STATISTICAL AND EVOLUTIONARY ASPECTS OF COMETARY ORBITS

J.A. FERNÁNDEZ
Departamento de Astronomia,
Facultad de Humanidades y Ciencias,
Tristán Narvaja 1674,
Montevideo, Uruguay

W.-H. IP
Max-Planck-Institut für Aeronomie
D-3411 Katlenburg-Lindau,
Federal Republic of Germany

ABSTRACT. The observed frequency of passages of Earth-crossing long-period (LP) comets (P > 200 yr) is about three per year for comets brighter than absolute magnitude $H_{10} \sim 10.5$. About one out of six LP comets is estimated to be new, i.e., making its first passage through the inner planetary region. The sample of observed LP comets shows an excess of retrograde orbits that may be accounted for by the shorter dynamical lifetimes of comets on direct orbits due to planetary perturbations. The original semimajor axes of new comets concentrate in the range $7 \times 10^3 \gtrsim a_{orig} \gtrsim 4 \times 10^4$ AU, which tells us about the region of the Oort cloud where forces other than planetary perturbations act with the greatest efficiency. Yet the distribution of original semimajor axes cannot tell us anything about the existence of a dense inner core of the Oort cloud. Besides planetary perturbations, passing stars, molecular clouds and the galactic tidal force also influence the dynamical evolution of Oort cloud comets. The observed distribution of the aphelion points of near-parabolic comets shows such a dependence on the galactic latitude. Molecular clouds and stars penetrating very deeply in the Oort cloud are found to give rise to major enhancements in the influx rate of new comets, known as comet showers, at average intervals of a few 10^7 yr.

An important issue to solve concerns how the frequency of comet passages varies with time, in particular as regards to the current level of comet appearances. Should we be passing through a highly intense phase, most aphelia of the incoming Oort comets would concentrate on the sky area where the strong perturber exerted its greatest effect. By contrast, the observed galactic latitude dependence of the aphelia suggests a dominant influence of the vertical galactic tidal force as compared with random strong perturbers. This seems to indicate that the frequency of comet passages is currently at, or near, its quiescent level. Whether intense comet showers are reflected in the impact cratering record is still a debatable issue. A periodicity of $\sim$ 26-30 Myr in the impact cratering rate is quite uncertain, owing to the small size of the sample of well-dated craters and the noise from background impact craters from asteroids.

The family of short-period (SP) comets (orbital periods P < 20 yr) has long been regarded as the dynamical end-state of new comets on low-inclination orbits captured by Jupiter. However, if SP comets came from a spherical population of comets (e.g., incoming new comets), we should expect to find a percentage of them on retrograde orbits, which contradicts the observations. An alternative hypothesis for the origin of most SP comets is

R. L. Newburn, Jr. et al. (eds.), Comets in the Post-Halley Era, Vol. 1, 487–535.

that they come from a trans-Neptunian comet belt. Extensive searches aimed at detecting faint slow-moving objects are required to assess the size of the comet population in the outer planetary region. Modeling of the transfer rate of comets from an outer belt to SP orbits gives transient populations between Saturn and Neptune on the order of $10^6 - 10^7$ bodies. This is roughly comparable to the upper limit set by the most recent searches of outer solar system bodies.

The impact crater production rate of comets, at the present time, can be estimated to be on the order of 10% of the value corresponding to asteroidal impacts. These estimates, however, are subject to large uncertainties in the brightness-mass relation of comets and crater scaling law. The Earth could have received about 2×10^{20} g of cometary material over the last 4 billion years — if the injection rate of new comets remained constant in the time interval. Within the context of H_2O inventory, the cometary influx should have rather minor effects. On the other hand, because of the paucity of H_2O content in the atmospheres of Venus and Mars, cometary impact could strongly modulate their water contents.

1. The Discovery Rate and Intrinsic Frequency of Passages of Long-Period Comets.

Comets coming to the vicinity of the Sun have quite different orbital periods, which reflect different dynamical ages. We will first analyze the so-called long-period (LP) comets with orbital periods P> 200 yr. There are 644 LP comets that have been observed up to the end of 1988. Among them, there is a special group of sun-grazers, known as the Kreutz family, that probably come from a single parent comet tidally disrupted by the Sun. The Kreutz family currently has 24 known members, among which 16 were discovered by the coronographs on board the SOLWIND and Solar Maximum Mission satellites, (see a review by Marsden 1990 and IAU Ciruculars 4793, 4815 and 4884 for the recently discovered comets SMM8, SMM9 and SMM10, respectively.)

Most LP comets are repeating passages through the inner planetary region. Yet, a fraction of about 15% may be coming for the first time from distant locations in the Oort cloud, judging by their very large original semimajor axes $a_{orig} \gtrsim 10^4$ AU (i.e., their values of $\underline{a}$ before being perturbed by the planets), corresponding to orbital periods $\underline{P} \gtrsim 10^6$ yr (see below). Oort (1950) called them "new" comets. Unfortunately, the original semimajor axes of many observed LP comets are unknown due to the scarcity of good astrometric measurements. The best determinations of a_{orig} for a rather large sample of LP comets have been performed mainly by Marsden and Everhart (Marsden et al. 1978, Everhart and Marsden 1983, 1987).

The discovery rate of Earth-crossing LP comets has remained nearly constant since the middle of the last century, with the exception of a small period between the two World Wars in which it dropped significantly, perhaps due to the scarcity of dedicated observers. This suggests to us that the discovery rate of LP comets with perihelion distances $q < 1$ AU is at present close to completeness. By contrast, the discovery rate of LP comets with $q > 1$ AU has been increasing for the last few decades, as a result of the use of Schmidt plates by professional astronomers (Fig. 1). Kresák and Pittich (1978) estimate that about 60% of all the LP comets crossing Earth's orbit are currently being discovered, with the rest being

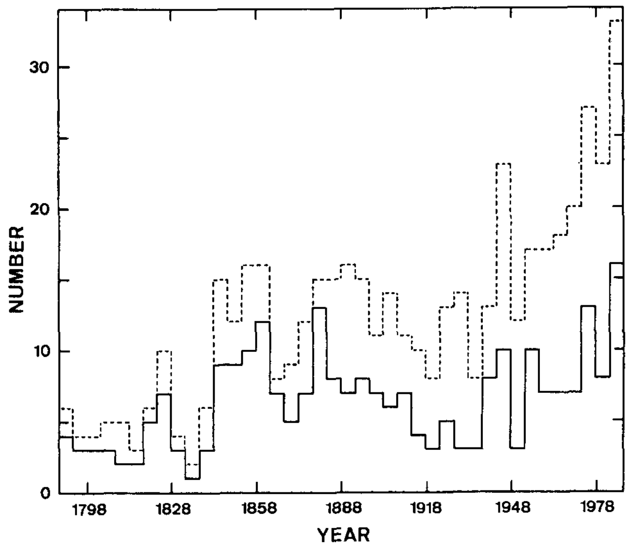

Fig. 1: The discovery rate of LP comets for the period 1788-1988, considering: (1) all the perihelion distances (dashed histogram), and (2) only LP comets on Earth-crossing orbits (solid histogram).

missed due to unfavorable observing geometries rather than to faintness. This is shown, for instance, by the Holetschek effect, by which LP comets passing perihelion when the Earth is on the opposite side of the Sun are less likely to be discovered (see also Everhart 1967).

By making allowance for missed comets, we can estimate the rate of passages of LP comets brighter than $H_{10} \sim 10.5$ in Earth-crossing orbits to be about 3 yr^{-1}. From the list of LP comets with $q < 1$ AU observed since 1850, we find that 15 are new ($a_{orig} > 10^4$ AU) and 74 evolved ($a_{orig} < 10^4$ AU). There are another 113 LP comets with indeterminate a_{orig} and the members of the Kreutz family that can be considered as a single comet. Among LP comets with determined a_{orig}, we find a ratio between the influx rates of evolved and new comets of $\dot{n}_{\mathrm{EVOL}}/\dot{n}_{\mathrm{NEW}} \simeq 5$. Assuming this ratio to be applicable to all the sample of Earth-crossing LP comets and that $\dot{n}_{\mathrm{LP}} = \dot{n}_{\mathrm{NEW}} + \dot{n}_{\mathrm{EVOL}}$, we obtain $\dot{n}_{\mathrm{NEW}} \simeq \frac{1}{6}\dot{n}_{\mathrm{LP}}$, which gives

$$\dot{n}_{\mathrm{NEW}}(q < 1) \sim 1 \text{ new comet every 2 yr.}$$

Taking into consideration that the distribution of perihelion distances of new comets may be rather uniform up to Jupiter's distance and that farther away it steadily increases, following the decrease of planetary perturbations (see Section 3), we derive a rate of passages of new comets with $q < 15$ AU of about 15 yr^{-1}. Within the sphere of 15 AU radius centered on the Sun, planetary perturbations are strong enough to remove a significant fraction of comets from the Oort cloud.

Everhart (1967) found that about 8000 LP comets with $q < 4$ AU reached perihelion in the previous 127 yr. It is difficult to derive how many of them would correspond to new comets, since the ratio $\dot{n}_{EVOL}/\dot{n}_{NEW}$ should increase for $q > 1$ AU, as comets may have much longer physical lifetimes. Nevertheless, a rough estimate would give about 1 to 2 new comets with $q < 1$ AU per year, i.e., 2 to 4 times greater than our estimate. We note that while we made a more or less straightforward determination, Everhart worked with discovery probability functions. From the study of LP comets that approached the Earth to within 0.2 AU in the last 300 yr, Kresák and Pittich (1978) derived a rate of 25 LP comets that annually penetrate Jupiter's region. This is about 1/3 of Everhart's result and is in a reasonable agreement with ours.

The above-mentioned results of frequency of comet passages refer to comets brighter than absolute magnitude $H_{10} \simeq 10.5$. A lack of fainter LP comets was noted by Kresák (1978) from the analysis of LP comets that approached the Earth in the last 300 yr. Indeed, Everhart (1967) already noted a change in the slope of the intrinsic distribution of absolute magnitudes at $H_{10} \sim 6$ and it becomes very uncertain at $H_{10} \sim 10.5$. His results tend to suggest a progressive lag in the number of LP comets fainter than $H_{10} \sim 6$ with respect to that expected from a simple extrapolation of the magnitude distribution of LP comets brighter than $H_{10} \sim 6$ (see also Hughes 1987). Sekanina and Yeomans (1984) have also analyzed the instances of passages of LP comets within 2500 Earth radii during the last 300 yr. They also find a lack of LP comets fainter than $H_{10} \sim 11$ among the Earth-approachers. They present two possible solutions for the intrinsic differential distribution of absolute magnitudes, which are reproduced in Fig. 2 together with the one proposed by Everhart (1967). Both their solutions suggest a sharp downturn at $H_{10} \sim 8$ *to* 10.

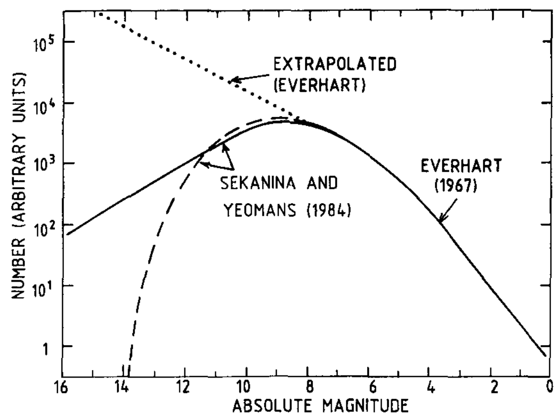

Fig. 2: Intrinsic differential distribution of absolute magnitudes of LP comets with some possible extrapolations for the fainter branch: (1) Everhart's (1967) solution, and (2) two possible extrapolations by Sekanina and Yeomans (1984) consistent with their derived rate of collisions of comets with the Earth (after Sekanina and Yeomans 1984).

The knowledge of the rate of passages of new comets is fundamental to assess the comet inventory of the Oort cloud. This problem has been addressed here, at least concerning the "detectable" LP comets brighter than $H_{10} \sim 10.5$. Some crucial questions remain open, such as, for instance: (1) Is there really a cutoff in the frequency of comet passages at $H_{10} \sim 10.5$? (2) Should this cutoff reflect an intrinsic property of the mass distribution of Oort cloud comets wherein smaller bodies are scarce for some unknown cosmogonic reason? (3) Or else do small comets exist in the Oort cloud, but disintegrate very quickly by interaction with the solar radiation, thus making their detection very difficult? If this were the case, we should further ask how significant the mass contribution of small comets is to the mass inventory of the Oort cloud.

2. Perturbing Forces Acting on Comets

A comet orbiting the Sun is subject to a set of perturbing forces that continuously change its osculating (instantaneous) orbital elements. When the comet passes through the planetary region, the equation of motion can be expressed as

$$\ddot{\vec{r}} = GM_{\odot}\frac{\vec{r}}{r^3} + \nabla R + \vec{J_r} + \vec{J_t} + \vec{J_n}, \tag{1a}$$

with the planetary disturbing function

$$R = G\sum_i m_i\left(\frac{1}{d_i} - \frac{xx_i + yy_i + zz_i}{r_i^3}\right) \tag{1b}$$

where G is the gravitational constant and $M_{\odot}$ the Sun's mass, m_i is the planetary masses, r and r_i are the heliocentric positions of the comet and the planets, respectively, d_i is the planet-comet distances, and (x, y, z) and (x_i, y_i, z_i) are the heliocentric coordinates of the comet and the planets, respectively. Finally, $\vec{J_r}, \vec{J_t}$, and $\vec{J_n}$ are the radial, tangential and normal components of the nongravitational (jet) accelerations, respectively. Equation (1) allows us to compute the original semimajor axes of LP comets with respect to the barycenter of the solar system. At distances $r \gtrsim 100$ AU, planets may be considered no longer able to perturb the comet's orbit.

Far from the Sun, say, at distances $r \gtrsim afew10^3$ AU, passing stars, molecular clouds and galactic tides start to have some influence on the dynamical evolution of comets.

We shall now briefly discuss the perturbing forces mentioned above.

2.1. PLANETS

When comets enter into the planetary region, they are perturbed by the planets. The angular orbital elements (inclination i, argument of perihelion ω and longitude of the ascending node Ω), and the perihelion distance q of comets on near-parabolic orbits experience only minor changes. By contrast, the orbital energy E or the reciprocal semimajor axis (1/a) — as it is sometimes taken instead, since $E \propto -1/a$ — can change drastically because ΔE is usually on the order of, or even larger than, E. A review on the effects of planetary perturbations on comet orbits has been presented elsewhere (Fernández and Jockers 1983).

Some previous works relevant to this subject are those by van Woerkom (1948), those by Bilo and van de Hulst (1960) and computational applications by Everhart (1968, 1976).

Comets in retrograde orbits meet the planets at larger relative velocities than those in direct orbits, so that the former ones will, on the average, be less perturbed. Let ΔE_R and ΔE_D be the typical energy changes after one perihelion passage for a comet in retrograde orbit and one in direct orbit, respectively. In general, we have $\Delta E_R < \Delta E_D$. Through successive perihelion passages, comets will random walk in the energy space so that they will evolve from an initial orbital energy $E_i \sim 0$ (corresponding to a near-parabolic orbit) to a final energy E_f (for instance, corresponding to a periodic comet), after an average number of revolutions

$$\Delta n_R \sim (E_f/\Delta E_R)^2 \tag{2a}$$

$$\Delta n_D \sim (E_f/\Delta E_D)^2 \tag{2b}$$

Since ΔE_R is about three times smaller than ΔE_D (Fernández 1981a), we obtain $n_R \sim 10n_D$. We should note that most comets will be ejected to interstellar space during their random walk in the energy space from E_i to E_f. In fact, from an initial population of N_o comets on near-parabolic orbits, the number N(n) still gravitationally bound to the solar system after n perihelion passages is (Everhart 1976)

$$N(n) \sim \frac{1}{2} N_o n^{-1/2} \tag{3}$$

2.2. NONGRAVITATIONAL FORCES

Nongravitational forces arise from the rocket reaction produced by the sublimating gases of the comet nucleus, as first pointed out by Whipple (1950). A non-rotating nucleus would only give a radial component J_r. Yet, in a more realistic situation of a rotating nucleus, thermal inertia will cause the region of maximum outgassing to shift towards the nucleus "afternoon" by a certain lag angle λ, giving rise to additional transverse and normal components, J_t and J_n. From numerical integrations of orbits of observed periodic comets, Marsden et al. (1973) found negligible values for the averaged normal component J_n.

The main nongravitational effect that can be detected in a periodic comet observed at previous apparitions is a delay or advance in the time of perihelion passage, with respect to that derived from purely gravitational methods, which reflects a change ΔP in its orbital period P. For instance, for the last few apparitions, P/Halley has arrived at its perihelion with an average delay of ΔP = 4.1 days. By using Gauss's equations, Rickman (1986) derives for ΔP the expression

$$\Delta P = \frac{6\pi}{\eta^2 a} \int_0^P \left(\frac{e \sin f}{\sqrt{1-e^2}} J_r + \frac{a\sqrt{1-e^2}}{r} J_t \right) dt, \tag{4}$$

where η is the mean motion, e the eccentricity and f the true anomaly. The radial and transverse components of the rocket force (per unit mass) are given by

$$J_r = \zeta v \dot{M}_c / M_c, \tag{5a}$$

$$J_t = \pm \zeta v \sin \lambda \dot{M}_c / M_c, \tag{5b}$$

where $\dot{M}_c$ is the loss rate of subliming volatiles, M_c the mass of the comet nucleus, v the mean molecular speed, and $\zeta < 1$ a correction factor that takes into account the spread of the outflowing molecules. The plus or minus sign is used in Eq. (5b) depending on whether the nucleus rotation is prograde or retrograde.

