JOINT COMMISSION VII

High-Redshift Galaxies

Commission 47 with 28, 44 & 48
Chairman & Editor: *K. Sato*

A UNIFIED PICTURE OF LARGE–SCALE STRUCTURE

NETA A. BAHCALL
Astrophysical Sciences
Princeton University
Princeton, NJ 08544-1001

ABSTRACT. A consistent picture of large-scale structure appears to be emerging from different types of observations including the spatial distribution of galaxies, clusters of galaxies, narrow pencil-beam surveys, and quasars. I describe these observations below. A network of large-scale superclusters, up to ~ 150 Mpc in scale, is suggested. The supercluster network surrounds low-density regions of similar scales, suggesting a "cellular" structure of the universe. (H_0 = 100 km /s/ Mpc is used).

1. Introduction

The existence of some large-scale structure in the universe has been known for over half a century. Shapley (1930) noticed a large remote "cloud of galaxies" in Centaurus, known today as the Shapley Supercluster: a ~ 50 Mpc structure that is rich and dense in clusters of galaxies (Raychaudhury *et al.* 1991). Zwicky, in 1937, noticed the very large galaxy concentration in Pisces, that also encompasses several clusters. Abell (1958) recognized that rich clusters of galaxies were themselves clustered into second order clustering, i.e., superclusters. The scales of the above superclusters reached tens of Mpc.

What is the nature of the large-scale structure? What is its shape and topology? While detailed answers to these questions await the results of large surveys, a great deal has been learned about large-scale structure in the last decade. I summarize some of these findings below. I show that a unified picture of large-scale structure is emerging from different types of observations: from the spatial distribution of galaxies, clusters of galaxies, narrow pencil-beam surveys, as well as the distribution of quasars and AGNs. The consistent picture suggests a network of large-scale superclusters, up to ~ 100 - 150 Mpc in scale, that surrounds lower density regions of similar scales. A"cellular" structure of the universe, similar to the "pancake" model discussed by Zeldovich and collaborators, is suggested by the data.

2. Superclusters

Early redshift surveys of galaxies have already revealed that superclusters are large systems that are flattened or filamentary in shape. Gregory and Thompson (1978) obtained a redshift survey of galaxies in the direction of the Coma cluster. They found the large, flattened Coma supercluster which is part of the recently named Great-Wall ,

J. Bergeron (ed.), Highlights of Astronomy, Vol. 9, 671–680.

extending to at least ~ 40 Mpc. The supercluster surrounds a large under-dense region of comparable size. Additional surveys by Gregory *et al.* 1981, and Chincarini *et al.* 1981 yielded similar results in the Hercules and Perseus superclusters. More recent galaxy redshift surveys (Giovanelli *et al.* 1986, de-Lapparent *et al.* 1986, da Costa *et al.* 1988) reveal similar large-scale superclusters surrounding low density regions.

Large-scale superclusters have been traced very successfully also by rich clusters of galaxies (Abell 1958, Bahcall and Soneira 1984). A complete catalog of superclusters - defined as clusters of clusters of galaxies - was constructed by Bahcall and Soneira (1984; hereafter BS84) from a complete redshift sample of rich Abell (1958) clusters to $z \leq 0.08$. The catalog identifies all superclusters that have a spatial density enhancement $f \geq 20$ times larger than the mean cluster density. The mean density of the Bahcall-Soneira superclusters is ~ 10^{-6} Mpc^{-3}, with an average mean supercluster separation of ~ 100 Mpc. The superclusters contain a large fraction of all clusters: ~ 54% at $f \geq 20$. The linear size of the largest superclusters are ~ 150 Mpc (e.g., Corona Borealis) and they are elongated in shape. The fractional volume of space occupied by the superclusters is very small: ~ 3% at $f \geq 20$.

A redshift-cone diagram of the superclusters in the declination slice $\delta = 0^\circ$ - 40° is presented in Figure 1. The mean separation of the superclusters, ~ 100 Mpc, is apparent in the diagram. We shall also see below (§3) that the superclusters appear to surround large, low-density regions such as the Bootes void as well as underdense regions seen in pencil-beam surveys. (For example, the Bootes void of Kirshner *et al.* 1981, is located in the region between the Hercules and Corona Borelais superclusters; Fig. 1.)

