ORIGINAL TABLES.

To the Editor of the Journal of the Institute of Actuaries.

DEAR SIR,—Annual Premiums, at H^M 3 per-cent, for Endowment Assurances on Two Joint Lives, are given in the following pages. The *Text-Book* graduation of the H^M Table has been adopted as most suitable for the present purpose, partly because the terms of duration are short as compared with whole life, but more especially for the reason that it follows Makeham's law, in virtue of which we can substitute two equal ages w for two unequal ages x and y: hence it is sufficient to tabulate the values for equal ages only. A simple inspection of the auxiliary Seniority Table enables us to make the substitution at once, and thus $P_{xy\overline{n}|}$ can easily be found: *e.g.*, if $P_{30,45\,\overline{20}|}$ is required; we have $x=30$ and $y=45$; then entering the table with 15 $(=y-x)$ we find 9·889, which must be added to 30 $(=x)$, giving 39·889 for the equivalent equal ages. Thus we have $P_{30,45\,\overline{20}|} = P_{39\,889,39\cdot889\,\overline{20}|}$,

Now turning to the Premium Table, we see that

$$P_{39,39\,\overline{20}|} = 5\cdot080$$

and $$P_{40,40\,\overline{20}|} = 5\cdot152$$

$$\overline{\cdot072}$$

$$\cdot072 \times \cdot889 \quad = \quad \cdot064$$

∴ $P_{39\cdot889,39\cdot889\,\overline{20}|} = P_{30,45\,\overline{20}|} = 5\cdot144$ the premium required.

The Seniority Table was constructed from the relation

$$c^x + c^y = 2c^w,$$

and consequently applies with considerable accuracy down to age 20 (see *Text-Book*, Part II, page 210, § 37), while above age 27 it is rigorously correct.

The formula employed in calculating the Premium Table was

$$\frac{D_{xx}}{N_{x-1,x-1} - N_{x+n-1,x+n-1}} - d,$$

the values being computed by us independently, and results duly compared. The proofs have also been carefully read over with the two manuscripts.

We have considered it unnecessary to tabulate the Single Premiums, as these can be so readily obtained from the Annual Premiums by means of Rothery & Ryan's Conversion Tables— *e.g.*: to find the Single Premium corresponding to the Annual Premium 4·752. Reference to Rothery & Ryan shows that ·61990 corresponds to 4·750 (occupying the same position on the adjacent page). Then we have merely to add $\frac{2}{6} \times$ the constant ·00029, say

·00010, to ·61990, giving ·62000, or 62·000, for the assurance per 100 as the Single Premium required.

<div align="center">We remain,</div>

<div align="right">Yours obediently,

A. S. HUME.
W. STOTT.</div>

Liverpool,
28 *September* 1901.

<div align="center">AUXILIARY TABLE,</div>

For finding the two Equal Ages (w) which may be substituted for two Unequal Ages (x) and (y) as determined by the equation $c^x + c^y = 2c^w$ (see Text-Book, *Part II, p. 210, Art. 37).*

Seniority of Elder Life $(y-x)$	Seniority of Equal Ages $(w-x)$	Seniority of Elder Life $(y-x)$	Seniority of Equal Ages $(w-x)$
1	0·511	26	19·384
2	1·046	27	20·302
3	1·602	28	21·227
4	2·182	29	22·158
5	2·783	30	23·095
6	3·406	31	24·037
7	4·050	32	24·983
8	4·715	33	25·934
9	5·400	34	26·890
10	6·104	35	27·848
11	6·827	36	28·811
12	7·567	37	29·776
13	8·325	38	30·745
14	9·100	39	31·716
15	9·889	40	32·689
16	10·694	41	33·665
17	11·513	42	34·643
18	12·344	43	35·623
19	13·188	44	36·604
20	14·044	45	37·588
21	14·911	46	38·572
22	15·788	47	39·558
23	16·674	48	40·545
24	17·569	49	41·533
25	18·473	50	42·523

Endowment Assurances for two Joint Lives (equal ages), Annual Premiums for 100, *payable in n Years or at First Death.—* H^M 3 *per-cent* (Text-Book).