Rickman (1986) has used this formulation to derive nuclear masses for P/Halley and P/Kopff.

The standard view (Whipple 1950) was that nongravitational forces arise from the transverse component J_t due to a rotationally induced thermal lag. This would be the case for a comet with a symmetric lightcurve (with respect to perihelion), with the result that the first term of Eq. (4), integrated over the whole orbital period P, would vanish. The only term that would remain in this case would be the one containing J_t. Yet most comet lightcurves (outgassing) are moderately or highly asymmetric, so the integral of the term containing J_r will no longer vanish. Indeed, it can be much greater than the integral of the term containing J_t. The probable dominant influence of asymmetries in comet lightcurves on nongravitational effects was noted by Rickman (1986) and recently developed by Yeomans and Chodas (1989) and Festou et al. (1990).

2.3. RANDOM STARS

Comets in the Oort cloud are perturbed by passing stars. The effect of stellar perturbations on comets was addressed by Fesenkov (1922) and independently developed by Öpik (1932) and Oort (1950). More recent work on this topic has been done by Rickman (1976), Weissman (1980a), Fernández (1980a) and Bailey (1983).

Let us evaluate the magnitude of a star's perturbation on a comet of semimajor axis $\underline{a}$ situated at a time-average heliocentric distance $\bar{r} = 1.5a$. The comet's orbital velocity thus becomes

$$v_c^2 = GM_\odot / 3a. \tag{6}$$

A passing star of mass M and relative velocity V will impart an impulsive change in v_c given by

$$\vec{\Delta v} = \vec{\Delta v}_c - \vec{\Delta v}_\odot \tag{7}$$

where $\vec{\Delta v}_c$ and $\vec{\Delta v}_\odot$ are the impulses received by the comet and the Sun from the passing star, respectively. They are given by

$$\vec{\Delta v}_c = K\vec{D}/D^2 \tag{8a}$$

$$\Delta \vec{v}_\odot = K\vec{D}_\odot / D_\odot^2 \tag{8b}$$

https://doi.org/10.1017/S0252921100109819 Published online by Cambridge University Press

where K = 2GM/V, and D and $D_\odot$ are the distances of closest approach to the comet and the Sun, respectively.

For the case of a star coming very close to the Sun, we have $D_\odot << D$ and, in general, $D_\odot << a$. Thus, Eq. (7) can be approximated by

$$|\Delta v| \simeq K/D_\odot \tag{9a}$$

When the star comes very close to the comet, we have instead $D << D_\odot$, which leads to

$$|\Delta v| \simeq K/D. \tag{9b}$$

For distant encounters, $\vec{D}$ and $\vec{D}_\odot$ become nearly parallel, so that Eq. (7) reduces to

$$|\Delta V| \simeq K\frac{r\cos\beta}{D_\odot^2}, \tag{10}$$

with β being the angle between $\vec{D}_\odot$ and $\vec{r}$.

During an orbital revolution of period P = $a^{3/2}$ years, a comet will be perturbed by many stars. Let $s(D_\odot)dD_\odot = 2n_o D_\odot dD_\odot$ be the rate of stellar passages at distances of closest approach to the Sun in the range $(D_\odot, D_\odot + dD_\odot)$. n_o is the stellar flux in the Sun's neighborhood of about 7 stars Myr^{-1} passing through a circle of 1-pc radius for an average encounter velocity with the Sun of V = 30 km sec^{-1}. The cumulative change in the orbital velocity of the comet during P, Δv_*, will be expressed as (Fernández and Ip, 1987)

$$\begin{aligned}\Delta v_*^2 &= K^2P\left[\int_{D_m}^{D_L} D_\odot^{-2}s(D_\odot)dD_\odot + r^2\cos^2\beta\int_{D_L}^{D_M} D_\odot^{-4}s(D_\odot)dD_\odot\right], \\ &\simeq 2K^2Pn_o\left[ln(D_L/D_m) + 1/6(\bar{r}/D_L)^2\right].\end{aligned} \tag{11}$$

$D_m = (2n_oP)^{-1/2}$ is the minimum distance of closest approach of a passing star expected during P. D_M is the maximum distance of a passing star that may have some dynamical influence. It can be taken as infinity without much error. D_L is a somewhat arbitrary boundary separating the regimes in which approximations (9a) and (10) apply. We have usually taken values between r and 2r giving differences of not more than 30% (Fernández 1980a). Finally, we have adopted an average value of $< \cos^2\beta >= 1/3$.

2.4. MOLECULAR CLOUDS

The importance of interstellar molecular clouds as major perturbers of the Oort cloud was first addressed by Biermann (1978). Afterwards, Napier and Clube (1979) and Napier and Staniucha (1982) argued that such encounters would be able to disrupt the Oort cloud. The dynamical effects of encounters with molecular clouds on Oort cloud comets were later elaborated in more detail by Bailey (1983) and Hut and Tremaine (1985).

Let us consider a penetrating encounter of the Sun with a molecular cloud, assumed to be spherical of uniform density, radius R_{cl} and mass M_{cl}. The impulsive change in the velocity of a comet at a distance r to the Sun is (Biermann 1978)

$$\Delta v_{cl} = \frac{2GM_{cl}}{v_{cl}} \frac{r}{b^2} \left[1 - (1 - \frac{b^2}{R_{cl}^2})^{3/2} \right] \sin\theta, \tag{12}$$

where $v_{cl} \sim 20$ km s^{-1} is the typical encounter velocity with molecular clouds, b the impact parameter, and θ the angle between $\vec{r}$ and $\vec{v}_{cl}$.

Hut and Tremaine (1985) argue that the energy change obtained from the impulse approximation of Eq. (12) could be overestimated when the encounter time is large compared with the orbital period of the comet, in other words, when the quantity $\frac{b}{v_{cl}P} \gtrsim 1$. This may be the case when the Sun encounters giant molecular clouds (GMCs) with radius $R_{cl} \sim 20$ pc (we further assume $R_{cl} \sim b$). Yet, these authors suggest that a certain degree of clumpiness present in the interior of GMCs might counteract somewhat the previous effect in such a way that Eq. (12) could still give reasonable results.

There is no precise answer yet on how frequently there are penetrating encounters of the solar system by molecular clouds. We should first note that there is a broad spectrum of molecular cloud sizes. The most devastating effects will occur when the solar system encounters a GMC, typically of a mass $M_{cl} \sim 5 \times 10^5 M_\odot$ and radius $R_{cl} \sim 20$ pc. Bailey (1983) estimates the number of such encounters to be in the range of 1 to 10 during the solar system lifetime (see also Torbett 1986). This is in good agreement with the value of 9 penetrating encounters with clouds of densities above $10^3 H_2$-molecules cm^{-3} at encounter velocities of $v_{cl} \sim 20$ km sec^{-1} quoted by Talbot and Newman (1977). Penetrating encounters with intermediate-size molecular clouds, with masses of a few $10^3 to 10^4 M_\odot$, will be more frequent, although less dramatic. The number density of these clouds may be about two orders of magnitude greater than that of GMCs (Drapatz and Zinnecker 1984), so that a penetrating encounter of the solar system with an intermediate-size molecular cloud might occur at average intervals of several 10^7 yr.

2.5. GALACTIC TIDAL FORCES

It has long been recognized that tides from the Galaxy may have a significant influence on the shape and extent of the Oort cloud. The simplest assumption to evaluate the dynamical effects of galactic tides is to consider that all the mass of the Galaxy, $M_G = 1.3 \times 10^{11} M_\odot$, is concentrated at its center (e.g., Chebotarev 1966). The Sun, orbiting the galactic nucleus at a distance of $r_G = 8.2$ kpc, will be surrounded by a region within which orbits will be dynamically stable against galactic tides. The outer boundary of this region will be defined by the condition that the relative velocity of a test particle with respect to the Sun becomes zero. The radius of this zero-velocity surface will be given by

$$r_t = \left(\frac{M_\odot}{3M_G} \right)^{1/3} r_G \simeq 200{,}000 AU. \tag{13}$$

A more realistic model of the Galaxy was assumed by Antonov and Latyshev (1972), in which the galactic potential was expressed as

https://doi.org/10.1017/S0252921100109819 Published online by Cambridge University Press

$$V_G = -\frac{1}{2}(\alpha x^2 + \gamma z^2), \tag{14}$$

where the x-axis is the radial direction directed away from the galactic nucleus and the z-axis is perpendicular to the galactic plane. $\alpha = 4A(A-B), A = 15$ kms^{-1} kpc^{-1} and B = - 10 kms^{-1} kpc^{-1} are the Oort constants describing galactic rotation, and $\gamma = -2\pi G\rho_{disk}$, with $\rho_{disk} = 0.185 M_\odot pc^{-3}$ (Bahcall 1984) being the density of the galactic disk. In essence, the first term of Eq. (14), describes the potential of the galactic nucleus, whereas the second term describes the potential of the galactic disk.

By applying Eq. (14), Antonov and Latyshev (1972) obtained a zero-velocity surface resembling a triaxial ellipsoid of semiaxes

$$\begin{aligned} x &= 293 \times 10^3 AU \\ y &= 196 \times 10^3 AU \\ z &= 152 \times 10^3 AU \end{aligned}$$

The z-axis turns out to be the shortest one (i.e., the shortest dynamical stability range along $\vec{z}$), which reflects the predominance of the galactic disk potential with respect to the potential of the galactic nucleus. Later numerical experiments by Smoluchowski and Torbett (1984) tend to confirm the previous results. The effect of the galactic disk potential on the dynamical evolution of Oort cloud comets has been further developed by Heisler and Tremaine (1986), Morris and Muller (1986) and Torbett (1986). Byl (1983) already noted that galactic effects were strongest on comets at mid-galactic latitudes.

The tidal force per unit mass, perpendicular to the galactic disk, acting on a comet at a galactic latitude ϕ is (Stothers 1984)

$$\left(\frac{dU}{dz}\right)_c - \left(\frac{dU}{dz}\right)_\odot = -4\pi G\rho_{disk} r sin\phi \hat{z}, \tag{15}$$

where U is the gravitational potential of the galactic disk, and $\hat{z}$ is the unit vector perpendicular to the galactic plane. The change in the comet's velocity Δv_{tide} during Δt will then be given by

$$\vec{\Delta v}_{tide} = 4\pi G\rho_{disk} r \sin\phi \Delta t \hat{z}. \tag{16}$$

Taking $\Delta t = P = 2\pi(GM_\odot)^{-1/2} a^{3/2}$ and a time-average heliocentric distance $r = \bar{r}$ = 1.5a, we obtain

$$\vec{\Delta v}_{tide} = 12\pi^2 G^{1/2} M_\odot^{-1/2} \rho_{disk} a^{5/2} \sin\phi \hat{z}. \tag{17}$$

3. The Frequency-Distributions of Perihelion Distances, Inclinations and Original Semimajor Axes.

We shall not review thoroughly the statistical properties of comet orbits. Some of them have already been discussed elsewhere (Fernández and Jockers 1983). We shall focus

https://doi.org/10.1017/S0252921100109819 Published online by Cambridge University Press

instead on the frequency distributions of perihelion distances (q), inclinations(i) and original semimajor axes (a_{orig}), which are of great relevance in understanding how the forces described above act on comets.

Neglecting the effect of planetary perturbations, it is easy to show that comets in very eccentric orbits will have a uniform distribution of perihelion distances q ("Opik 1966). To this purpose, let us consider the transverse velocity of one such comet

$$v_r^2 = v_c^2 \sin^2\theta \simeq v_c^2\theta^2 \simeq \frac{2\mu q}{r^2}, \tag{18}$$

where $\mu = GM_\odot$, v_c is the comet's velocity and θ is the angle formed between v_c and r. From eq. (18) we obtain

$$v_c^2\theta d\theta = \frac{\mu}{r^2}dq. \tag{19}$$

If such highly eccentric comets belong to a population of Oort cloud comets whose velocity vectors v_c have been randomized by stellar perturbations, their angles θ will be distributed according to

$$f_\theta(\theta)d\theta = \frac{\sin\theta}{2}d\theta \sim \frac{\theta d\theta}{2}. \tag{20}$$

By combining eqs. (19) and (20) we obtain

$$f_q(q)dq = \frac{\mu}{2v_c^2 r^2}dq. \tag{21}$$

As v_c and r are independent of q, the distribution of perihelion distances $f_q(q)$ turns out to be uniform.

The consideration of planetary perturbations leads to a distribution of perihelion distances different from that expressed by Eq. (21). Oort comets coming closer to the Sun than Jupiter and Saturn are very likely to be removed from the Oort cloud after one perihelion passage. By contrast, comets with larger perihelion distances (say, q $\gtrsim$ 12-15 AU) are much less perturbed, so they can remain in the Oort cloud for many revolutions. The effect is a depletion of velocity vectors $\vec{v}_c$ close to the radius vector $\vec{r}$, defining an almost empty volume in the velocity phase space known as a "loss cone" (to be more precisely defined below). In other works, fewer Oort cloud comets will cross the barrier of Jupiter and Saturn as compared with those passing by the outer planetary region. Numerical experiments carried out by Fernández (1982) and Weissman (1985a) confirm the previous conclusion (see Fig. 3). Both numerical experiments follow the dynamical evolution of test particles under the perturbing action of random stars and planets modeled in a simple way. The difference is that Fernández follows the evolution through many passages ("LP comets"), whereas Weissman restricts the study to the first passage ("new comets"). Despite some quantitative differences, there is a good qualitative agreement between both models. As shown in Fig. 3, the distribution of perihelion distances stays more or less flat up to Jupiter's distance; then it increases slowly up to $q \sim 10 - 12$ AU; beyond that the increase is steeper. Weissman (1985a) found that the q-distribution will tend to level off at $q \gtrsim 100$ AU.

https://doi.org/10.1017/S0252921100109819 Published online by Cambridge University Press

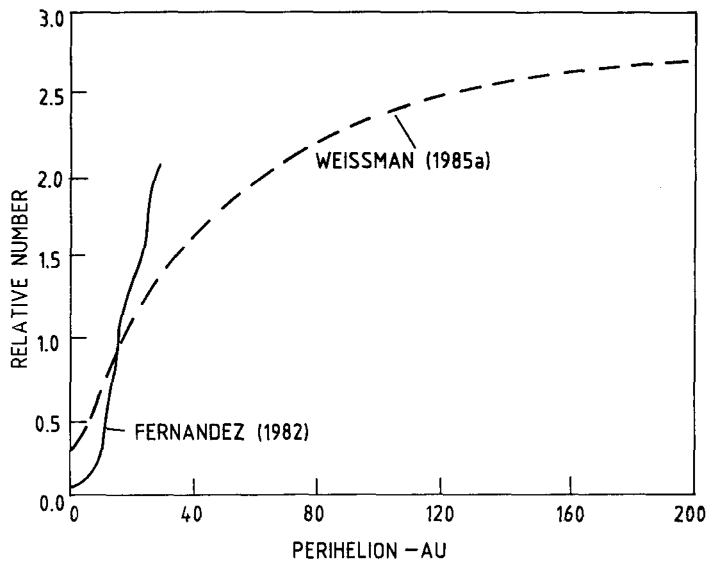

Fig. 3: Computed distribution of perihelion distances of the incoming new comets ($a > 10^4$ AU), according to the models developed by Fernández (1982) and Weissman (1985a).

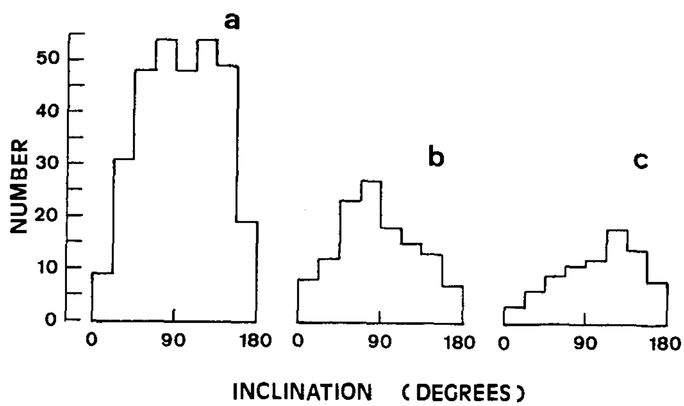

Fig. 4: Frequency distribution of inclinations of the LP comets discovered between 1759-1988. The sample has been divided into three subsets according to the following ranges of perihelion distances: (a) $q \leq 1.1$ AU, (b) $1.1 < q \leq 2$ AU, and (c) $q > 2$ AU.

https://doi.org/10.1017/S0252921100109819 Published online by Cambridge University Press

In principle, comets isotropically distributed in the Oort cloud will come to the planetary region from random directions, whereby the resultant i-distribution would have to follow a sine-law. Yet it is now a well-established observational fact that an excess of retrograde orbits is found among the sampled LP comets (Porter 1963). To analyze this apparent "anomaly", we have considered the sample of 516 LP comets discovered between 1759 and the end of 1988. The comet orbital data through 1985 have been taken from Marsden's (1986) catalogue and from IAU Circulars from 1986 on. We have discarded the LP comets discovered before 1759 because they are presumably contaminated with periodic comets, as shown by the strong excess of comets on low-i orbits present among them. We have also discarded comets 1833, 1949 III and 1963 IX because, according to Marsden (1986), they are suspected of being periodic. Finally, comet 1882 II has been taken as the single representative of the Kreutz family of Sun-grazing comets.