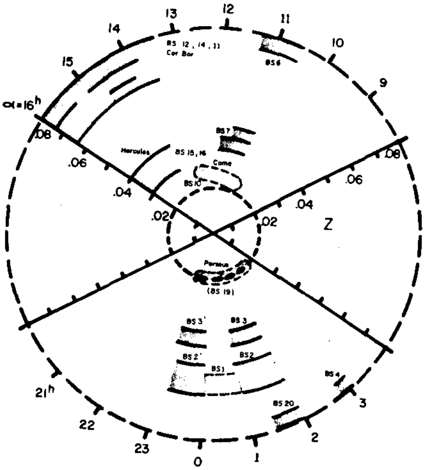

Figure 1. Redshift-cone diagram of the Bahcall-Soneira superclusters in the $\delta = 0^\circ$ - 40° slice (Bahcall 1991). The Coma-Hercules supercluster union is the Great Wall.

How do these superclusters compare with the structures found by galaxy redshift surveys? In Figure 2 I superimpose the supercluster contours from Figure 1 on top of the cumulative galaxy redshift map from the CfA survey (Geller and Huchra 1989), plotted on the same scale. It is clear that the superclusters identified by the clustering of clusters highlight well the main large-scale systems seen in the galaxy survey in the overlap region. In particular, the union of the Coma and Hercules superclusters of Figure 1 constitute the "Great-Wall" seen in the CfA survey, as well as in the earlier Gregory and Thompson survey. The Great-Wall is thus a merging of two BS superclusters, with a total extent of ~150Mpc and thickness of $\lesssim$ 10 Mpc. This extent and flattened shape is comparable to the other large superclusters in the BS catalog; for example, the Corona Borealis supercluster is another such Great-Wall considerably greater and richer than Coma-Hercules. It is located behind a large void in Bootes. This comparison of the galaxy and cluster distribution indicates that the large-scale structure traced by both galaxies and rich clusters is consistent with each other; both find the same superclusters. While the rich clusters are most efficient in finding the largest-scale structures, the galaxies are essential for tracing the small-scale connectedness to the larger scales.

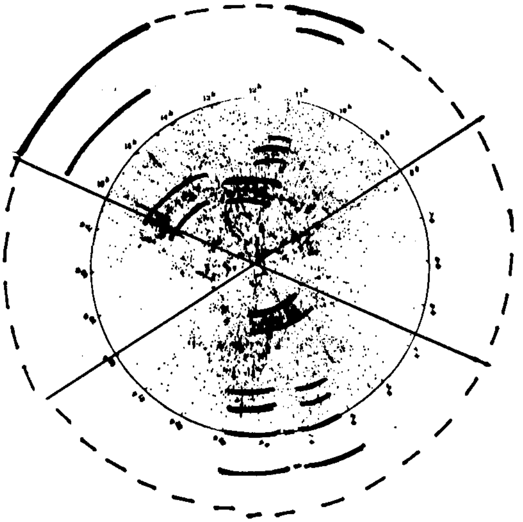

Figure 2. The Bahcall-Soneira supercluster contours of Fig. 1 superpsoed on the CfA galaxy redshift distribution. The BS superclusters highlight the main galaxy superclusters in the CfA survey. The Great-Wall is the union of the Coma-Hercules superclusters.

A rather different method of finding superclusters is that used by Lynden-Bell *et al.* (1988) utilizing peculiar velocity information to infer the existence of large massive superclusters such as the Great Attractor. The estimated mass of the Great Attractor, ~ 5 x 10^{16} $M_\odot$ (Lynden-Bell *et al.* 1988) is comparable to that of the large Bahcall-Soneira superclusters. The Great Attractor does not appear however to contain rich clusters.

In summary, we see that clusters, galaxies, and velocity fields (as well as pencil-beam surveys; §4), appear to trace similar superclusters. These superclusters are the largest systems yet observed. Their sizes extend to ~ 150^2 x 20 Mpc^3, and their mass is estimated to be ~ 2 - 10 x 10^{16} $M_\odot$ (e.g., Bahcall 1988). This mass is comparable to the mass of ~ 20 - 50 rich clusters. The superclusters, Great Walls, and Great Attractors appear to all be similar systems. There are some indications that the supercluster distribution is not random. Bahcall and Burgett (1986) suggest positive correlations among superclusters on scales ~ 100 - 150 Mpc.

