5. For  $\Gamma_i$ , read  $\Gamma_1$ . [read respectively XIV 62, XIV 63, XIV 64.]

Page 163, line 7. For  $\Gamma_i$ , read  $\Gamma_i$ .

Page 165, line 24. For  $(\Lambda, E_1, E_2)$ , read  $(\Lambda, E'_1, E'_2)$ ; and in the latter part of the line, for

Page 165, line 27. For «  $\Lambda, E_1, E_2$  », read «  $\Lambda, E'_1, E'_2$  ». [ $E_1$ , read  $E'_1$ ].

Page 177. In 2.41, for  $i^n$ , read  $i^m$ .

Page 189, line 18. For « relation numbers », read « relation-numbers ».

Page 193, line 21. For « Structure », read « Structures ».

Page 197, line 23 from the bottom. After « of », insert « the ».

Page 205, line 6 from the bottom. For « AND'S », read « AND's ».

Page 208, line 20. For « methodes », read « méthodes »; and for « de », read « du ».

Page 208, line 23. For « deductive », read « déductive ».