The sample of LP comets has been divided into three subsets according to different ranges of perihelion distances as indicated in Fig. 4. There is an overall excess of retrograde orbits in the sample, and it is also present in the subsets for $0 < q \leq 1.1$ AU and $q > 2$ AU. Only the subset for $1.1 < q \leq 2$ AU shows a predominance of direct orbits that Fernández (1981b) explains as being due to selection effects. Thus, comets in direct orbits with q somewhat larger than the orbital radius of the Earth spend a significantly larger amount of time in the Earth's neighborhood, owing to their smaller encounter velocities, making them easier to discover. By contrast, there is no obvious selection effect able to explain an excess of retrograde orbits among LP comets at the two extreme ranges of the considered perihelion distances.

In our opinion, there is an important dynamical reason that explains the observed excess of retrograde orbits. As shown in Section 2 (see also Fernández 1981b), comets coming to the planetary region in direct orbits are subject to stronger planetary perturbations than those in retrograde orbits. The latter ones will consequently last longer in orbits bound to the solar system.

The frequency distribution of original orbital energies (or original 1/a) shows a spike at $1/a \sim 0$ (near-parabolic orbits) that led Oort (1950) to the concept of a comet cloud surrounding the solar system. The 1/a-distribution has been reviewed later by other authors (e.g., Weissman 1985b). Yet we consider it worthwhile to analyze directly the distribution of a_{orig}, as it can give clues about the spatial distribution of the Oort cloud comets.

The histogram for the a_{orig} -distributions of a sample of 130 LP comets with $10^3 < a_{orig} < 10^5$ AU, taken from Marsden et al. (1978) and Everhart and Marsden (1983, 1987), is shown in Fig. 5. The curve superposed on the histogram follows the law $f(a) \propto a^{-2}$, as should be expected for comets diffusing in the energy space by planetary perturbations (van Woerkom 1948). A departure from the above law, under the form of an excess of comets, is already observed at a_{orig} somewhat smaller than 10^4 AU. The range $7 \times 10^3 \lesssim a_{orig} \lesssim 4 \times 10^4$ AU corresponds to the region of the Oort cloud wherein forces other than planetary perturbations (galactic tides, passing stars and molecular clouds) act on comets with the greatest efficiency.

From the observed frequency of passages of new comets, Weissman (1990) has derived a population of $\sim 0.4-1.3\times10^{12}$ comets (brighter than $H_{10} = 11$) for the dynamically active Oort cloud. However, we note that this cannot be more than a rough estimate, even if we reach an accurate knowledge of the forces acting on Oort cloud comets. The fundamental

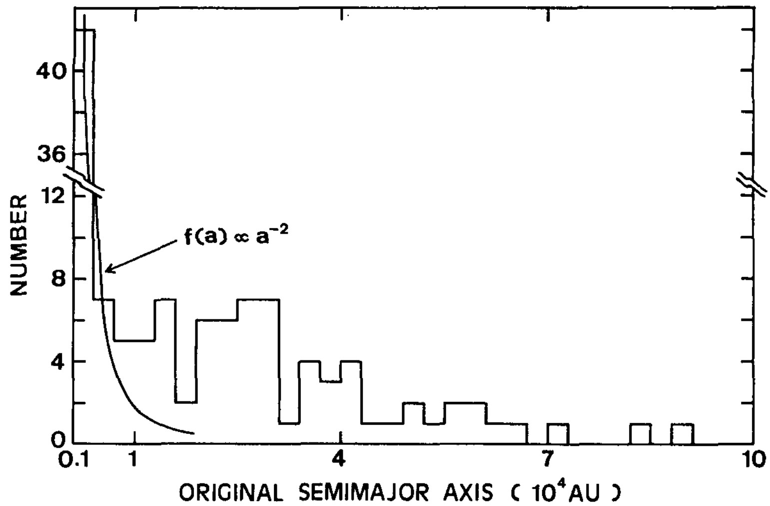

Fig. 5: Frequency distribution of original semimajor axes of observed LP comets as computed by Marsden et al. (1978) and Everhart and Marsden (1983, 1987).

problem is to know the intrinsic radial distribution of comets in the Oort cloud. We will briefly analyze this problem later.

4. Observed Anisotropies in the Directions of Aphelia

This is a long-standing problem going back to the last century (see Hurnik 1959 and earlier references therein). An obvious irregularity in the distribution of aphelion points arises from the unequal coverage of the Northern and Southern hemispheres, by which most discovered LP comets have perihelia on the Northern hemisphere. Possible associations with the galactic plane or the apex of the solar motion have been discussed on several occasions (e.g., Tyror 1957, Oja 1975, Hasegawa 1976, Bogart and Noerdlinger 1982). On the other hand, Kresák (1975) has argued that the observed irregularities could be attributed to recent close approaches of passing stars.

Departures from randomness in the distribution of aphelion points have been further analyzed by Biermann et al. (1983) and Lüst (1984). From the analysis of a selected sample of 80 new comets, they find clear departures from expected Poisson or binomial distributions showing up as sky areas with an anomalous large number of comets ("aphelion clusterings"). Lüst (1984) also noted the existence of a belt of $\pm$ 10^o latitude around the galactic plane almost devoid of aphelia of new and young LP comets.

The distribution of aphelion points on the celestial sphere of 142 new and young LP comets with $a_{orig} > 500$ AU is shown in Fig. 4. The more evolved LP comets with $a_{orig} < 500$ AU have probably been perturbed by the planets in such a way that their aphelia have lost "memory" of their original locations. LP comets with original hyperbolic orbits have also been removed from the sample. We can see the strong preference of the aphelion points for mid-galactic latitudes. This effect has already been noted by Delsemme and Patmiou (1986), who attributed it to the perturbing action of the vertical galactic tidal force on Oort cloud comets. Their conclusion seems to be essentially correct, since this force strongly depends on the galactic latitude (see the next section).

From an analysis of the angular momenta of the original orbits of new and young comets, Delsemme (1989) has found a strong anisotropy, which mostly comes from a lens-shaped cluster of 29 aphelia. He attributed this anisotropy to the recent passage of a brown dwarf of mass $\sim$ 30 times Jupiter's mass on an unbound, hyperbolic orbit. By estimating a number of brown dwarfs about 60 times greater than the number of ordinary visible stars in the solar neighborhood, he derived a frequency of encounters with brown dwarfs moving slowly enough to cause appreciable dynamical effects, say, $\lesssim 0.1$ km s^{-1}; the frequency he derived was about one every 7 Myr. It is still possible that Delsemme's clustering is an artifact of his model. The assumed cluster comets mingle with neighboring comets in such a way that it is difficult to discriminate between their membership (see Fig. 6). For instance, the northern tip of Delsemme's clustering falls on a region of at least equal or even greater concentration of aphelion points. The southern tip falls on the mid-galactic latitude zone where the aphelion points have the greatest concentration, which should be taken into consideration. No doubt this is an issue that deserves further analysis.

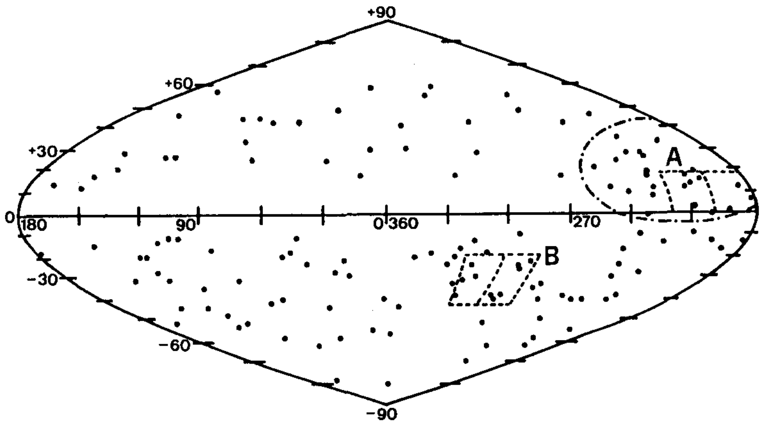

Fig. 6: Distribution of aphelion points of 142 "young" comets with $a_{orig} > 500$ AU on the celestial sphere. Galactic coordinates are considered. A and B denote two zones with excess areas. The dash-dotted curve encircles the excess region found by Lüst (1984). Delsemme's (1986) lens-shaped clustering stretches between zones A and B.

https://doi.org/10.1017/S0252921100109819 Published online by Cambridge University Press

It is not totally correct to apply binomial or Poisson distribution functions on the whole sky, because of the clear preference of cometary aphelia for mid-galactic latitudes, which by itself sets a departure from randomness. To avoid this problem as far as possible, we have divided each galactic hemisphere into three equal-area belts along the galactic equator; their boundaries are $(0, \pm 19^o47)$, $(\pm 19^o47, \pm 41^o81)$, $(\pm 41^o81, \pm 90^o)$. Two possible excess areas of cometary aphelia, denoted by A and B, were found in the equatorial and mid-galactic latitude belts of Fig. 6. The area A is within the excess area found by Lüst (1984) and partially overlaps the northern tip of Delsemme's clustering. The area B, located on the southern mid-galactic belt, contains some of Delsemme's cluster comets. Areas A and B may be associated with aphelion clusterings, but not necessarily, since they could be related to a single trigger mechanism.

5. Injection Rate of Oort Cloud Comets into the Planetary Region

Under the action of passing stars, molecular clouds and galactic tides, the orbital angular momentum of Oort cloud comets is continuously changed in such a way that some of them will enter the planetary region. Once this happens, they will be quickly removed from the Oort cloud by planetary perturbations. As shown in Section 1, these comets should reduce their perihelion distances to values $q < q_L$, where $q_L \simeq 15$ AU. Oort cloud comets of a given semimajor axis $\underline{a}$ will have an associated empty region in the velocity phase space known as a "loss cone" (Hills, 1981). The loss cone will have an axis following the solar direction and an angular radius $2F_q^{1/2}$, where F_q would be the fraction of thermalized Oort cloud comets with $q < q_L$ if no losses were assumed (see below).

For a comet at heliocentric distance r on a near-parabolic orbit, i.e., $e \sim 1$, the comet's orbital angular momentum per unit mass can be approximately given by

$$h = (2\mu q)^{1/2} \tag{22}$$

where $\mu = GM_\odot$. After one orbital revolution P, the comet will change its angular momentum by an amount Δh, given by

$$\Delta h = \Delta v_T \times r \tag{23}$$

where $\Delta v_T = \Delta v \times cos\Psi$ is the transverse component of the velocity increment, Δv, during P, and Ψ is the angle between $\vec{\Delta v}$ and its transverse component.

The distribution of perihelion distances of thermalized comets is uniform for small values of q (Öpik 1966). Thus, the distribution of angular momenta will follow the law

$$\varphi(h)dh \propto h\, dh. \tag{24}$$

As said before, comets with angular momentum in the range $0 < h < h_L = (2\mu\, q_L)^{1/2}$ will be quickly removed from the Oort cloud.

Without too much error, we can assume that Eq. (24) is valid for all the range of q, up to $q = a$ (circular orbits), for which we get a corresponding $h_{\rm MAX} = (\mu a)^{1/2}$. Thus, the fraction F_q of Oort cloud comets with $q < q_L$ can be obtained from

$$F_q = \int_o^{h_L} \varphi(h)dh \Big/ \int_o^{h_{MAX}} \varphi(h)dh = \frac{2q_L}{a}, \tag{25}$$

which turns out to be in agreement with the figure obtained by Hills (1981). He used results first derived for binary stars in statistical equilibrium, showing that the fraction of binaries with eccentricities between e and unity is $(1 - e^2)$. He applied the same analysis to the different pairs formed by the Sun and each one of the Oort cloud comets.

Due to the perturbation Δh of the angular momentum of Oort comets of semimajor axis $\underline{a}$, a fraction f of the loss cone will be kept filled at any time. We have

$$f = \int_o^{\Delta h} \varphi(h)dh \Big/ \int_o^{h_L} \varphi(h)dh = \frac{\Delta h^2}{h_L^2}. \tag{26}$$

By introducing the corresponding values of Δh (Eq. 23) and h_L and assuming the comet to be always at an average distance $r = 1.5a$, we get

$$f = 1.12\,\frac{a^2 \Delta v_T^2}{\mu q_L}. \tag{27}$$

This expression turns out to be dimensionally equivalent to the one obtained by Hills (1981) using the concept of a "smear cone," which basically compares the impulse velocity imparted to the comet by the perturbing star with the comet's orbital velocity.

Appropriate expressions for Δv_T, for the different forces considered before, are:
(a) for stellar perturbations:

$$\Delta v_T^2 = \Delta v_*^2 < \cos^2 \Psi > = \frac{2}{3}\,\Delta v_*^2, \tag{28a}$$

where Δv_*^2 is given in Eq. (11).
(b) for the tidal force of the galactic disk:

$$\begin{aligned} \Delta v_T &= \Delta v_{tide} \cos\phi \\ &= 6\pi^2 G^{1/2} M_\odot^{-1/2} \rho_{disk}\, a^{5/2} \sin 2\phi. \end{aligned} \tag{28b}$$

The fraction f can be subject to temporary changes as a result of the passage of a strong perturber as, for instance, a molecular cloud.
In this case we have
(c) for a molecular cloud:

$$\Delta v_T = \Delta v_{cl} \cos\theta, \tag{28c}$$

with Δv_{cl} being expressed in Eq. (12).

By introducing either Eq. (28a), (28b) or (28c) into Eq. (27), we get the fraction of the loss cone filled with comets due to the cumulative effect of stellar perturbations (f_{star}), the action of the vertical galactic tidal force (f_{tide}), or the impulsive change due to the passage of a molecular cloud (f_{mc}), respectively. We note that whereas f only depends on $\underline{a}$ for stellar perturbations, it also has an angular dependence for galactic tides and

https://doi.org/10.1017/S0252921100109819 Published online by Cambridge University Press

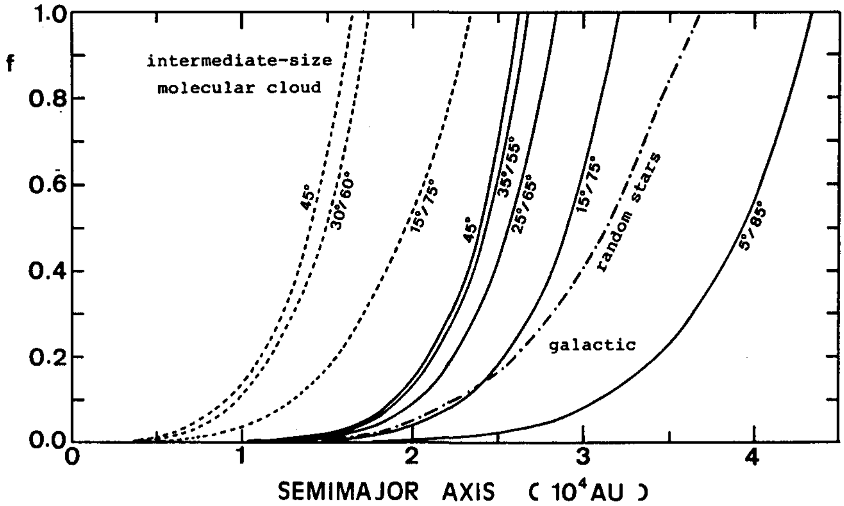

Fig. 7: Fraction of the loss cone filled with comets due to perturbations from random stars (dash-dotted curve), the tidal force of the galactic disk (solid curves for different galactic latitudes), or a penetrating encounter with an intermediate-size molecular cloud (dashed curves for different angles between the Sun-comet vector and the cloud's relative velocity).

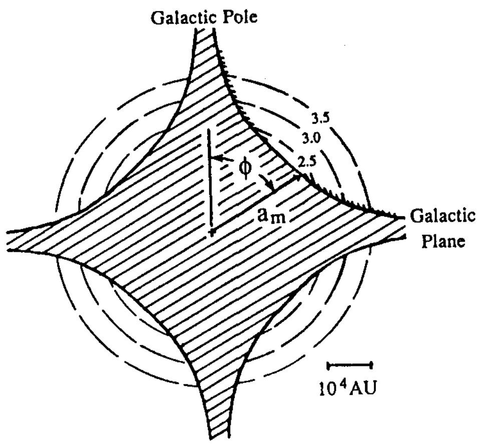

Fig. 8: Oort cloud comets with semimajor axes outside the shaded region can lose sufficient angular momentum in a single revolution to be injected into the visible region ($q < 3$ AU) by the tidal action of the galactic disk (from Morris and Muller 1986).

https://doi.org/10.1017/S0252921100109819 Published online by Cambridge University Press

molecular clouds (we can add a very close stellar passage). This is an important point when trying to explain the observed anisotropies in the aphelion directions.