3. Superclusters Around Voids

The BS84 supercluster catalog was used by Bahcall and Soneira (1982) to study the area around the large, ~ 60 Mpc diameter void of galaxies in Bootes (Kirshner *et al.*

1981). The largest, densest superclusters are located near and around the area devoid of galaxies (~ 14.5h + 50°). In the redshift-cone diagram of Figure 1, the void is located between Hercules (part of the Great-Wall), and Corona Borealis (the next Great-Wall). It is interesting to note that the overdensity of galaxies observed by Kirshner *et al.* (1981) on both redshift sides of the void, at $z \simeq 0.03$ and $z \simeq 0.08$, coincide in redshift space with these two surrounding superclusters. This suggests that the large superclusters surround the galaxy void (at $z \simeq 0.05$), and that the halos of their galaxy distribution account for the overdensities observed $\gtrsim$100 Mpc away by Kirshner *et al.* This connection provides a strong indication of large halos (~ 150 Mpc) to rich superclusters.

Previous observational evidence (Gregory and Thompson, 1978, Gregory *et al.* 1981, Chincarini *et al.* 1981) together with these results, as well as similar conclusions regarding comparisons with pencil-beam surveys (§4) and large galaxy redshift surveys (§2) suggest that galaxy voids are generally associated with surrounding galaxy excesses; the bigger the void, the stronger may be the related excess (see also §4 and §5).

4. Pencil-Beam Surveys

Recent observations of the redshift distribution of galaxies in narrow (~ 40 arcmin.) pencil-beam surveys to $z \lesssim 0.3$ (Broadhurst *et al.* 1990; hereafter BEKS) reveal a highly clumped and apparently periodic distribution of galaxies. The distribution features peaks of galaxy counts with an apparently regular separation of 128 Mpc, with few galaxies between the peaks. What is the origin of this clumpy, periodic distribution of galaxies? What does it imply for the nature of the large-scale structure and the properties discussed above? Bahcall (1991) investigated these questions observationaly, by comparing the specific galaxy distribution with the distribution of known superclusters.

Bahcall showed that the observed galaxy clumps originate from the tails of the large BS superclusters. When the narrow-beams intersect these superclusters, which have a mean separation of ~ 100 Mpc, the BEKS galaxy distribution is reproduced.

The redshift distribution of the superclusters in the $\delta = 0° - 40°$ slice (Figure 1) is plotted as a histogram (shaded area) in Figure 3. This distribution is superimposed on the galaxy distribution of BEKS. It is apparent from Figure 3 that the supercluster distribution and the BEKS galaxy distribution are essentially identical for $z \lesssim 0.1$. It indicates that the galaxy clumps observed in the pencil-beam survey originate from these superclusters as the beam crosses the superclusters' surface. The main superclusters that contribute to the clumps are indicated in Figure 3. For example, the first northern clump originates from the Coma-Hercules supercluster (= the Great-Wall); the second northern clump is mostly due to the large Corona Borealis supercluster (BS12).

The narrow-beam survey of BEKS is directed toward the north and south galactic poles. Some of the BS superclusters coincident with the BEKS peaks are located at projected distances of up to ~ 50 - 100 Mpc from the poles. This suggests, similar to the Bootes void analysis (§3), that the high-density supercluster regions are embedded in larger halo surfaces, ~ 100 Mpc in size, and that these large structures surround large underdense regions. The observed number of clumps and their mean separation are consistent with the number density of superclusters and their average extent.

The narrow widths of the BEKS peaks are consistent with, and imply, flat superclusters. From simulations of superclusters and pencil-beams, Bahcall and Miller (1991) find that the observed peak-widths distribution is consistent with that expected of randomly placed superclusters with $\lesssim$ 20 Mpc width (and ~ 150 Mpc extent) (Fig. 4).

