Correspondence

DEAR EDITOR,

Re: Dicing decimal digits (*Math. Gaz.* July 1997)

The Blest method of producing decimal digits by throwing two dice can be improved by throwing a die and a coin together. We add 4 to the die score if the coin is heads, and -1 if the coin is tails. As before, the die is thrown again if it comes up as 6.

This removes the problems of distinguishing the two dice or confusing the two rules.

Yours sincerely,

JEREMY KING

Tonbridge School, Tonbridge, Kent TN9 1JP

DEAR EDITOR,

I offer a couple of quick thoughts triggered by the articles in July 1997's excellent issue of the *Gazette*.

1. There is a surprising connection between Keith Lloyd's article on the Pell equation $x^2 - 3y^2 = 1$ and H. W. Bitton's quest for nice cubics in Note 81.25. Let u, v be integers satisfying $u^2 - 3v^2 = 1$. Then the cubic $f(x) = (u - 2v)x^3 + 2x^2 + (u + 2v)x$ is readily shown to have the nice rational roots $\frac{v-1}{u-2v}$, 0, $\frac{v+1}{2v-u}$ and its derivative has the nice rational roots $\frac{u-2}{3(u-2v)}$, $\frac{u+2}{3(2v-u)}$.

2. I heartily agree with J. A. Scott (note 81.33) that convexity arguments deserve all the publicity they can get: as Walter Rudin once observed, "Many of the most common inequalities in analysis have their origin in the notion of convexity". Just as the AM–GM inequality has a neat proof using convexity, so the Cauchy-Schwarz inequality has a similarly succinct one. Let $X = \sum x_i^2$, $Y = \sum y_i^2$ where (without loss of generality) none of the x_i are 0. Then

$$\left(\sum x_i y_i\right)^2 = X^2\left(\sum \frac{x_i^2}{X} \cdot \frac{y_i}{x_i}\right)^2$$

$$\leqslant X^2\left(\sum \frac{x_i^2}{X}\left(\frac{y_i}{x_i}\right)^2\right)$$

by convexity if $f(x) = x^2$ with weights $\frac{x_i^2}{X}$.

$$= XY$$

Moreover, there is equality if all the points involved, $\frac{y_i}{x_i}$, are equal.

I also enjoyed the article [1] in the November 1997 *Gazette* and offer two brief observations:

https://doi.org/10.1017/S0025557200206992 Published online by Cambridge University Press

(i)

$$\left.\begin{aligned} x &= e^{-y\pi/2}\cos(x\pi/2) \\ y &= e^{-y\pi/2}\sin(x\pi/2) \end{aligned}\right\} \Rightarrow \left\{\begin{aligned} x^2 + y^2 &= e^{-y\pi} \\ x &= y\cot(x\pi/2) \end{aligned}\right\}$$

$$\Rightarrow x = \cos\left(\frac{x\pi}{2}\right)\exp\left(-\frac{\pi x \tan(x\pi/2)}{2}\right)$$

so x can be found by fixed-point iteration or Newton-Raphson.

(ii) A couple of interesting articles on similar themes occurred in the *Gazette* in 1983 [2, 3]. Both of these cite Macintyre [4] for a proof that (i^i) converges.

References

1. Greg Parker and Steve Abbott, Complex power iterations, *Math. Gaz.* **81** (November 1997) pp. 431-434.
2. P. J. Rippon, Infinite exponentials, *Math. Gaz.* **67** (October 1983) pp. 189-196.
3. Peter L. Walker, Iterated complex radicals, *Math. Gaz.* **67** (December 1983) pp. 269-273.
4. A. J. Macintyre. Convergence of i^i, *Proc. Amer. Math. Soc.* **17**, (1966) p. 67.

Yours sincerely,

NICK LORD
Tonbridge School, Tonbridge TN9 1JP

DEAR EDITOR,

Since Bill Richardson showed kind concern for the state of my health in his Presidential Address (reprinted in the *Gazette* of November 1997), I should like to say that I feel fitter now than I did when I wrote to him at the end of 1996. Although I have Parkinson's disease, I continue to enjoy the normal activities of life such as hill-walking and Scottish dancing. More importantly, I am still lecturing part-time and doing as much geometry as ever. If I fail yet again to appear at the 1998 Conference it will be because I am planning a trip to New Zealand!

Yours sincerely,

JOHN RIGBY
Mathematics Institute, University of Wales, Cardiff CF2 4YH

https://doi.org/10.1017/S0025557200206992 Published online by Cambridge University Press