For the different perturbing agents of the Oort cloud, the fraction of the loss cone filled with comets as a function of $\underline{a}$ is shown in Fig. 7. What we can consider as permanent perturbers of the Oort cloud — passing stars and galactic tides — will partially or totally fill the respective loss cones of comets with a range of semimajor axes from $a \simeq 1.2 \times 10^4$ AU to $a \simeq 3.5 \times 10^4$ AU. We note a strong dependence of f_{tide} on the galactic latitude. For $a \leq 1.2 \times 10^4$ AU, the loss cone will usually be empty, i.e., no Oort cloud comets will usually come to the inner planetary region from the inner core of the Oort cloud. We can compare our results with those obtained by Morris and Muller (1986), which are shown in Fig. 8. Comets outside the shaded region can lose sufficient angular momentum to enter the 'detectability' zone within $q \leq 3$ AU due to the tidal force of the galactic disk. We note the same dependence on $\sin 2\phi$.

Stronger perburbers, like molecular clouds or very close stellar passages, can temporarily reduce the limit of filled loss cones. We note that an encounter with a molecular cloud will also give rise to a strong angular dependence, in such a way that the smallest $\underline{a}$ at which the loss cone is filled corresponds to comets whose radius vectors are at an angle of 45° with respect to the encounter velocity (see Fig. 7). A similar angular dependence can be found for a very close stellar passage (see below). For $a \geq 3.5 \times 10^4$ AU, the respective loss cones will be filled at any time, so that we should expect a steady supply of comets from the outer portions of the Oort cloud, independently of occasional strong perturbers or the galactic latitude. This result is in good agreement with that found by Heisler and Tremaine (1986) using a different analytical procedure based on the computation of the mean square change in angular momentum per revolution due to stellar perturbations.

It is interesting to compare the range of $\underline{a}$ where the respective loss cones pass from empty to full with the observed range of original $\underline{a}$ of new comets (cf. Fig. 5). In general, we note a good correspondence. As shown in Fig. 6, Oort cloud comets ($10^4 \leq a_{orig} \leq 3.5 \times 10^4$ AU) concentrate at the mid-galactic latitudes at which loss cones fill at smaller $\underline{a}$. Thus, loss cones of comets at $\phi = 45°$ are filled at values of $\underline{a}$ about a factor of two smaller than those corresponding to Oort cloud comets close to the galactic equator ($\phi = 5°$).

Having derived the fraction of the loss cone filled with comets, we are in a position to compute the injection rate of Oort cloud comets. For this purpose, we have to assume a certain radial distribution of comets in the Oort cloud, of the kind

$$\Gamma(a)\, da \propto a^{-\alpha} da, \tag{29}$$

where the index α is unknown. Reasonable values of α are in the range of 2 to 4, depending on the degree of central condensation (Bailey 1983, Duncan et al. 1987, Fernández and Ip 1987).

Let us consider separately the different forces acting on Oort comets. The comet influx rate due to the cumulative effect of random stars will thus be given by

$$\dot{n}_{star} = \int_{a_1}^{+\infty} \Gamma(a)\, F_q\, f_{star}(a)\, \frac{1}{P}\, da, \tag{30}$$

where $a_1 \simeq 1.5 \times 10^4$ AU is the limiting semimajor axis such that $f_{star} \sim 0$ for $a < a_1$.

https://doi.org/10.1017/S0252921100109819 Published online by Cambridge University Press

Analogously, the influx rate of Oort cloud comets due to the tidal force of the galactic disk is

$$\dot{\eta}_{tide}(\phi) = \frac{\cos\phi}{2} \int_{a_2(\phi)}^{+\infty} \Gamma(a)\, F_q\, f_{tide}(a,\phi)\, \frac{1}{P}\, da, \tag{31}$$

where $a_2(\phi)$ has the same meaning as a_1. As a numerical example, we have that $a_2(\pi/4) \sim 10^4$ AU. The overall comet influx rate for all galactic latitudes will thus be given by

$$\dot{n}_{tide} = \int_{-\pi/2}^{+\pi/2} \dot{\eta}_{tide}(\phi)\, d\phi. \tag{32}$$

Equations (30) to (32) express the steady component of the injection rate of Oort cloud comets. The injection rate due to the vertical galactic tidal force will dominate for most galactic latitudes, except for an approximate $\pm 10°$ belt around the galactic equator, where it falls below the level of perturbing stars.

The sudden increase in the injection rate of Oort cloud comets caused by a penetrating encounter with a molecular cloud can be expressed as

$$\dot{\eta}_{mc}(\theta) = \frac{\cos\theta}{2} \int_{a_3(\theta)}^{a_{fill}} \Gamma(a) F_q\, f_{mc}(a,\theta)\, \frac{1}{P}\, da, \tag{33}$$

where $a_3(\theta)$ has the same meaning as $a_2(\theta)$ and a_1. For $a > a_{fill}$, the respective loss cones are already filled, so the encounter will not introduce more comets into them. We stress that the comet influx rate is strongly directional. The overall influx rate will thus be given by

$$\dot{n}_{mc} = \int_{-\pi/2}^{+\pi/2} \dot{\eta}_{mc}(\theta)\, d\theta. \tag{34}$$

Every 30 Myr or so, a star will pass at about 10^4 AU from the Sun, thus strongly perturbing the Oort cloud. The Oort cloud comets most strongly perturbed will be those along the star's path (Fernández and Ip 1987). We find a double cone-shaped volume with its vertex at the point of closest approach to the Sun wherein all loss cones will be filled (Fig. 9). Let us assume that all comets falling in the shaded disk of width dz and volume $\pi D^2 dz$ have roughly the same semimajor axis $\underline{a}$ and average heliocentric distance $r \sim 1.5a$. Due to the star's close passage, the respective loss cones will pass from a fraction $[1 - f(a)]$ already filled to be completely filled. The corresponding increase in the comet influx rate will be expressed as

$$\dot{n}_{close} = 2 \int_o^{z_{MAX}} \gamma(r) \pi D^2 F_q [1 - f(a)]\, \frac{1}{P}\, dz, \tag{35}$$

where $\gamma(r) \propto r^{-\propto+2}$ is the number of the density of comets, which is related to the distribution function $\Gamma(a)$, and $r^2 = z^2 + D_{\odot}^2$. Furthermore, for small values of r, we have $f(a) \sim 0$ (we note that this has implicitly been assumed in the case of a molecular cloud). For $\propto \geq 2$, we can take z_{MAX} as infinity without too much error. We note that the sudden influx of Oort cloud comets, triggered by a close stellar passage, will have aphelion directions heavily concentrated along the star's path and towards the point of closest approach.

https://doi.org/10.1017/S0252921100109819 Published online by Cambridge University Press

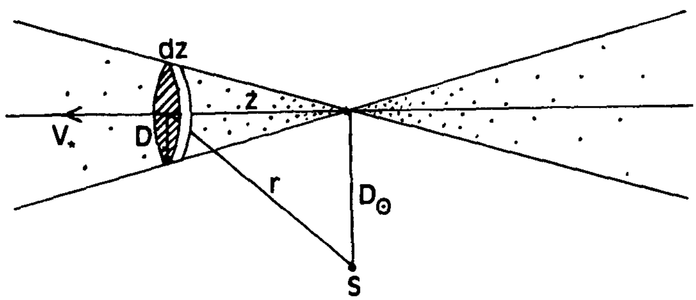

Fig. 9: Double cone along the star's path enclosing a region of the Oort cloud in which all loss cones get totally filled.

Table 1 shows relative values of influx rates of Oort cloud comets as caused by different perturbers and for different degrees of radial concentration of Oort cloud comets, going from weak ($\alpha = 2$) to strong ($\alpha = 4$). The comet influx rate caused by random passing stars is taken as unity in all the cases. As shown, tidal forces of the galactic disk are a factor of about 1.5 to 2 – depending on α – more effective than random passing stars.

As mentioned above, since vertical tides predominate, the influx rate of Oort comets will be strongly dependent on the galactic latitude. It is highest at $\phi = 45°$ and goes down to zero at $\phi = 0°$ and $\pm 90°$. As shown in Fig. 10, it is only slightly dependent on the radial structure of the Oort cloud.

Table 1. Influx Rate of Oort Cloud Comets

Perturber	α 2	3	4
Random stars	1.00	1.00	1.00
Intermediate-size molecular cloud	19.5	124.	1364.
Vertical galactic tidal force	1.49	1.72	2.18
Close stellar passage ($D_\odot = 10^4$AU)	18.4	62.1	191.

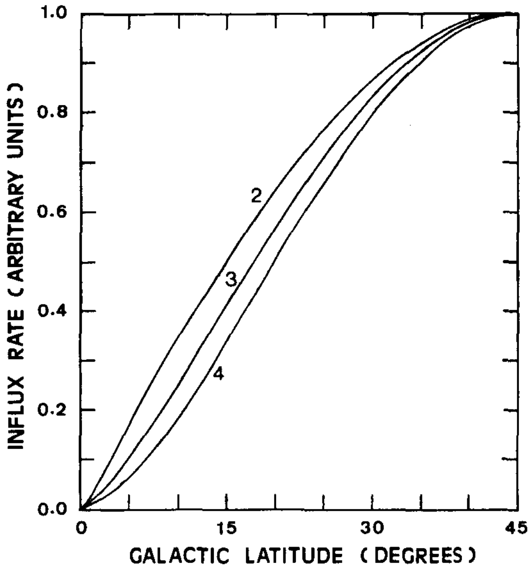

Fig. 10: Influx rate of Oort cloud comets caused by the tidal force of the galactic disk as a function of the galactic latitude. The attached numbers stand for three different exponents of assumed radial distributions of Oort cloud comets as defined in the text.

The sudden increase in the influx rate of Oort cloud comets due to a penetrating encounter with an intermediate-size molecular cloud ($M \sim 5 \times 10^3 M_\odot$) could be as high as $\sim 10^3$ if the Oort cloud were heavily concentrated. Yet, contrary to random passing stars and tidal forces that act continuously on Oort cloud comets, such encounters may occur at most at intervals of several 10^7 yr (see Section 2.4). More devastating effects would be expected from penetrating encounters with GMCs, but they are much less frequent, perhaps a few during the solar system lifetime (cf. Section 1). A close stellar passage at $D_\odot \sim 10^4$ AU can cause a temporary increase in the influx rate of Oort cloud comets comparable to that caused by a penetrating encounter with an intermediate-size molecular cloud, at least for $\alpha \lesssim 3$. The frequency of occurrence of both events might also be similar.

6. Temporal Variation of the Frequency of Comet Passages: Comet Showers.

The dependence of the distribution of aphelion points of new comets on the galactic latitude suggests that the tidal force of the galactic disk at present plays a dominant role in bringing comets to the planetary region (Fig. 11). From time to time, penetrating encounters with molecular clouds or very close stellar passages may trigger sudden increases in the influx rate of Oort comets of short duration – a few million years – that Hills (1981) called "comet showers". During these periods, the concentration of aphelion points on certain sky areas may overcome the galactic effect.

The fact that certain clusterings of aphelion points are still observable (cf. Section 4) indicates that the effects of the latest encounters with molecular clouds and/or very close stellar passages have not disappeared completely. Biermann et al. (1983) have argued that the number of comet aphelia belonging to clusterings is comparable to, but somewhat smaller than, the number of those that do not appear to belong to any group. We basically agree with this conclusion, though the observed galactic dependence in the distribution of aphelion points suggests to us that clustered comets (whose locations should not depend on the galactic latitude) should clearly represent less than half of all the sampled comets.

Comet showers have been considered to be responsible for biological mass extinctions for which Raup and Sepkoski (1984) claimed a periodicity of 26 Myr in their occurrence (see a recent discussion by Hut et al. 1987). There is no obvious terrestrial cause able to explain such periodicity, if it proves to be correct, which should be by no means taken for granted. Alvarez et al. (1980) have argued that the iridium enrichment found in the layer at the Cretaceous–Tertiary boundary was caused by the impact of an asteroid with the Earth. The end of the Cretaceous (66 Myr ago) was the time when the dinosaurs and, in general, nearly three-quarters of all the species alive at that time disappeared.

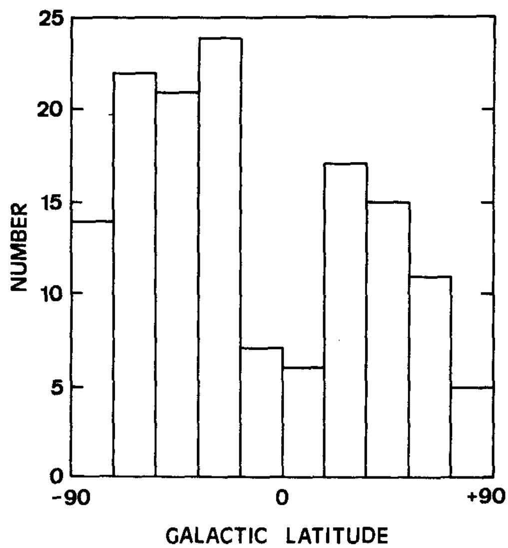

Fig. 11: Number of Oort cloud comets in equal-area strips of the celestrial sphere parallel to the galactic equator. The histogram contains the sample of 142 'young' comets defined in Fig. 4.

A periodicity in the ages of well–dated impact craters, with similar period and in phase with biological mass extinctions, has been presented by Alvarez and Muller (1984) as a strong evidence in favor of the occurrence of periodic comet showers. According to the authors, during a comet shower, several comets would collide with the Earth, thus increasing the frequency of occurrence of impact craters, and at the same time triggering a mass extinction.

Several mechanisms for triggering periodic comet showers have been proposed. We can mention: (1) a solar companion star Nemesis on a 26–Myr-period orbit (Whitmire and Jackson 1984; Davis et al. 1984), (2) excursions of the solar system through the galactic plane (Rampino and Stothers 1984); and (3) an as–yet undetected trans–Neptunian planet X that perturbs a comet ring. We shall not discuss the dynamical feasibility of such mechanisms, which have already been reviewed by other authors, e.g., Tremaine (1986). What we shall do is to analyze briefly the assumed periodicity in the impact cratering rate, which is crucial for our understanding of the production and intensity of comet showers and, in the last instance, to give support to any of the above–proposed mechanisms.

The age distribution of the 23 best–dated impact craters with diameters $\geq$10 km and ages $<$250 Myr has been analyzed by Stothers (1988). He obtains a best–fitting cycle of $\sim$30 Myr. A first look suggests two immediate problems: (1) the crater sample is very scant, and (2) the quoted errors in the estimated ages are rather large ($\approx \pm$ 6 Myr) for the older craters. In this regard, Heisler and Tremaine (1989) have found from Monte Carlo simulations that if the root-mean-square (r.m.s.) error is greater than about 13% of the assumed period – or about 4 Myr in our case – then the periodic data can no longer be distinguished from random data at the 90% confidence level. Furthermore, Fogg (1989) has recently argued that even if a periodicity does exist, the actual period may be blurred by a background impact flux from asteroids. Another important objection has been raised by Weissman (1985c), who argued that out of 8 identified impactor compositions, 6 are differentiated or highly differentiated meteorite types, which suggests an asteroidal nature rather than a cometary one. Comets presumably are very fluffy bodies, so it is possible that many of them will not survive a passage through the Earth's atmosphere as a single compact body. Aerodynamic stresses can be large enough to crush even kilometer-size bodies into a large number of fragments (Melosh 1981). If this is the case, they would not leave a single large crater, but, rather, many small craters or a shallow one that may be quickly eroded.

Even if the hypothesis of the collisions with comets as the cause of mass extinctions proves to be incorrect, it will not mean a rejection of the idea of the occurrence of comet showers. The only thing that will change is the notion that they would necessarily have to be reflected in the impact cratering record and in the extinction of species. If comet showers are triggered by random close encounters with stars or molecular clouds, they will not be periodic, even though they will have a characteristic frequency of, perhaps, a few 10^7 yr. Numerical models developed by Heisler et al. (1987) show very nicely how the frequency of comet passages fluctuates with time, interspersed with sporadic major enhancements of up to more than one order of magnitude.

The relaxation period of the Oort cloud to return to the quiescent stage after a comet shower has been triggered can last a few million years, with a tail of surviving shower comets lasting a few tens of millions of years. Numerical models simulating this process have been developed by Fernández and Ip (1987) and Hut et al. (1987). Profiles of comet showers obtained by Fernández and Ip (1987) are reproduced in Fig. 12. Such a long survival may guarantee that comet clusterings on the sky will be observed at almost any time. As we have seen in Section 4, this is what actually happens. Yet the fact that the aphelion distribution of new comets follows a pattern reflecting the influence of the galactic disk potential suggests that the frequency of comet passages is at present close to its bottom level. Comets injected during a shower might greatly exceed the steady supply of Oort comets, brought mainly by galactic tides, in such a way that the galactic dependence in the distribution of aphelion points could be severely weakened.

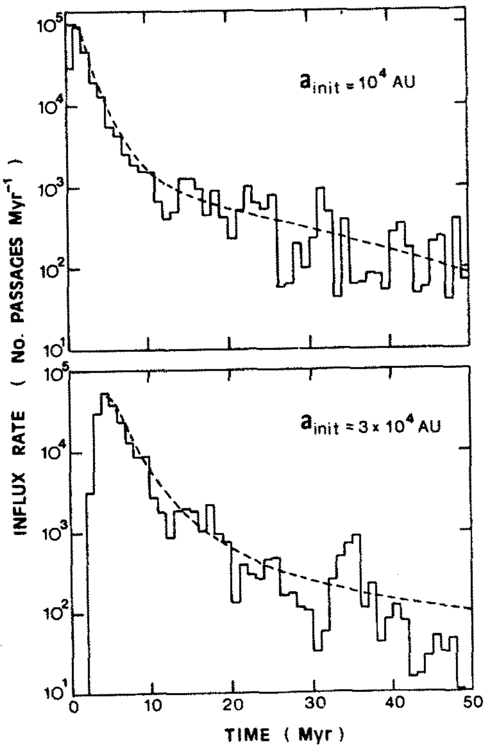

Fig. 12: Numerical simulations showing the time evolution of comet showers, expressed as the number of passages with $q < 1.1$ AU per Myr, after injection of 10^5 hypothetical comets with the initial semimajor axes indicated at the upper right of the graphs from Fernández and Ip (1987).