The BS superclusters exhibit weak positive correlations on scales ~ 100 - 150 Mpc (§2). This implies that the superclusters, and thus their related galaxy clumps, are not randomly distributed but are located in some weakly correlated network of superclusters and voids, with typical mean separation of ~ 100 Mpc. This picture is consistent with statistical analyses of the BEKS distribution (eg. Kurki-Suonio 1990, Ikeuchi and Turner 1991, Park and Gott 1991, Bahcall and Miller 1991), as well as with the observational data presented in sections 2 and 3. The apparent periodicity in the galaxy distribution is expected to be greatly reduced when pencil-beams in various directions are combined.

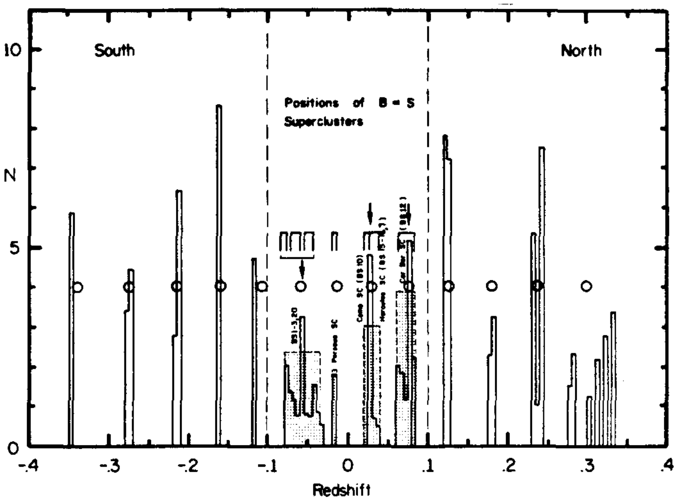

Figure 3. Histograms of the redshift distribution of the Bahcall-Soneira superclusters in the slice $\delta = 0^{\circ} - 40^{\circ}$ (shadded area), for $z \lesssim 0.1$, superposed on the BEKS galaxy distribution (corrected for selection; BEKS preprint). The specific location and names of the BS superclusters are marked (Bahcall 1991).

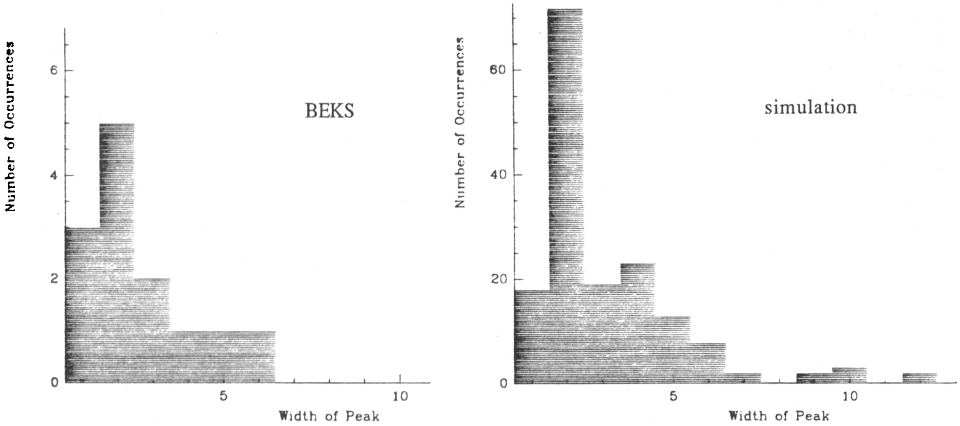

Figure 4. Histogram of the width distribution of galaxy peaks in the BEKS (left) and simulated (right) surveys. The simulated survey contains superclusters of dimensions 150 x 150 x 20 Mpc^3 (Bahcall and Miller 1991). The peak-widths are in 10 Mpc units.

5. Cellular Model of Large-Scale Structure

The observational data described above suggest a "cellular" structure in the universe (e.g. a Zeldovich "pancake" model), in which large-scale flattened superclusters surround low-density regions. Such a model was simulated by Bahcall, Henriksen and Smith (1989), where galaxies were placed on surfaces of randomly placed shells, and clusters were placed at shell intersections. It was found that such a "cellular" model produced cluster correlations that are consistent with observations, showing the large increase in correlation strength (§6) from galaxies to clusters. The model galaxy correlations are also consistent with observations, even showing the tail of weak positive correlations at large separations recently reported by the APM survey (Maddox *et al.* 1990). These results suggest that the observed strong cluster correlation function may be due to the global geometry in which clusters are positioned on randomly placed shells or similar structures; the typical structure size is best fit with a radius of ~ 20 Mpc. Similar simulations based on the explosion model for shell formation were also carried out by Weinberg *et al.* (1989) with similar results.