7. Dynamical Stability of Comets in the Oort Cloud

Comets may be removed from the Oort cloud either because they enter the planetary region, where they are strongly perturbed by the planets, or because they get enough energy from external perturbers to overcome the gravitational field of the Sun. Most Oort comets at present probably have their perihelia outside the planetary region (i.e., outside their

https://doi.org/10.1017/S0252921100109819 Published online by Cambridge University Press

respective loss cones), so we can consider them quite stable dynamically. Yet the situation might have been quite different in the early solar system if comets formed, for instance, in the Uranus and Neptune accretion zones (Kuiper 1951; Safronov 1972; Fernández 1980a). Such early comets would have been within their loss cones. Contrary to the case of Oort comets injected into their loss cones reviewed before, we have to analyze now how early comets were removed from their loss cones.

The transverse velocity, v_T, of a comet on a near-parabolic orbit is approximately related to its perihelion distance by the following expression

$$v_T^2 \simeq \frac{2GM_\odot q}{r^2}. \tag{36}$$

Thus, to change q by an amount Δq, the comet will have to suffer a change in its transverse velocity given by

$$\frac{\Delta v_T}{v_T} \sim \frac{1}{2}\frac{\Delta q}{q}. \tag{37}$$

To be removed from the planetary region, the change in its transverse velocity should be at least on the order $\Delta v_T \sim v_T$. Appropriate expressions for Δv_T are given by Eqs. (28a), (28b) and (28c), corresponding to stellar perturbations, the tidal force of the galactic disk and a penetrating encounter with a GMC (assumed to be of $M_{cl} \sim 5 \times 10^5 M_\odot$ and radius $R_{cl} \sim 20pc$), respectively.

By considering the r.m.s. change in Δv_T over the age of the solar system, $T_{ss} = 4.5 \times 10^9 yr$, we obtain the semimajor axes of comets for which $(\Delta v_T)_{rms} \simeq v_T$ as a function of their perihelion distance (Fig. 13). Such comets will by now have their perihelia removed from the planetary region. For instance, for $q = 30$ AU, the corresponding semimajor axis will be $a \simeq 3.3 \times 10^3$ AU. As Fig. 13 shows, passing stars are the most efficient perturbers of the inner core of the Oort cloud, say, $r < 10^4$ AU, because of their ability to penetrate it. This is in qualitative agreement with Bailey's (1986a) conclusion that stars dominate the dynamical evolution of comets for $a < 2 \times 10^4$ AU. We note that the closest passage of a star to the Sun expected during its lifetime is $(2n_0T_{ss})^{-1/2} \simeq 1.5 \times 10^3$ AU.

A strong change of q will generally be accompanied by a large change of the inclination i, so one would expect that comets for which $(\Delta v_T)_{rms} \sim v_T$ will have the orientations of their orbital planes nearly randomized by now. Even if comets started out their dynamical evolution in orbits close to the ecliptic plane, a full randomization of their orbital planes should have been reached for $a > 3000-4000$ AU, as a result of the action of very close stellar passages. Indeed, from numerical simulations including planetary perturbations, random stars and galactic tides, Duncan et al. (1987) found randomization of orbital planes for $a > 5000$ AU.

Once the r.m.s. change of the comet's velocity reaches the escape velocity, the comet will be lost to the interstellar space, i.e., when

$$(\Delta v)_{rms} = v_{esc} = \left(\frac{2GM_\odot}{r}\right)^{1/2}, \tag{38}$$

where Δv is given by Eqs. (11), (12) or (17), depending on whether the considered

https://doi.org/10.1017/S0252921100109819 Published online by Cambridge University Press

perturbers are random stars, GMCs, or galactic tides, respectively. The values of a at which Eq. (38) is fulfilled are shown in Table 2.

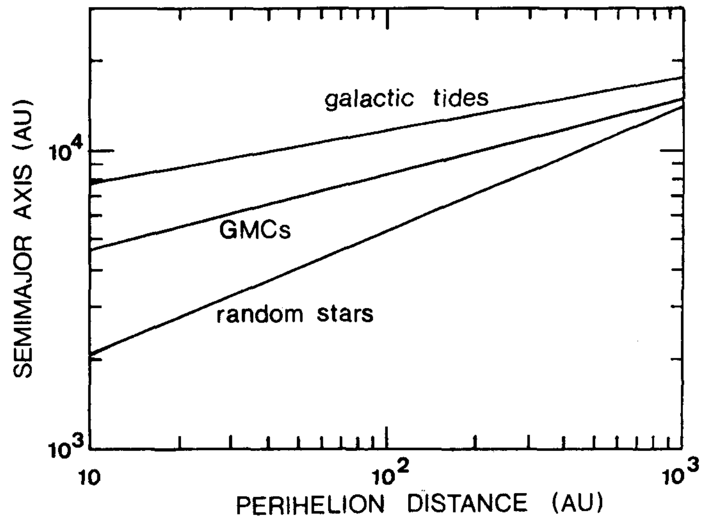

Fig. 13: Semimajor axes of comets as a function of their perihelion distance for which the rms change in the transverse velocity, Δv_T, over the solar systems lifetime equals v_T. Three different kinds of perturbers of the Oort cloud are considered.

From Table 2 we see that penetrating encounters with GMCs and the tidal force of the galactic disk are of fundamental importance in defining the stability boundary of the Oort cloud at $a \sim 3 \times 10^4$ AU, or a radius $\sim 6 \times 10^4$ AU for very eccentric orbits. This turns out to be significantly smaller than the boundary originally found by Oort (1950) and later recomputed by Weissman (1980a), who only considered the effect of stellar perturbations. Our theoretical estimate can be compared with the observed maximun separations between members of wide binary stars in the Galaxy, which are just on the order of a few 10^4 AU (Bahcall and Soneira 1981).

The fact that we define a stability radius does not mean that beyond it, the Oort cloud is empty, since the cloud will be continuously replenished with comets from the inner Oort cloud. This is seen in the distribution of original semimajor axes of new comets (cf. Fig. 5), in which nearly 40% have $a > 3 \times 10^4$ AU. Nevertheless, we should expect a significant drop

https://doi.org/10.1017/S0252921100109819 Published online by Cambridge University Press

Table 2. Semimajor Axis at which Oort Cloud Comets Get Escape Velocities Over $T_{ss} = 4.5 \times 10^9$yr

Perturber	a (AU)
Random stars	1.3×10^5
Tidal force of the galactic disk	3.0×10^4
GMCs	$(3.0 \pm 1.0) \times 10^{4*}$

*NOTE: The uncertainty for a possible range of penetrating encounters is between 1 and 10 (Bailey 1983).

in the number of Oort cloud comets for larger a as comets have ever decreasing dynamical lifetimes in this region.

We note that new comets with $a \gtrsim 3 \times 10^4$ AU show the same trend to concentrate at mid–galactic latitudes as those with $a < 3 \times 10^4$ AU. Yet under the action of stellar perturbations, the former ones will have their respective loss cones filled, independently of the galactic latitude (cf. Fig. 7), which seems to be in contradiction with the observations. This might indicate that new comets with $a > 3 \times 10^4$ AU have reached such large semimajor axes only in the recent past (perhaps due to the perturbations of the outer planets), which allows them to keep a "memory" of their galactic dependence in the incoming directions.

In short, from a dynamical viewpoint, we can define three regions in the Oort cloud: (1) an inner core (semimajor axes $a < 4000 - 5000$ AU), still showing a strong concentration of comets towards the ecliptic plane (provided that they formed in the protoplanetary disk); (2) a spherical inner Oort cloud ($4000 < a < 30,000$ AU), containing comets dynamically stable over periods comparable to the solar system's age; (3) an outer Oort cloud ($a > 3 \times 10^4$ AU), where comets should have been ejected over time spans comparable to, or smaller than, the solar system's age. Therefore, Oort comets belonging to this region have been there for $t < 4.5 \times 10^9$yr, or even much less than that, given their galactic dependence.

8. End-States of Long-Period Comets

Dynamical ejection is by far the main loss mechanism. For LP comets reaching the inner planetary region, Jupiter is the main perturber. As LP comets random–walk in the orbital energy-phase space, most of them will finally jump from the potential well to hyperbolic orbits.

As seen before, the number of LP comets still bound to the solar system after n perihelion passages goes down as $n^{1/2}$. Thus, an original population of N_{NEW} new comets injected into the inner planetary region will give rise to an average number of passages of evolved LP comets given by (Fernández 1985)

$$n_{EVOL} = \sum_{n=1}^{n_{MAX}} N(n) \simeq N_{NEW}\, n_{MAX}^{1/2}, \tag{39}$$

where $N(n)$ is given by Eq. (3) and n_{MAX} is the maximum number of perihelion passages. For small-q comets we should expect that physical causes limit n_{MAX}.

The ratio n_{EVOL}/N_{NEW} should be equivalent to the ratio between the frequency of passages of evolved LP comets and new comets: $\dot{n}_{EVOL}/\dot{n}_{NEW}$. As shown in Section 1, this ratio is ~ 5 for Earth-crossing LP comets. Therefore, we obtain

$$n_{MAX} = \left(\frac{\dot{n}_{EVOL}}{\dot{n}_{NEW}}\right)^2 = 25. \tag{40}$$

Sublimation of volatiles may be another possible loss mechanism of cometary nuclei, although the number of revolutions that a freely sublimating comet nucleus with $q \sim 1$ AU can perform before being completely sublimated may be one to two orders of magnitude greater than the upper limit given by Eq. (40) (Cowan and A'Hearn 1979). Formation of insulating dust mantles may further prolong the physical lifetimes of comets. On the other hand, splitting of comet nuclei may contribute to their disintegration process, although it is not clear that by itself this leads directly to their destruction. On the contrary, it can produce daughter comets, as exemplified by the well known sungrazer group, comets P/Neujmin 3 and P/Van Biesbroeck which probably originated from one single comet that broke up around 1840 (Carusi et al. 1984) and LP comets Levy 1988e and Shoemaker-Holt 1988g, which have very similar orbital elements, also suggesting a common origin (MPC 13304).

At $n_{\max} \sim 25$ revolutions, about 90% of the original comet population would have already been lost to interstellar space, as derived from Eq. (3). If $n_{\max} \sim 25$ has a meaning at all as an upper limit, physical elimination has to account only for the remaining $\sim 10\%$ of the original population. Weissman (1980b) attributes the elimination of $\sim 28\%$ of the incoming LP comets to random disruption, i.e., about a factor of three larger than our estimate. The disagreement can be explained as being due to the different procedures used to estimate such percentages and to the different roles assigned to nuclear splitting as a direct cause of physical loss of comets.

Direct collision with a planet, its ring system or penetration within its Roche limit leading to tidal disruption constitutes a very rare phenomenon. For a LP comet on a randomly oriented orbit, Weissman (1980b) computes a combined probability of collision with any of the planets of 1.3×10^{-7} per perihelion passage (understood as a passage within its Roche radius).

The process of catastrophic collisions with interplanetary boulders deserves a closer look since it might be of a certain significance as an end-state of some LP comets. For an average random LP comet of $q \sim 1$ AU, Fernández (1990) finds a typical collisional lifetime of $N_{coll} \sim 5 \times 10^4$ revolutions, bearing in mind that a wide range of values around N_{coll} are possible due to different orbital inclinations as well as uncertainties in a comet's size, its internal strength and the number and size distribution of interplanetary boulders. The probability of catastrophic collision with an interplanetary boulder is thus $P_{coll} \sim N_{coll}^{-1} \sim 2 \times 10^{-5}$ per perihelion passage, with an uncertainty of about one order of magnitude. Taking into consideration that a LP comet can perform an average number of five revolutions, we

https://doi.org/10.1017/S0252921100109819 Published online by Cambridge University Press

obtain a probability of collisional disruption of about 10^{-4} per LP comet reaching the region of the terrestrial planets (say, with $q \leq 2$ AU). Even though this process should be considered as marginal as an end-state, as compared with other physical processes like sublimation or nuclear splitting, nevertheless it turns out to be about two orders of magnitude more probable than a planetary collision.

A LP comet can be transferred to a SP orbit after an usually long dynamical process that can take up to thousands of revolutions. Thus, if $\Delta E \sim 3 \times 10^{-3}$ AU^{-1} is the typical energy change of a Jupiter-crossing LP comet on a low-inclination orbit (Fernández and Ip, 1983b), then the average number of revolutions required to pass from the Oort cloud ($E \sim 0$) to a typical SP comet orbit with $P < 20$ yr ($E_{SP} \geq 0.136$ AU^{-1}) will be

$$n = \frac{E_{sp}^2}{\Delta E^2} \simeq 2 \times 10^3. \tag{41}$$

The physical processes described before, namely sublimation, nucleus splitting or collisional disruption, may prevent a LP comet with a perihelion distance of less than a few AU from reaching a SP orbit. The dynamical process of transfer to SP orbits should then be left to more distant LP comets, say, with perihelion distances $q \geq 4$ AU. Everhart (1972) has found a capture zone in the phase space $4 < q < 6$ AU and $0 < i < 9°$ with an efficiency of one captured comet per $\sim 10^2$ injected comets in near-parabolic orbits. Everhart followed the dynamical evolution of a sample of hypothetical comets for up to 10^3 revolutions. It is very likely that the prolongation of the study to a larger number of revolutions would result in the capture of comets with larger i as they evolve slowly and therefore need more revolutions to reach SP orbits (see below).

The previous analysis shows that the transfer to SP orbits does not have much relevance as an end-state of LP comets. Assuming that a capture probability of $f_c \simeq 10^{-2}$ holds for the range of inclinations $0 < i < 30°$, corresponding to most observed SP comets, the ratio of captured SP comets to new comets will be $f_c \times f_{i<30} \simeq 6.7 \times 10^{-4}$. $f_{i<30} = 0.067$ is the fraction of new comets with $i < 30°$ under the assumption of random orientations of their orbital planes. Therefore, less than one comet in 10^3 will end up on a SP orbit, provided that physical losses are neglected. As seen before, for $q \leq 2$ AU, disintegration processes should set an upper limit to the comet lifetime shorter than the dynamical time scale for reaching a SP orbit, so the capture ratio should drop well below the value derived above.

9. The Origin of the Short-Period Comet Family: A Comet Source at the Outer Fringe of the Planetary Region.

The idea that SP comets come from the capture of LP comets by Jupiter grew stronger during the past decade, based on Everhart's (1972) fundamental work showing that LP comets can be more efficiently transferred to SP orbits after a long diffusion process rather than by a single close encounter with Jupiter. Yet there were some persisting difficulties. In particular, there seemed to be too many SP comets as compared with the theoretical

https://doi.org/10.1017/S0252921100109819 Published online by Cambridge University Press

prediction, if they came from captures by Jupiter (Joss 1973, Fernández and Ip 1983b). Delsemme (1973) explained the observed steady-state number of SP comets on the basis of a source of 3×10^4 to 10^5 intermediate period comets with perihelia $4 < q < 6$ AU. This proposal simply shifts the problem to trying to justify the existence and size of such an intermediate source. Thus, it seems necessary to consider other mechanisms for bringing comets to SP orbits. One possibility is that giant planets other than Jupiter also contribute to the capture of LP comets (Kazimirchak-Polonskaya 1976, Bailey 1986). Another possibility is that SP comets come from a trans-Neptunian belt, as suggested by Mendis (1973) and then developed by Fernández (1980b), Fernández and Ip (1983b) and Duncan et al. (1988).

On cosmogonic grounds, Kuiper (1951) argued that comets formed out of the nebular material contained in the region between 30 and 50 AU, from where they were scattered to the Oort cloud, first, by the perturbations of Pluto and then by the perturbations of Neptune once they became Neptune-crossers (c.f. also a later discussion by Safronov 1972). The concept of a trans-Neptunian comet belt was later elaborated by Cameron (1962) and Whipple (1964). Hamid et al. (1968) looked into the apparitions of P/Halley of 1835 and 1910 to see whether there was any evidence for perturbations produced by a hypothetical comet belt beyond Neptune. This idea was further elaborated numerically by Fernández and Ip (1983a), who showed that a residual population of icy planetesimals might have remained in the Uranus-Neptune zone. Using a two-planet map to model the evolution of initially circular test particle orbits in the solar system , Duncan et al. (1989) have recently found dynamically stable bands between Saturn and Uranus and between Uranus and Neptune that might contain residual planetesimals from the protoplanetary disk.

Duncan et al. (1988) have shown that SP comets captured from an original population of LP comets with random inclinations would have a fraction of retrograde orbits. Since SP comets on retrograde orbits are not observed, Duncan et al. conclude that they should come from a low-inclination comet belt. The authors considered samples of hypothetical comets starting out at Neptune's zone. To speed up the orbital integration, they multiplied the masses of the Jovian planets by factors up to 40. However, this procedure of computing the orbital diffusion has been criticized by Stagg and Bailey (1989) on the basis that it favors the moderate energy changes over the strong ones. They argued that the very rare strong energy changes should be dominant when the actual planetary masses are adopted because, in this case, the orbital diffusion becomes so slow that they are very likely to occur. By contrast, when the planetary masses are greatly increased, "moderate" perturbations can produce captures to SP orbits without any "strong" perturbation.

To check the previous results, we have performed a simple numerical simulation using Öpik's two-body formulation by considering two cases that reflect the two proposed origins for the SP comet family: (1) an original spherical population of comets, as would be the case if the captured comets came from a spherical Oort cloud, and (2) a flattened comet population ($i < 20°$), as would be the case if comets came from a trans-Neptunian comet belt. As an example, we show in Fig. 14 the kind of i-distributions we obtain for the test comets that end up on SP orbits ($P < 20$ yr). For case (1), about 25% of the computed SP comets are on retrograde orbits, which is in qualitative agreement with Duncan et al.'s (1988) result. The time scale for comets to evolve to SP orbits was typically a few 10^8 yr. About 1/3 of all the comets that ended up with $q < 2.5$ AU have intermediate-period orbits ($20 < P < 200$ yr). For case (2), we have not obtained SP comets with $i > 70°$ in

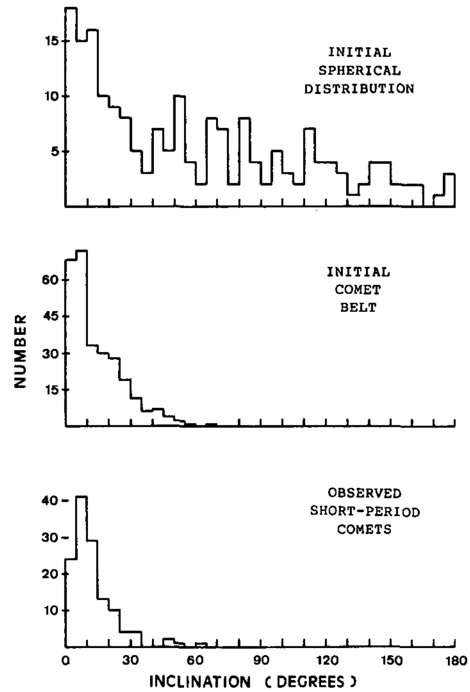

Fig. 14: Computed inclination-distributions of SP comets ($P < 20$ yr) obtained from two hypothetical samples of comets assumed to be captured by Neptune, as compared with the one for the observed SP comets (bottom). The distribution at the top was derived from an initial spherical population of comets (as, e.g., Oort cloud comets), whereas the one in the middle was derived from an initially flat distribution such as that assumed for the comet belt.

any case. This is in better agreement with the observed i-distribution of SP comets, as can be compared in Fig. 14. In this case, the time scale for comets to evolve to SP orbits is several 10^7 yr. For case (2), we only recorded those comets reaching perihelion distances $q < 1.5$ AU.

An original comet population with random orientations of the orbital planes gives a significant fraction of SP comets on retrograde orbits, which contradicts observations. Yet some intermediate-period comets on retrograde orbits, like P/Halley, might have come from such an original comet population. Since Oort cloud comets are continuously being injected into the planetary region, an inevitable consequence would be the capture of some of them by the giant planets into SP and intermediate-period orbits. This may explain cases like P/Halley and P/Tempel-Tuttle. For the remaining comets so far discovered and most intermediate-period comets on direct low-inclination orbits, some sort of comet belt, as described by case (2), seems to be the most probable source.

As discussed before, residual icy planetesimals from the protoplanetary disk might have remained tightly bound to the solar system. In some cases, they might have kept their primordial near-circular orbits as long as they did not closely interact with Neptune or Uranus. Others might have evolved to eccentric orbits, although they would still be far from where they could be subject to perturbers outside the solar system; thus, these planetesimals would maintain their primordial low-inclination orbital planes. All these bodies would then form a flat structure, close to the ecliptic plane, stretching from the outer planetary region to several 10^3 AU, where perturbations from passing stars and molecular clouds would be strong enough to have randomized the planetesimals'orbital planes over the age of the solar system. This structure should be more properly called a "comet disk" than a "comet belt," given its large extent. Figure 15 gives an overall picture of the space distribution of comets as a synthesis of several current ideas on the subject.

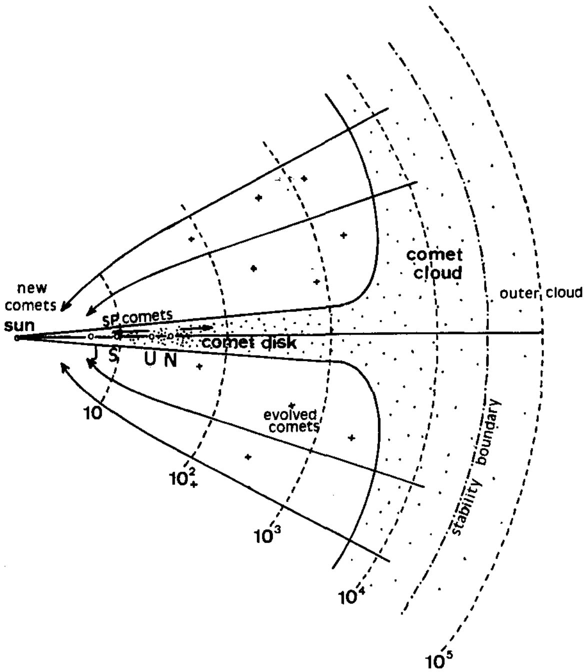

Fig. 15: A schematic picture of the current spatial distribution of comets around the solar system if they formed in the protoplanetary disk. A comet disk might still be present, stretching from the region of the outer planets to a few 10^3 AU. Farther away, the action of external perturbers should have by now randomized the orbital planes of comets. New comets probably come from the Oort cloud, whereas most SP comets may come from the comet disk. The spherical volume a few 10^3 AU in radius should contain evolved LP comets (crosses). The dashed circles indicate distances to the Sun in AU.

10. The Comet Population in the Outer Planetary Region.

At present we know of only two bodies pertaining to an outer solar system population: Pluto – possibly a large planetesimal formed in a heliocentric orbit (McKinnon and Mueller 1988) – and Chiron that moves between the orbits of Saturn and Uranus. This is too limited a sample to permit any meaningful derivation of the size of a trans-Jovian comet population. We have to use some indirect pieces of evidence to infer something about such a population, as, for instance, the required input rate of comets to keep the SP comet family in the steady state. Thus, Delsemme (1973) estimated a transient population of 3×10^4to10^5 intermediate-period comets with perihelia in the range of 4 to 6 AU, whereas Duncan et al. (1988) estimated a transient population of about 10^5 bodies inside the orbit of Uranus. From our simulations we derive steady-state numbers of about 10^5 bodies in the Jupiter-Saturn region, and $\sim 2 \times 10^6$ bodies in the Uranus-Neptune region, under the assumption that SP comets come from an outer comet belt or disk (Ip and Fernández, 1990).

A massive comet belt or disk might be detectable by its combined flux in the far infrared through searches from space such as that carried out by the IRAS satellite (Bailey 1983, Jackson and Killen 1988). It also would be possible to detect the brightest members of comet populations either between the orbits of the giant planets or beyond Neptune.

Only a few limited searches for outer solar system (OSS) bodies have been undertaken, starting with the one carried out by Clyde Tombaugh that led to the discovery of Pluto in 1930. Through a 14-year search at the Flagstaff Observatory, Tombaugh was able to cover a large fraction of the northern sky down to an absolute photographic magnitude $m_{pg} \sim 18$, but no OSS bodies other than Pluto showed up. Luu and Jewitt (1988) have recently carried out a deep survey, close to the ecliptic plane, searching for OSS bodies, which drew negative results. This allows us to set upper limits to the number of bodies in the outer solar system. For objects with apparent magnitudes $m_v \leq 20$, Luu and Jewitt find that their number density should be $\Sigma < 1.7 \times 10^{-2}$ deg^{-2} for slow-moving objects located beyond Saturn ($r > 10$ AU).

An even more comprehensive search of OSS bodies with the 48-in. Palomar Schmidt Telescope has recently been reported by Kowal (1989), covering an 8-year period (1976-1985). He surveyed a sky area of 6400 deg^2 in the ecliptic region as compared with only 297 deg^2 of Luu and Jewitt's survey for the same limiting magnitude of $m_v = 20$. Kowal has obtained a number density of $\sum \sim 1.56 \times 10^{-4}$ deg^{-2} trans-Jovian objects brighter than $m_v = 20$, which turns out to be about two orders of magnitude smaller than the upper limit set by Luu and Jewitt. Yet Kowal's result should be taken with extreme care since it was based on a single discovery (Chiron) throughout the studied period.

From the previous results, let us try to estimate upper limits, n_{up}, to the size of comet populations at different range of heliocentric distances. To this end, let us consider a belt that extends 10° on either side of of the ecliptic, whose area is approximately $360 \times 20 = 7200 deg^2$, in which there will be a maximum number of objects with $m_V \lesssim 20$ given by

$$N_{MAX} \gtrsim 7200 \times \Sigma \simeq 10^{1\pm1} \tag{42}$$

https://doi.org/10.1017/S0252921100109819 Published online by Cambridge University Press

Let us assume a size distribution of bodies

$$s(R)dR \propto R^k dR, \tag{43}$$

where the index k is still somewhat uncertain. For a population of bodies subject to a process of collisional fragmentation, $k \sim 4$, as for the case of intermediate-size main-belt asteroids (Hughes 1982, Donnison and Sudgen 1984). A photographic survey of faint asteroids carried out by Ishida et al. (1984) tends to confirm the previous value applicable to asteroids with diameters > 25 km. We will apply the value $k = 4$ for the population of OSS bodies, even though we should bear in mind that they might have had a different formation and collisional history.

Let us consider the upper limits of size populations for three zones: (1) Saturn-Uranus, typical r = 15 AU; (2) Uranus-Neptune, typical r = 25 AU; and (3) a trans-Neptunian belt, typical r = 40 AU.

The radius-magnitude relation is

$$pR^2 = 2.25 \times 10^{16} r^2 \Delta^2 10^{0.4(m_\odot - m)}, \tag{44}$$

where p is the geometric albedo of the body; we assume that p = 0.1, R is the radius expressed in kilometers, r and Δ are the heliocentric and geocentric distances of the body, respectively, expressed in AU. For distant objects, we can assume that $r \sim \Delta$ and $m_\odot = -26.74$ is the apparent visual magnitude of the Sun. For the limiting magnitude $m_v = 20$, we get the corresponding limiting radii, R_L, shown in Table 3 for zones (1), (2) and (3).

Table 3. Outer Solar System Bodies

Average r (AU)	R_L (km)	Maximum Number	Upper Limit to the Population Mass (g)
15	48	$1.1 \times 10^{6\pm1}$	$9.0 \times 10^{22\pm1}$
25	133	$2.4 \times 10^{7\pm1}$	$1.8 \times 10^{24\pm1}$
40	340	$3.9 \times 10^{8\pm1}$	$3.4 \times 10^{25\pm1}$

If the size range of the outer solar system bodies is $R_{min} < R < R_{max}$, we find that the total number of bodies that defines the upper limit to the size of the comet population is

$$n_{up} = \int_{R_{\min}}^{R_{\max}} s(R)dR \simeq n_{m\leq 20} \left(\frac{R_{\min}}{R_L}\right)^{1-k} \tag{45}$$

https://doi.org/10.1017/S0252921100109819 Published online by Cambridge University Press

If we adopt $R_{\min} = 1$ km, we obtain the maximum numbers of bodies shown in Table 3. Considering the most restrictive upper limit found by Kowal, the number of objects for the Saturn-Uranus zone turns out to be on the order of the value obtained by Duncan et al. (1988) to keep the SP comet population in the steady state.

The corresponding upper limits to the masses of the population of the outer solar system bodies are obtained from

$$M_{up} = \frac{4}{3}\,\pi\rho C \int_{R_{\min}}^{R_{\max}} R^3\, s(R)\, dR. \tag{46}$$

where C is a proportionality factor. After some algebra, we obtain

$$M_{UP} = \frac{4}{3}\,\pi\rho\, \frac{(k-1)n_{m\leq 20}}{R_L^{1-k}}\, \frac{R_{\max}^{4-k} - R_{\min}^{4-k}}{(4-k)}\, \frac{1}{[1-(R_L/R_{\max})^{4-k}]}. \tag{47}$$

The corresponding values of M_{UP} are shown in Table 3. We have adopted $\rho = 1$ g cm^{-3} and $R_{\max} = 500$ km for the zones of Saturn-Uranus and of Uranus-Neptune and $R_{\max} = 10^3$ km for the trans-Neptunian belt. The values of $R_{\max}$ were chosen with the understanding that larger bodies would have been quickly discovered from ground-based observations.

The results of Table 3 show that Luu and Jewitt's survey has not been extensive enough to detect population masses in the outer planetary region on the order of 1 to 10 times the mass of the asteroids or on the order of Mars's mass in the trans-Neptunian comet belt. The latter mass is comparable to the limit estimated by Hamid et al. (1968) from the lack of observed effects on Halley's motion. However, the most stringent condition set by Kowal's survey would give an upper limit to the comet belt mass not greater than a few times the mass of Ceres.

The consequence of transferring comets from an outer belt to SP orbits is to have transient comet belts between the outer planets. It is possible that these transient belts might mingle with stable bands of primordial cometesimals left there after the planetary formation. The existence of such stable bands has been analyzed by Duncan et al. (1989). Our numerical results show that the steady-state populations in the Saturn-Uranus and Uranus-Neptune zones should be on the order of 10^6 bodies. This is about two orders of magnitude smaller than the constraint imposed by the negative detection of the outer solar system bodies by Luu and Jewitt, as quoted in Table 3, or on the order of the estimated number of bodies there if we follow the most restrictive result found by Kowal.

11. Cometary Impacts on Planetary Surfaces and Atmospheres

11.1 CRATER SIZE DISTRIBUTION AND SCALING LAW

One important consequence of the injection of comets from the Oort cloud and the trans-Neptunian belt into the inner solar system has to do with impacts on planetary surfaces. For planets with substantial atmospheres, like the present Earth and Venus, hypervelocity atmospheric interaction of cometary nuclei composed of fluffy structures might

https://doi.org/10.1017/S0252921100109819 Published online by Cambridge University Press

lead to fragmentation of the objects before hitting the ground. The chance of large surface cratering thus might be reduced. In other words, the terrestrial crater records could be biased towards impact events of asteroidal origin because of such selection effects. For more complete records registering both cometary and asteroidal impacts, the lunar and Mercurian surface would be better suited for this purpose. In the following, we shall therefore use the lunar record as a benchmark in the examination of comet impact histories of other planetary objects.

One important ingredient in the study of impact craters is the scaling law between the kinetic energy (and hence the mass and impact velocity of the projectile) and the diameter (D) of the crater created. Several different expressions have been derived (see Shoemaker and Wolfe 1982). A more recent study by Holsapple and Schmidt (1982) shows that the mass of the projectile can be related to the density (ρ) and surface gravity (g) of the target body, impact velocity (V), and density (δ) of the projectile in the following fashion

$$m(D) = 0.089 \left(\frac{\rho}{V}\right)^{6/5} g^{3/5} \delta^{-1/5} D^{18/5} \tag{48}$$

In the case of the Moon, with g = 161 cm s^{-2} and $\rho = 2.9$ g cm^{-3}, we find

$$m(D) = 3.31 \left(\frac{16.1\,\mathrm{km/s}}{V}\right)^{6/5} \left(\frac{2}{\delta\, gm^{-3}}\right)^{1/5} D^{18/5} \tag{49}$$

where D is in units of meters and m in units of grams. In Fig. 16, the m-D relations for three cases are illustrated: (a) the Apollo-Amor objects with $\delta = 3.0$ g cm^{-3} and v = 16.6 km s^{-1}; (b) the short-period comets with $\delta = 0.5$ g cm^{-3} and v = 26.0 km s^{-1}; and (c) the long-period comets with $\delta = 0.5$ g cm^{-3} and v = 56 km s^{-1}. With the same mass, the corresponding values of D created by these three different types of stray bodies do not differ by more than 30%.

According to Strom and Neukum (1989), the crater diameter (D), in units of kilometers, and the cumulative crater frequency, in units of km^{-2}, follow a simple power law relation of $N \propto D^{-\alpha}$. Between $D \approx 5$ km and 50 km, $\alpha \approx 2$, and for $D < 5$ km and $D > 50$ km, $\alpha \approx 3$. That these two curves exhibit close similarities is an indication that, as a first approximation, they might have been subject to bombardment processes by the same populations of stray small bodies during the history of the Solar System. There are, in fact, several groups of small objects that might be relevant in this connection. First, there are the planetesimals left behind after the formation of the terrestrial planets. The bulk of the planetesimals would be cleared away via collisions and planetary gravitational scattering within a time scale of 100 million years . However, some of these inner solar system planetesimals and a long-lived component in the outer planetary region might be able to survive a few hundred million years or more and could be one of the sources of the late-heavy bombardment event terminated at about 3.8 billion years ago (Wetherill 1975). The next group of bombarding bodies is the Apollo-Amor objects referring to the Mars- and Earth-crossing populations of asteroids (and defunct cometary nuclei). According to the observational statistics (Helin and Shoemaker 1979, Wetherill and Shoemaker 1982, Bailey and Stagg 1988), the total Earth-crossing Apollo-Amor objects with absolute visual magnitude H_{10} brighter than 18 can be estimated to be

$$n(<18) = 800 \pm 300.$$

Using the brightness-size relation given in Eq.(44), we find that the mass of a C-type asteroid with $H_{10} = 18$ is about 4.5×10^{15} g and 8.7×10^{14} g for a S-type object (see Shoemaker et al. 1979). Note that $p_v \approx 0.02$-0.065 for the C-type objects and $p_v \approx 0.065$-0.23 for the S-type objects.