6. The Cluster Correlation Function

The clustering of a large fraction of clusters ($\gtrsim$ 50%) in superclusters is the cause of the strong correlation function observed among clusters. The cluster correlation function is stronger than the galaxy correlation function by a factor of ~ 15 (Bahcall and Soneira 1983, Bahcall 1988); the correlations yield, respectively, $\xi_{cc}(R \geq 1) \simeq 300\ r^{-1.8}$ for richness $R \geq 1$ clusters versus $\xi_{gg} \simeq 20\ r^{-1.8}$ for galaxies. Many different samples and catalogs of clusters have now been analyzed, *all* yielding consistent results with the correlations above (Klypin and Kopylov 1983, Shectman 1985, Postman *et al.* 1986, Bahcall *et al.* 1986, Huchra *et al.* 1990, Lahav *et al.* 1989 for X-ray selected clusters, West and van den Bergh 1991 for cD selected clusters, Postman *et al.* 1991).

All observational determinations of the correlations of rich clusters, *for richness class $R \geq 1$*, yield correlation scales that are in the range $r_o \simeq 22 \pm 2$ Mpc for $R \geq 1$ clusters (where $\xi(r) = Ar^{-1.8} = (r/r_o)^{-1.8}$). This includes different catalogs (Abell, Zwicky, Shectman), as well as X-ray selected clusters and cD selected clusters. The correlation results do not appear to be significantly influenced by systematics or projection effects.

It has also been shown (Bahcall and Soneira 1983, Bahcall 1988) that the cluster correlation function is richness-dependent: the correlation amplitude increases with the richness of the galaxy clusters. This richness dependence is presented in Figure 5.

An approximate relation describing this dependence is $\xi_i(N_i) \simeq (20\ N_i^{2/3})\ r^{-1.8}$, (Bahcall 1988), where N_i is the Abell (1958) richness of cluster i and $\xi_i(N_i)$ is the correlation function of clusters of richness N_i. This richness-dependent correlation appears to hold well. The newly determined cluster correlation function of the APM survey (Dalton *et al.* 1991) is consistent with the prediction of the richness-dependent cluster correlations; their correlation scale of ~ 13 Mpc is consistent with that expected for the poorer richness threshold of the APM clusters (Bahcall and West 1991).

A second dependence of the cluster correlations is observed as a function of the mean space density, n (or separation, $d \propto n^{-1/3}$) of the clusters (Figure 6). This dependence, $\xi \propto d^{1.8}$, yields a *universal* dimensionless correlation function, when normalized by the mean separation of clusters: $\xi_i(d_i) \simeq 0.2\ (r/d_i)^{-1.8}$ (Szalay and Schramm 1985, Bahcall 1988). Equivalently, the correlation-scale is approximated by: $r_{o,i} \simeq 0.4\ d_i$.

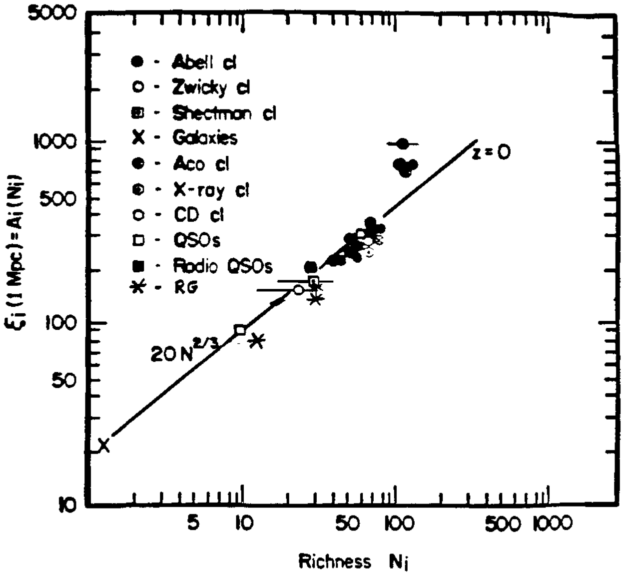

Figure 5. The richness-dependent cluster correlation function. Data points include different samples and catalogs of clusters, as well as X-ray selected and cD clusters. Quasars and radio-galaxies, represented by their parent-groups (§7), are also shown.

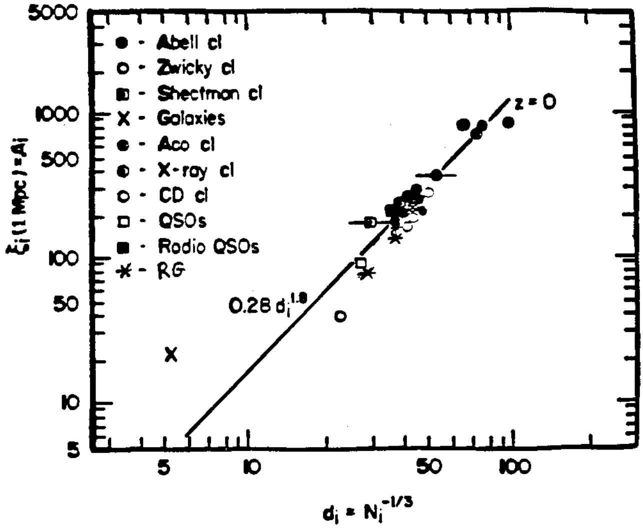

Figure 6 The *universal* dimensionless cluster correlations. Other notations as in Fig. 5.

Again, this dependence appears to hold well for all systems studies so far. The new APM clusters, with a mean density *four* times larger than the $R \geq 1$ clusters (2.4×10^{-5} Mpc^{-3} versus 0.6×10^{-5} Mpc^{-3}, respectively), fit well this *predicted* relation.

The universal dimensionless cluster correlation function is consistent with a fractal structure in the distribution of clusters. This may simply reflect the "cellular" geometry discussed in the previous sections in which clusters placed at "cell" intersections represent a similar, scale-invariant structure when normalized by their mean-separation.

7. Quasars/AGNs and Large-Scale Structure

Observations over the last several years reveal that quasar positions are correlated in space (Iovino and Shaver 1991; Shanks *et al.* 1988; Kruszewski 1985; Fang *et al.* 1985; Chu and Zhu 1988; Crampton *et al.* 1989). The quasar correlation function is stronger than that of bright galaxies but weaker than the correlation of the richest clusters. The quasars therefore trace large-scale structure in the universe in an intermediate manner between galaxies and rich clusters. Some large groups - or superclusters - of quasars have also been reported (Clowes and Campusano, 1991, Crampton *et al.* 1989); these findings are consistent, qualitatively, with the positive quasar correlations discussed above.

What is the origin of the observed quasar correlation and its implied large-scale structure? Bahcall and Chokshi (1991a) investigated the data and suggest that the quasar correlations may reflect the same large-scale structure traced by groups and clusters of galaxies provided the quasars are preferentially located in these high density systems.

Using observational studies of the galaxy environment around nearby quasars to $z \lesssim 0.7$ (Yee and Green 1987, Boyle *et al.* 1988, Ellingson *et al.* 1991), Bahcall and Chokshi (1991a) estimated the mean richness of the average parent-group around the quasars. They find that optically selected quasars are located in small groups of average richness ~ $10L*$ galaxies (as compared with $\simeq 65L*$ galaxies for richness $R = 1$ clusters). The optically selected quasars have the same correlation function as expected for these small groups using the richness-dependent cluster correlation function (Figure 5). Radio quasars are located in richer groups of ~ $30L*$ galaxies on average at $\bar{z} \sim 0.6$, having the stronger correlations expected for these richer groups (Figure 5).

The quasar correlations thus agree well with the universal richness-dependent cluster correlation function, as well as with the universal dimensionless cluster correlation (Figure 6), provided the quasars are in groups of the average richness observed above. This suggests that the quasar correlations are due to the groups in which they are located, thereby displaying the same large-scale structure traced by their parent groups. The agreement of the quasar correlations with the universal relations provides a unified model for large-scale correlations for galaxies, clusters, and quasars.