The impact probability of the Earth-crossing asteroids with the Earth is on the order of (Bailey and Stagg 1988)

$$P_A(\oplus) = (5.0 \pm 2.5) \times 10^{-9} \mathrm{yr}^{-1}$$

In the case of impact with the Moon, the collision probability would be reduced by a factor proportional to the ratio of their gravitational cross-sectional areas, i.e.,

$$P_A(\mathrm{Moon}) = P_A(\oplus)/21.3$$

With an average impact velocity of $< V_i > = 16.6$ km s^{-1}, the Apollo-Amor objects with V(1,0) $\approx$ 18 would produce a crater diameter D $\approx$ 20 km on the Moon. The corresponding crater frequency at present time thus would be

$$N_A(D > 20km) = n_A P_A(Moon) = 5.4 \times 10^{-7} yr^{-1}. \quad (50)$$

In comparison, the lunar record given by Neukum (1975) and Strom and Neukum (1989) yields

$$N_A(D > 20km)_{obs} = 10^{-6} \times 4\pi(1738)^2/10^9\, yr^{-1} = 3.8 \times 10^{-8}\, yr^{-1}. \quad (51)$$

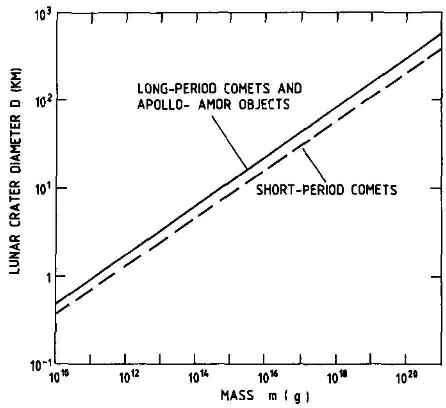

Fig. 16: The lunar crater diameters as a function of the mass of the impacting objects according to the Schmidt and Holsapple crater scaling law.

https://doi.org/10.1017/S0252921100109819 Published online by Cambridge University Press

The estimated cratering rate by the Apollo-Amor asteroids on the Moon exceeds the observed value by a factor of 14. Since errors and uncertainties involved in the derivations of Eq.(51) could amount to a factor of 5 (Bailey and Stagg 1988), the above discrepency may be resolved after more detailed analyses. This discussion nevertheless highlights the intrinsic uncertainties in relating the crater size frequency distributions to the number and size distributions of the impacting bodies.

The next category of stray bodies creating craters on planetary surfaces is naturally the long-period and short-period comets. The main difficulty in the examination of the crater production rates from these objects has to do with the mass-magnitude relation, which has been generally expressed in the following form,

$$log(m) = b_2 + b_1 H_{10} \qquad (52)$$

where m is in units of kilograms and H_{10} is the absolute magnitude of the comet. Different authors give very different b_1 and b_2 values (Allen 1963, Fernández 1982, Weissman 1983, Hughes 1987, Bailey and Stagg 1988). The determination of the H_{10} value of a comet depends on fitting its brightness variation as a function of power law in solar distance,

$$H = H_{10} + 5 \log_{10} \Delta + 10 \log_{10} r \qquad (53)$$

where H is the apparent magnitude of a comet when it is at heliocentric distance r and geocentric distance Δ. In the above equation, an inverse fourth-power dependence of the comet's brightness on r is assumed (Everhart 1967). Since the coma activities and evolutionary history of a comet have strong effects on the brightness variation, Eq.(53) certainly can not be applied to all cases. Before a comprehensive survey is performed, of distant comets at large solar distances (> 3 AU) where outgassing process generally cease, no more improvements can perhaps be made in this area. As a working assumption, in our derivation of cometary nucleus mass, we shall use the values adopted by Bailey and Stagg (1988): b_1 = -0.5 and b_2 = 16.9. As emphasized by these authors, such "average" values could lead to a potential error in the mass estimate upward or downward by a factor of 5.

11.2 SIZE DISTRIBUTION OF COMETS

The application of the brightness-mass relation given by Bailey and Stagg (1988)

$$\log(m) = +16.9 - 0.5 H_{10}\,\mathrm{kg} \qquad (54)$$

would suggest that the cumulative number of the long-period comets would have a power law dependence of $N(m) \propto m^{-0.58}$ for $m < 10^{14}$ kg and $N(m) \propto m^{-1.16}$ for $m > 10^{14}$ kg. The difference in the mass spectral index is due to the kink at $H_{10} \approx 6$ in the brightness distribution (see Fig. 2). In comparison, the main-belt asteroids have a mass distribution given by $N(m) \propto m^{-1}$, at least for diameters < 25 km (cf. Section 10). The impact craters on planetary surfaces therefore reflect the bombardment effects of various kinds of projectiles with different mass distributions and impact velocities. The separation of the contributions from different groups is difficult, however. Estimates of the proportion of crater production by long-period comets, for example, vary from just a few percent (Bailey and Stagg 1988) to over 50% (Hut and Tremaine 1986). We can make an independent check based on some previous results (Ip and Fernández 1988). There, we found that, on the average, there would be about 48 new comets (with $H_{10} < 11.0$) per year entering the Solar System within the orbit of Neptune. For lunar craters, a lunar crater diameter of D $\approx$ 20 km would correspond to an absolute magnitude of $H_{10} < 8.0$. The influx of new comets of such H_{10} value can be estimated to be 10 per year. The impact probability (P_i) with the Earth per new comet with perihelion distances inside 30 AU has been estimated to be 2×10^{-8} (Ip and Fernández, 1988). The crater production rate for D $\approx$ 20 km on the lunar surface due to long-period comets is thus on the order of 1.6×10^{-8} yr $^{-1}$. Since the impact rate from short-period comets has generally been computed to be a factor of 2 to 5 larger than that from long-period comets, our exercise suggests that cometary impacts should contribute to 10% to 20% of the total crater production. It should be noted that if the contribution from comet showers is included, the percentage could be appreciably increased.

11.3 VOLATILE INJECTION TO PLANETARY ATMOSPHERES:

11.3.1. *Present.* With a mass distribution of

$$\frac{dn}{dm} = Am^{-0.58} \quad \text{for} \quad m < 10^{17}\,\text{g}$$

we find that the average mass of the long-period comets would be $< m > = 3 \times 10^{16}$ g, with the corresponding absolute magnitude equal to $H_{10} = 7$. A reasonable estimate for the volatile mass from long-period comets accreted by terrestrial planets over the last 4×10^9 years (if the new "LP comet" injection rate is assumed to be constant) is:

$$\Delta m(\text{Earth}) = 48 \times (10/300) \times 2 \times 10^{-8} \times < m > \times 4 \times 10^9 = 1.3 \times 10^{19}\text{g}. \qquad (55)$$

The total volatile contribution from all comets (short-period and long-period ones) is therefore $1.3 \times 10^{19} \times 4 = 5.2 \times 10^{19}$ g. In comparison, the present terrestrial atmospheric mass (78% N_2, 21% O_2, and < 4% H_2O) is 5×10^{21} g and the ocean water mass is 1.3×10^{24} g. Even though the late-phase injection of comets into the terrestrial atmosphere during the last 4 billion years is insufficient to affect the global inventory of the hydrospheric water budget, significant effects nevertheless could result in the short-term changes in the physical/biological condition immediately after each cometary impact.

The situation with Venus and Mars is very different from the terrestrial case because of the very low water content in the atmospheres of both planets. The current atmospheric water content at Venus is on the order of 10^{19} g (20 ppm); the H_2O mixing ratio thus could be strongly modulated by cometary impacts. As mentioned before, for new comets with $H_{10} \leq 11.0$, the influx rate would be 48 per year inside 30 AU; the corresponding time interval between long-period comet impacts at Venus is thus $\Delta t \approx 1/(\dot{N}P_i) \approx 2.1 \times 10^6$ years (see Ip and Fernández 1988). Each impact separated by Δt can bring about a potential perturbation to water abundance. Next, according to the idea of cometary showers (Hills 1981, Fernández and Ip 1987, Hut et al. 1987), the shower time interval should be about 2×10^7 years and the concentrated cometary influx during the shower period would be enhanced by a similar factor. In other words, a total mass of about 10^{18} g of cometary material could be delivered to Venus via long-period comets in just a few million years during such a cometary shower. The present-day atmospheric water content is consequently closely coupled to the bombardment effect of stray comets.

As for Mars, its CO_2-rich atmosphere contains no more than 2×10^{15} g of H_2O. The impact of a small comet of 1-km radius would be enough to cause a major change in the atmospheric water content. The cumulative effect of a cometary shower should be very significant even for the CO_2 atmosphere (the total CO_2 mass is about 6.7×10^{18} g in the Martian atmosphere).

11.3.2. *Past.* Even though the present rate of mass accretion from comet impacts is relatively low, the bombardment events during the formative phase of the planetary system could be much more significant. For example, it has been estimated that in the accretion/scattering process of the planetesimals in the Uranus-Neptune zone, the total mass accreted by Jupiter from the Uranus-Neptune zone objects would be about 0.2-2 $M_\oplus$ (Fernández and Ip 1981, 1983a). For this population of icy planetesimals, about 1% of them will be intercepted by the Earth (and 0.5% by Venus and 0.2% by Mars). The potential mass input (with 50% in H_2O) to the primordial terrestrial atmosphere is thus on the order of 6×10^{24} to 6×10^{25} g (i.e., 4 to 40 oceanic masses). Thus, in association with the accretion of Uranus and Neptune and the buildup of the cometary Oort cloud, a full-scale hydrosphere could have been established on the Earth (and on Venus and Mars) as a result of the early phase of cometary impacts terminated approximately 4 billion years ago. These estimates should be considered as upper limits, however, since we have neglected the potential effect of atmospheric erosion during cometary impacts.

The issues of the initial existence and eventual disappearance of water on the surfaces of Venus and Mars are of great interest to the study of the origin and evolution of the atmospheres of these two terrestrial planets. The total water content in the atmosphere of Venus is equivalent to a global layer 20-cm thick, in contrast to oceans of an average depth of 3-km covering the surface of the Earth. The reported enhancement of the D/H ratio in the lower atmosphere of Venus by a factor of 100 in comparison with the value derived for the terrestrial ocean water (Donahue et al. 1982) has been used as an argument for the presence of much higher water content in the past history. This inference is based on the assumption that the deuterium and hydrogen should have different escape rates from the atmosphere (Kumar et al. 1983). Similarly, an enhancement factor for the D/H ratio of about 6 ± 3 in the upper atmosphere of Mars also could be indicative of a history of large-scale atmospheric loss (Owen et al. 1988). More precise measurements of the D/H ratios at Venus and Mars, as well as a better understanding of the pertinent fractionation process in atmospheric escape, are required to verify this scenario.

Finally, it should be pointed out that a much smaller degree of volatile deposition has

https://doi.org/10.1017/S0252921100109819 Published online by Cambridge University Press

been derived by Pollack and Yung (1980) using the observed crater frequency distribution on the Moon's ancient terrain (they found a total impacting mass of about 10^{21} g). A recent study using the Schmidt and Holsapple (1982) mass-scaling law for the crater diameters has led Chyba (1987) to the conclusion that the Earth should have acquired about one oceanic mass of water from cometary impacts between 4.5 and 3.8 billion years ago. Since the lunar crater record might have been partially degraded, the actual amount of water depostion on Earth could have been larger. Chyba's evaluation, which is independent of cometary flux models, is therefore not in disagreement with our estimate of 4 to 40 oceanic masses of H_2O deposition during the early phase of cometary impacts. It should be noted that, however, the discrepancy between these two estimates might have to be explained in another way because Chyba's estimate is based on the largest lunar basis of which erosion effect might be small.

12. Concluding Remarks

Summing up, we would like to emphasize some significant new contributions to our understanding of comet dynamics as well as some still unsolved problems:

1) The frequency of passages of LP comets on Earth-crossing orbits is estimated to be about 3 yr^{-1}, although this only refers to comets brighter than $H_{10} \sim 10.5$. Whether fainter — mostly undetected — LP comets contribute significantly to the total mass inventory of the Oort cloud is still an open question.

2) The tidal force of the galactic disk plays a dominant role in driving comets to the planetary region. From time to time, random encounters with molecular clouds or close stellar passages may overcome the galactic effect.

3) The onset of comet showers may result from passing molecular clouds or stars penetrating very deeply in the Oort cloud ($r \lesssim 10^4$ AU). The enhancement in the frequency of comet passages during a shower will depend on the radial distribution of Oort cloud comets.

4) The distribution of aphelion points of young and new comets on the celestial sphere shows a galactic dependence, suggesting that the frequency of comet passages is currently at, or near, its background level.

5) There is no firm evidence in favor of the proposed periodicity of $\sim$ 26 Myr in the terrestrial cratering record attributable to cometary impacts during showers. It is even possible that craters produced during a comet shower cannot leave any distinguishable enhancement against the heavy crater background from colliding asteroids.

6) Most short-period comets of the Jupiter's family might come from an outer comet belt or disk. This is suggested by their highly flattened distribution of orbital inclinations, which seems to be incompatible with a comet source having a spherical distribution.

7) The volatile contribution to the Earth from colliding comets could have been sufficient to supply a few ocean masses of water during the late heavy bombardment that ended $\sim$ 3.8 byr ago. Comets colliding with Venus or Mars can significantly modulate the water content of their dry atmospheres.

Acknowledgments

We thank M. Bailey for a critical reading of the manuscript and P.R. Weissman and another anonymous referee whose critical reviews helped us to improve the manuscript. One of us (JAF) acknowledges financial support from the Programa de Desarrollo de las Ciencias Básicas (PEDECIBA). We are also grateful to Mrs. I. Gehne for assistance in preparation of the manuscript.

6. References

Allen, C.W. (1963) Astrophysical Quantities (second edition), Athlone Press, London, p.155.

Alvarez, L.W., Alvarez, W., Asaro, F., and Miches, H.V. (1980) "Extraterrestrial cause for the Cretaceous-Tertiary extinction," Science 208, 1095-1108.

Alvarez, W., and Muller, R.A. (1984) "Evidence from crater ages for periodic impacts on the Earth," Nature 308, 718-720.

Antonov, V.A., and Latyshev, I.N. (1972) "Determination of the form of the Oort cometary cloud as the Hill surface in the galactic field," in G.A. Chebotarev, E.I. Kazimirchak-Polonskaya and B.G. Marsden (eds.), The Motion, Evolution of Orbits, and Origin of Comets, IAU Symp. No. 45, Reidel, Dordrecht, Springer-Verlag, New York, pp. 341-345.

Bahcall, J.N. (1984) "Self-consistent determination of the total amount of matter near the Sun," Astrophys. J. 276, 169-181.

Bahcall, J.N., and Soneira, R.M. (1981) "The distribution of stars to V = 16th magnitude near the north galactic pole: Normalization, clustering properties, and counts in various bands," Astrophys. J. 246, 122-135.

Bailey, M.E. (1983) "The structure and evolution of the Solar System comet cloud," Mon. Not. Roy. Astron. Soc. 204, 603-633.

Bailey, M.E. (1986) "The near-parabolic flux and the origin of short period comets," Nature 324, 350-352.

Bailey, M.E., and Stagg, C.R. (1988) "Cratering constraints on the inner Oort cloud; steady-state models," Mon. Not. Roy. Astron. Soc. 235, 1-32.

Biermann, L. (1978) "Dense interstellar clouds and comets," in A. Reiz and T. Anderson (eds.), Astronomical Papers Dedicated to Bengt Stromgren, Copenhagen Observatory, p. 327.

Biermann, L., Huebner, W.F., and Lüst, R. (1983) "Aphelion clustering of 'new' comets: Star tracks through Oort's cloud," Proc. Natl. Acad. Sci. USA 80, 5151-5155.

Bilo, E.H., and van de Hulst, H.C. (1960) "Methods for computing the original orbits of comets," Bull. Astron. Inst. Neth. 15, 119-127.

Bogart, R.S., and Noerdlinger, P.D. (1982) " On the distribution of orbits among long-period comets," Astron. J. 87, 911-917.

Byl, J. (1983) "Galactic perturbations on near-parabolic cometary orbits," Moon and Planets 29, 121-137.

Cameron, A.G.W. (1962) "The formation of the Sun and the planets," Icarus 1, 13-69.

Carusi, A., Kresák, L., Perozzi, E., and Valsecchi, G.B. (1984) Long-Term Evolution of Short-Period Comets, Instituto Astrofísical Spaziale Internal Report 12 Rome.

Chebotarev, G.A. (1966) "Cometary motion in the outer solar system," Soviet Astron. - AJ 10, 341-344.

Chyba, C.F. (1987) "The cometary contribution to the oceans of primitive Earth," Nature 330, 632-635.