The recently observed superclusters of quasars ($\gtrsim 100$ Mpc in size) are consistent with this picture. According to this scenario, quasars inhabit groups or clusters of galaxies which themselves trace the large superclusters detected to scales of ~ 150 Mpc (§ 1 - 6). The quasars therefore highlight the same superclusters.

Radio-galaxies, like quasars, are strongly clustered in space (Peacock and Miller 1988; Peacock and Nicholson 1991). Intermediate power radio-galaxies are clustered more strongly than individual galaxies but weaker than rich clusters. The radio-galaxies therefore do not trace randomly the general distribution of galaxies.

Bahcall and Chokshi (1991b) investigated the richness of the environment around the radio-galaxies from works of Hill and Lilly (1991) and Prestage and Peacock (1988), and compared it with the observed correlation strengths. The results are shown in Figures

5,6. The data appears to be consistent with the richness-dependent cluster correlation function, as well as with the universal dimensionless correlations when the mean-separation of the parent-groups is used.

The above suggests that, like the quasars, the radio-galaxy clustering arises from their preferential location in galaxy groups. Radio-galaxies, and quasars, may thus be a good tracer of superclusters in the universe, especially at intermediate to high redshifts.

8. Conclusions

A unified picture is emerging regarding the phenomenology of large-scale structure in the universe using different tracers: galaxies, clusters, pencil-beam surveys, velocities, quasars, and radio-galaxies.

Large-scale superclusters are observed to scales of ~ 150 Mpc in the distribution of galaxies , clusters of galaxies, and probably quasars and AGNs. The same superclusters are traced well by galaxies and by rich clusters. The superclusters appear to be flattened systems, with dimensions of up to ~ 150^2 x 20 Mpc^3; their mean space density is low: ~ 10^{-6} Mpc^{-3}, and their mean separation is ~ 100 Mpc.

Great-Walls, Great-Attractors, and the generic Superclusters are all similar structures with different names. They appear to surround large under-dense regions of comparable sizes. These superclusters are the main origin of the galaxy peaks observed at ~ 100 - 150 Mpc intervals in narrow pencil-beam surveys. The peaks originate when the narrow beam crosses the large-scale superclusters. It is suggested that superclusters are not randomly distributed in space but rather are weakly correlated on large scales. A network system of superclusters is suggested by the data; a "cellular", or Zeldovich "pancake" type model may provide an approximate representation of the observations. Understanding the detailed topology of the structure will require considerably larger redshift samples of galaxies and clusters than currently available.

A richness-dependent cluster correlation function and a universal dimensionless cluster correlation appear to represent well the available data for galaxies, groups, and clusters, as well as quasars and radio-galaxies. The predictive power of these relations has succeeded, since new data appear to be consistent with these predictions.

REFERENCES

Abell, G.O. (1958) *Ap.J. Suppl.* **3**, 211.
Bahcall, N.A. (1988) *Ann. Rev. Astron. Astrophys.* **26**, 631.
___________(1991) *Ap.J.* **376**, 43.
Bahcall, N.A. and Burgett, W.S. (1986) *Ap.J. Letters* **300**, L35
Bahcall, N.A. and Chokshi, A. (1991a) *Ap.J. Letters* (in press).
______________________(1991b) *M.N.R.A.S.* (submitted).
Bahcall, N.A., Henriksen, M.J. and Smith, T.E. (1989) *Ap.J. Letters* **346**, L45.
Bachall, N.A. and Soneira, R.M. (1982) *Ap.J. Letters* **258**, L17.
______________________(1983) *Ap.J.* **270**, 20.
______________________(1984) *Ap.J.* **277**, 27.
Bahcall, N.A. and Miller, N. (1991) *Ap.J.* (to be submitted).
Bahcall, N.A. and West, M. (1991) *Ap.J.* (to be submitted).
Boyle, B.J., Shanks, T. and Yee, H.K.C. (1988) "Large Scale Structure of the Universe" IAU Symp. 130, p.576 (J. Audouze *et al.* eds.).
Broadhurst, T.J., Ellis, R.S., Koo, D.C. and Szalay, A. (1990) *Nature* **343**, 726.

Chincarini, G., Rood, H.J. and Thompson, L.A. (1981) *Ap.J. Letters* **249**, L47.
Chu, Y.Q. and Zhu, X.F. (1988) *A.A.* **205**, 1.
Clowes, R.G. and Campusano, L.E. (1991) *M.N.R.A.S.* **249**, 218.
Crampton, D., Cowley, A.P. and Hartwick, F.D.A. (1989) *Ap.J.* **345**, 59.
da Costa, L.N. *et al.* (1988) *Ap.J.* **327**, 544.
Dalton, G.B., Efstathiou, G., Maddox. S.J. and Sutherland, W.J. (1991) preprint.
de Lapparent, V., Geller, M.J. and Huchra, J.P. (1986) *Ap.J. Letters* **302**, L1.
Ellingson, E., Yee, H.K.C. and Green, R.F. (1991) *Ap.J.* **371**, 49.
Fang, L.Z., Chu, Y.Q. and Zhu, X.F. (1985) *Astrophys. Space Sci.* **115**, 99.
Geller, M.J. and Huchra, J.P. (1989) *Science* **246**, 897.
Giovanelli, R., Haynes, M. and Chincarini, G. (1986) *Ap.J.* **300**, 77.
Gregory, S.A. and Thompson, L.A. (1978) *Ap.J.* **222**, 784.
Gregory, S.A., Thompson, L.A. and Tifft, W. (1981) *Ap.J.* **243**, 411.
Groth, E. and Peebles, P.J.E. (1977) *Ap.J.* **217**, 385.
Hill, G.J. and Lilly, S.J. (1991) *Ap.J.* **367**, 1.
Huchra, J.P., Henry, J., Postman, M. and Geller, M.J. (1990) *Ap.J.* **365**, 66.
Iovino, A. and Shaver, P. (1991) A.S.P. Conference Series, in press.
Ikeuchi, S. and Turner, E.L. (1991) *M.N.R.A.S.*, (in press).
Kirshner, R.P., Oemler, A. Jr., Schechter, P.L. and Shectman, S.A. (1981) *Ap.J. Letters* **248**, L57.
Klypin, A.A. and Kopylov, A.I. (1983) *Sov. Astron. Letters* **9**, 41.
Kruszewski, A. (1985) preprint.
Kurki-Suonio, H., Mathews, G.J. and Fuller, G.M. (1990) *Ap.J. Letters* **356**, L5.
Lahav, O., Edge, A.C., Fabian, A.C. and Putney, A. (1989) *M.N.R.A.S.* **238**, 881.
Lynden-Bell, D., Faber, S., Burstein, D., Davies, R., Dressler, A., Terlevich, R. and Wegner, G. (1988) *Ap.J.* **326**, 19.
Maddox, S.J., Efstathiou, G., Sutherland, W. and Loveday, J. (1990) *M.N.R.A.S.* **242**, 43.
Oort, J. (1983) *Ann. Rev. Astron. Astrophys.* **21**, 373.
Park, C. and Gott, J.R. (1991) *M.N.R.A.S.* (in press).
Peacock, J.A. and Miller, L. (1988) "Optical Surveys for Quasars" A.S.P. Conference Series **2** (Osmer *et al.*, eds.).
Peacock, J.A. and Nicholson, D. (1991) preprint.
Postman, M., Geller, M.J. and Huchra, J.P. (1986) *A.J.* **91**, 1267.
Postman, M., Huchra, J.P. and Geller, M.J. (1991) *Ap.J.* (in press).
Prestage, R.M. and Peacock, J. (1988) *M.N.R.A.S.* **230**, 131.
Raychaudhury, S., Fabian, A.C., Edge, A., Jones, C. and Forman, W. (1991) *M.N.R.A.S.* **248**, 101.
Shanks, T., Boyle, B.J. and Peterson, B. (1988) "Optical Surveys for Quasars" A.S.P. Conference Series **2**, 244 (Osmer *et al.*, eds.).
Shapley, H. (1930) Harvard Obs. Bull. No. **874**,9.
Shectman, S. (1985) *Ap.J. Suppl.* **57**, 77.
Szalay, A. and Schramm, D.N. (1985) *Nature* **314**, 718.
Weinberg, D., Ostriker, J.P. and Dekel, A. (1989) *Ap.J.* **336**, 9.
West, M. and van den Bergh, S. (1991) *Ap.J.* **373**, 1.
Yee, H.K.C. and Green, R.F. (1987) *Ap.J.* **319**, 28.