Cowan, J.J., and A'Hearn, M.F. (1979) "Vaporization of comet nuclei: Light curves and life times," Moon and Planets 21, 155-171.

Davis, M., Hut, P., and Muller, R.A. (1984) "Extinction of species by periodic comet showers," Nature 308, 715-717.

Delsemme, A.H. (1973) "Origin of the short-period comets," Astron Astrophys. 29, 377-381.

Delsemme, A.H. (1986) "Cometary evidence for a solar companion?," in R. Smoluchowski, J.N. Bahcall, and M.S. Matthews (eds.), The Galaxy and the Solar System, Univ. of Arizona Press, Tucson, pp. 173-203.

Delemme, A.H. (1989) "Whence come comets?," Sky & Telesc. March, 260-264.

Delemme, A.H., and Patmiou, M. (1986) "Galactic tides affect the Oort cloud: An observational confirmation," in Proc. 20th ESLAB Symp. on the Exploration of Halley's Comet, Heidelberg, ESA SP-250, pp. 409-412.

Donahue, T.M., Hoffman, J.H., Hodges, R.R., Jr., and Watson, A.J. (1982) "Venus was wet: A measurement of the ratio of deuterium to hydrogen," Science 216, 630-633.

Donnison, J.R., and Sugden, R.A. (1984) "The distribution of asteroids diameters," Mon. Not. Roy. Astron. Soc. 210, 673-682.

Drapatz, S., and Zinnecker, H. (1984) "The size and mass distribution of Galactic molecular clouds," Mon. Not. Roy. Astron. Soc. 210, 11p-14p.

Duncan, M., Quinn, T., and Tremaine, S. (1987) " The formation and extent of the solar system comet cloud," Astron. J. 94, 1330-1338.

Duncan, M., Quinn, T., and Tremaine, S. (1988) "The origin of short-period comets," Astrophys. J. Letts. 328, L69-L73.

Duncan, M., Quinn, T., and Tremaine, S. (1989) "The long-term evolution of orbits in the Solar System: A mapping approach," CITA preprint.

Everhart, E. (1967) "Intrinsic distributions of cometary perihelia and magnitudes," Astron. J. 72, 1002-1011.

Everhart, E. (1968) "Change in total energy of comets passing through the Solar System," Astron. J. 73, 1039-1052.

Everhart, E. (1972) "The origin of short-period comets," Astrophys. Lett. 10, 131-135.

Everhart, E. (1976) "The evolution of comet orbits," in B. Donn, M. Munna, W. Jackson, M. A'Hearn, and R. Harrington (eds.), The Study of Comets, IAU Coll. No. 25, NASA SP-393, pp. 445-464.

Everhart, E., and Marsden, B.G. (1983) "New original and future cometary orbits. III," Astron, J. 93, 753-754.

Fernández, J.A. (1980a) "Evolution of comet orbits under the perturbing influence of the giant planets and nearby stars," Icarus 42, 406-421.

Fernández, J.A. (1980b) "On the existence of a comet belt beyond Neptune," Mon. Not. Roy. Astron. Soc. 192, 481-491.

Fernández, J.A. (1981a) "New and evolved comets in the solar system," Astron. Astrophys. 96, 26-35.

Fernández, J.A. (1981b) "On the observed excess of retrograde orbits among long-period comets," Mon. Not. Roy. Astron. Soc. 197, 265-273.

Fernández, J.A. (1982) "Dynamical aspects of the origin of comets," Astron. J. 87, 1318-1332.

Fernández, J.A. (1985) "The formation and dynamical survival of the comet cloud," A. Carusi and G.B. Valsecchi (eds.), Dynamics of Comets: Their Origin and Evolution, Reidel, Dordrecht, pp. 45-70.

Fernández, J.A. (1990) "Collisions of comets with meteoroids," in C.-I. Lagerkvist, H. Rickman, B.A. Lindblad, and M. Lindgren (eds.), Asteroids, Comets, Meteors III, University of Uppsala, pp. 309-312.

Fernández, J.A., and Ip, W.-H. (1981) "Dynamical evolution of a cometary swarm in the outer planetary region," Icarus 47, 470-479.

Fernández, J.A., and Ip, W.-H. (1983a) "On the time evolution of the cometary influx in the region of the terrestrial planets," Icarus 54, 377-387.

Fernández, J.A., and Ip, W.-H. (1983b) "Dynamical origin of the short-period comets," in C.-I. Lagerkvist and H. Rickman (eds.), Asteroids, Comets, Meteors, University of Uppsala, pp. 387-390.

Fernández, J.A., and Ip, W.-H. (1987) "Time-dependent injection of Oort cloud comets into Earth-crossing orbits," Icarus 71, 46-56.

Fernández, J.A., and Jockers, K. (1983) "Nature and origin of comets," Rep. Prog. Phys. 46, 665-772.

Fesenkov, V.G. (1922) "Sur les perturbations séculaires dans le mouvement des cometes non péiodiques par des étoiles voisines," Publ. Russian Astrophys. Obervatory 1, 186-195.

Festou, M., Rickman, H., and Kamél, L. (1990) "The origin of nongravitational forces in comets," in C.-I. Lagerkvist, H. Rickman, B.A. Lindblad, and M. Lindgren (eds.), Asteroids, Comets, Meteors III, University of Uppsala, pp. 313-316.

Foog, M.J. (1989) "The relevance of the background impact flux to cyclic impact/mass extinction hypotheses," Icarus 79, 382-395.

Hamid, S.E., Marsken, B.G., and Whipple, F.L. (1968) "Influence of a comet belt beyond Neptune on the motions of periodic comets," Astron. J. 73, 727-729.

Hasegawa, I. (1976) "Distribution of the aphelia of long-period comets," Publ. Astron. Soc. Japan 28, 259-276.

Helin, E.F., and Shoemaker, E.M. (1979), "The Palomar planet-crossing asteroid survey 1973-1978," Icarus 40, 321.

Heisler, J. and Tremaine, S. (1986) "The influence of the galactic tidal field on the Oort comet cloud," Icarus 65, 13-26.

Heisler, J. and Tremaine, S. (1989) "How dating uncertainties affect the detection of periodicity in extinctions and craters," Icarus 77, 213-219.

Hills, J.G. (1981) "Comet showers and the steady-state infall of comets from the Oort cloud," Astron. J. 86, 1730-1740.

Holsapple, K.A., and Schmidt, R.M. (1982) "On the scaling of crater dimensions 2, Impact processes," J.Geophys. Res. 87, 1849-1870.

Hughes, D.W. (1982) "Astroidal size distribution," Mon. Not. Roy. Astron. Soc. 199, 1149-1157.

Hughes, D.W. (1987) "Cometary magnitude distribution and the fading of comets," Nature 325, 231-232.

Hurnik, H. (1959) "The distribution of the directions of perihelia and the orbital poles of non-periodic comets," Acta Astron. 9, 207-221.

Hut, P., Alvarez, W., Hansen, T., Kauffman, E.G., Keller, G., Shoemaker, E.M. and Weissman, P.R. (1987) "Comet showers as a cause of mass extinctions," Nature 329, 118-126.

Hut, P., and Tremaine, S. (1985) "Have interstellar clouds disrupted the Oort comet cloud?," Astron. J. 90, 1548-1557.

Ip, W.-H., and Fernández, J.A. (1988) "Exchange of condensed matter among the outer and terrestrial protoplanets and the effect on surface impact and atmospheric accretion," Icarus 74, 47-61.

Ip, W.-H., and Fernández, J.A. (1990) "Steady-state injection of short-period comets from the trans-Neptunian cometary belt," submitted to Icarus.

Ishida, K., Mikami, T., and Kosai, H. (1984) "Size distribution of asteroids," Publ. Astron. Soc. Japan 36, 357-370.

Jackson, A.A., and Killen, R.M. (1988) "Infrared brightness of a comet belt beyond Neptune," Earth, Moon, Planets 42, 41-47.

Joss, P.C. (1973) "On the origin of short-period comets," Astron. Astrophys. 25, 271-273.

Kazimirchak-Polonskaya, E.I. (1976) "Review of investigations performed in the U.S.S.R. on close approaches of comets to Jupiter and the evolution of cometary orbits," in B. Donn, M. Mumma, W. Jackson, M. A'Hearn, and R. Harrington (eds.), The Study of Comets, NASA SP-393, pp. 490-536.

Kowal, C.T. (1989) "A solar system survey," Icarus 77, 118-123.

Kresák, L. (1975) "The bias of the distribution of cometary orbits by observational selection," Bull. Astron. Inst. Czech. 26, 92-111.

Kresák, L. (1978) "The comet and asteroid population of the Earth's environment," Bull. Astron. Inst. Czech 29, 114-125.

Kresák, L., and Pittich, E.M. (1978) "The intrinsic number density of active long-period comets in the inner solar system," Bull. Astron. Inst. Czech 29, 299-309.

Kuiper, G.P. (1951) "On the origin of the solar system," in J.A. Hynek (ed.), Astrophysics, McGraw-Hill, New York, pp. 357-427.

Kumar, S., Hunten, D.M., and Pollack, J.B. (1983) "Nonthermal escape of hydrogen and deuterium from Venus and implication for loss of water," Icarus 55, 369.

L"ust, R. (1984) "The distribution of the aphelion directions of long-period comets," Astron. Astrophys. 141, 94-100.

Luu, J.X., and Jewitt, D. (1988) "A two-part search for slow-moving objects," Astron. J. 95, 1256-1262.

Marsden, B.G. (1986) Catalogue of Cometary Orbits. (fifth edition), IAU Central Bureau for Astron. Telegrams, Cambridge, Mass.

Marsden, B.G. (1990) "The sungrazing comet group. II," Astron. J. (in press).

Marsden, B.G., Sekanina, Z., and Everhart, E. (1978) "New osculating orbits for 110 comets and analysis of original orbits," Astron. J. 83, 64-71.

Marsden, B.G., Sekanina, Z., and Yeomans, D.K. (1973) "Comets and nongravitational forces. V.," Astron. J. 78, 211-225.

McKinnon, W.B., and Mueller, S. (1988) "Pluto's structure and composition suggest origin in the solar, not a planetary, nebula," Nature 335, 240-243.

Melosh, H.J. (1981) "Atmospheric breakup of terrestrial impactors," in P.H. Schultz, and R.B. Merrill (eds.), Multipling Basins, Proc. Lundar Planet Sci. 12A, pp 29-35.

Mendis, D.A. (1973) "The comet-meteor stream complex," Astrophys. Space Sci. 20, 165-176.

Morris, D.E., and Muller, R.A. (1986) "Tidal gravitational forces: The infall of 'new' comets and comet showers," Icarus 65, 1-12.

Napier, W.M., and Clube, S.V.M. (1979) "A theory of terrestrial catastrophism," Nature 282, 455-459.

Napier, W.M., and Staniucha, M. (1982) "Interstellar planetesimals - I, Dissipation of a primordial cloud of comets by tidal encounters with massive nebulae," Mon. Not. Roy., astron. Soc. 198, 723-735.

Neukum, G. (1975) "Mars: A standard crater curve and possible new time scale," Science 194, 1381-1387.

Oja, H. (1975) "Perihelion distribution of near-parabolic comets," Astron. Astrophys. 43, 317-319.

Oort, J.H. (1950) "The structure of the cloud of comets surrounding the solar system and a hypothesis concerning its origin," Bull. Astron. Inst. Neth. 11, 91-110.

Öpik, E.J. (1932) "Note on stellar perturbations on nearly parabolic orbits," Proc. Am. Acad. Arts. Sci. 67, 1659-183.

Öpik, E.J. (1966) "Sun-grazing comets and tidal disruption," Irish Astron. J. 7, 141-161.

Owen, T., Maillard, J.P., de Bergh, C., and Lutz, B.L. (1988) "Deuterium on Mars: The abundance of HDO and the value of D/H," Science 240, 1767-1769.

Porter, J.G. (1963) "The statistics of comet orbits," in B.M. Middlehurst and G.B. Kuiper (eds.), The Moon, Meteorites and Comets, The University of Chicago Press, pp. 550-572.

Pollack, J.B., and Yung, Y.L. (1980) "Origin and evolution of planetary atmospheres," Ann. Rev. Earth Planet. Sci. 8, 425-487.

Rampino, M.R., and Stothers, R.B. (1984) "Terrestrial mass extinctions, cometary impacts and the sun's motion perpendicular to the galactic plane," Nature 308, 709-712.

Raup, D.M., and Sepkoski, J.J. (1984) "Periodicity of extinctions in the geologic past," Proc. Natl. Acad. Sci. USA 81, 801-805.

Rickman, H. (1976) "Stellar perturbations of orbits of long-period comets and their significance for cometary capture," Bull. Astr. Inst. Czech. 27, 92-105.

Rickman, H. (1986) "Masses and densities of comets Halley and Kopff," in The Comet Nucleus Sample Return Mission Proc. Workshop, Canterbury, UK, ESA SP-249, pp. 195-205.

Safranov, V.S. (1972) "Ejection of bodies from the solar system in the course of the accumulation of the giant planets and the formation of the cometary cloud," in G.A. Chebotarev, E.I. Kazimirchak-Polonskaya and B.G. Marsden (eds.), The Motion, Evolution of Orbits, and Origin of Comets, I.A.U. Symp. No. 45, Reidel, Dordrecht, Springer-Verlag, New York, pp. 329-334.

Sekanina, Z., and Yeomans, D.K. (1984) "Close encounters and collisions of comets with the Earth," Astron. J. 89, 154-161.

Shoemaker, E.M., Williams, J.G., Helin, E.F., and Wolfe, R.F. (1979) "Earth-crossing asteroids: orbital classes, collision rates with Earth, and origin," in Asteroids, T. Gehrels (ed.), Univ. of Arizona Press, Tucson, pp. 253.

Shoemaker, E.M., and Wolfe, R.F. (1982) "Cratering time scales for the Galilean satellites, in Satellites of Jupiter, D. Morrison (ed.), Univ. of Arizona Press, Tucson, pp. 277-339.

Smoluchowski, R., and Torbett, M. (1984) "The boundary of the solar system," Nature 311, 38-39.

Stagg, C.P., and Bailey, M.E. (1989) "Stochastic capture of short-period comets," Mon. Not. Roy. Astron. Soc. 241, 507.

Stothers, R.B. (1984) "Mass extinctions and missing matter," Nature 311, 17.

Stothers, R.B. (1988) "Structure of Oort's comet cloud inferred from terrestrial impact craters," The Observatory 108, 1-9.

Strom, R.G., and Neukum, G. (1988) "The cratering record on Mercury and the origin of impacting objects," in Mercury, Eds. F. Vilas, C.R. Chapman and M.S. Matthews, Univ. of Arizona Press, Tucson, 336-373.

Talbot, R.J., and Newman, M.J. (1977) "Encounters between stars and dense interstellar clouds," Astrophys. J. Suppl. Ser. 34, 295-308.

Torbett, M.V. (1986) "Injection of Oort cloud comets to the inner solar system by galactic tidal fields," Mon. Not. Roy. Astron. Soc. 223, 885-895.

Tremaine, S. (1986) "Is there evidence for a solar companion star?," in R. Smoluchowski, J.N. Bahcall, and M.S. Matthews (eds.), The Galaxy and the Solar System, Univ. of Arizona Press, Tucson, pp. 409-416.

Tyror, J.G. (1957) "The distribution of the directions of perihelia of long-period comets," Mon. Not. Roy. Astron. Soc. 117, 369-379.

Van Woerkom, A.J.J. (1948) "On the origin of comets," Bull. Astr. Inst. Neth. 10, 445-472.

Wetherill, G.W. (1975) "Late heavy bombardment of the moon and terrestrial swarm subsequent to the formation of the Earth and the Moon," In Proc. 8th Lunar Sci. Conf., pp. 1-16.

Wetherill, G.W., and Shoemaker, E.M. (1982) "Collision of astronomically observable bodies with the Earth," in Geological Implications of Impacts of Large Asteroids and Comets on the Earth, Geological Soc. Amer. Sp. Pap., 190, 1.

Weissman, P.R. (1980a) "Stellar perturbations of the cometary cloud," Nature 288, 242-243.

Weissman, P.R. (1980b) "Physical loss of long-period comets," Astron. Astrophys. 85, 1919-196.

Weissman, P.R. (1983) "The mass of the Oort cloud," Astron. Astrophys. 118, 90-94.

Weissman, P.R. (1985a) "Dynamical evolution of the Oort cloud," in A. Carusi and G.B. Valsecchi (eds.), Dynamics of Comets: Their Origin and Evolution, Reidel, Dordrecht, pp. 87-96.

Weissman, P.R. (1985b) "Cometary dynamics," Sp. Sci. Rev. 41, 299-349.

Weissman, P.R. (1985c) "Terrestrial impactors at geological boundary events: Comets or asteroids," Nature 314, 517-518.

Weissman, P.R. (1990) "The Oort cloud," Nature (in press).

Whipple, F.L. (1950) "A comet model. I. The acceleration of comet Encke," Astrophys. J. 111, 375-394.

Whipple, F.L. (1964) "Evidence for a comet belt bezond Neptune," Proc. Natl. Acad. Sci. 51, 711.

Whitmire, D.P., and Jackson, A.A. (1984) "Are periodic mass extinctions driven by a distant solar companion?," Nature 308, 713-715.

Yeomans, D.K., and Chodas, P.W. (1989) "An asymmetric outgassing model for cometary nongravitational accelerations," Astron. J. 98, 1083-1093.