Duration	5	6	7	8	9	10	11	12
Ages								
20 20	18·789	15·547	13·238	11·511	10·172	9·105	8·237	7·516
21 21	18·817	15·574	13·264	11·537	10·198	9·131	8·262	7·541
22 22	18·839	15·597	13·287	11·559	10·220	9·153	8·283	7·562
23 23	18·857	15·616	13·305	11·577	10 238	9 171	8·302	7·580
24 24	18·872	15·631	13·320	11·594	10·254	9·187	8·318	7·597
25 25	18·885	15·643	13·333	11·606	10·268	9·201	8·332	7·612
26 26	18·895	15·654	13·345	11 619	10·281	9·215	8 347	7 627
27 27	18·906	15·666	13·357	11·632	10·294	9·229	8 361	7 643
28 28	18·917	15·678	13·370	11·645	10·309	9·245	8·377	7·660
29 29	18·929	15·690	13·384	11·660	10·324	9·260	8·394	7·677
30 30	18·941	15·704	13·398	11·675	10·340	9·277	8·413	7·697
31 31	18·956	15·719	13·415	11·692	10 359	9·297	8·434	7·719
32 32	18·971	15·735	13·432	11·711	10·379	9·319	8·455	7·742
33 33	18·986	15·753	13·451	11·732	10·400	9·342	8·480	7·768
34 34	19·004	15·772	13·472	11·754	10·424	9·366	8·507	7·796
35 35	19·024	15·794	13·496	11·779	10·451	9·395	8·537	7·828
36 36	19·046	15·818	13·521	11·806	10·479	9·425	8·569	7·862
37 37	19·069	15·843	13·548	11·836	10·511	9·458	8·604	7·898
38 38	19·096	15·872	13·578	11·868	10·546	9·495	8·643	7·940
39 39	19·123	15·903	13·612	11·904	10·583	9·536	8·685	7·985
40 40	19·154	15·937	13·649	11·943	10·625	9·580	8 732	8 034
41 41	19·188	15·973	13·689	11·985	10·671	9·628	8·783	8·088
42 42	19·227	16 015	13·733	12·034	10·721	9·681	8·840	8·148
43 43	19·267	16·059	13·781	12·084	10·776	9·739	8·901	8·213
44 44	19·312	16·108	13·834	12·141	10·836	9·804	8·969	8·284
45 45	19·361	16·162	13·892	12·203	10·902	9·873	9·043	8·362
46 46	19·415	16·221	13·956	12·271	10·975	9·951	9·125	8·449
47 47	19·474	16·286	14·026	12·346	11·055	10·036	9·215	8·543
48 48	19·539	16·357	14·102	12·429	11·143	10·129	9·313	8·648
49 49	19·611	16·435	14·188	12·519	11·239	10·231	9·421	8·762
50 50	19·689	16·520	14·279	12·618	11·344	10·343	9·540	8·888
51 51	19·776	16·616	14·381	12·727	11·461	10·467	9·672	9·026
52 52	19·869	16·718	14·493	12·847	11·588	10·602	9·815	9·178
53 53	19 974	16·832	14·615	12·979	11·729	10·752	9·973	9·345
54 54	20·087	16·956	14·750	13·124	11·883	10·916	10·147	9·528
55 55	20·211	17·093	14·898	13·282	12·052	11·096	10·338	9 730
56 56	20·348	17·243	15·060	13·456	12·239	11·294	10·548	9·952
57 57	20·497	17·407	15·239	13·649	12·444	11·512	10·779	10 196
58 58	20·663	17·589	15·436	13·860	12 670	11·754	11·034	10·465
59 59	20·844	17·788	15·652	14 092	12·919	12·017	11·314	...
60 60	21·043	18·008	15·890	14·348	13·193	12·308		...

2 E 2

Endowment Assurances for two Joint Lives (equal ages), Annual Premiums for 100, *payable in n Years or at First Death.—* H^M 3 *per-cent* (Text-Book).

Duration	13	14	15	16	17	18	19	20
Ages								
20 20	6·909	6·392	5·947	5·559	5·220	4·920	4·655	4·418
21 21	6·934	6·416	5·970	5·583	5·244	4·945	4·680	4·444
22 22	6·955	6·438	5·992	5·605	5·266	4·967	4·703	4·467
23 23	6·973	6·457	6·011	5·625	5·286	4·988	4·724	4·488
24 24	6·991	6·474	6·029	5·643	5·305	5·008	4·744	4·510
25 25	7·006	6·490	6·046	5·661	5·323	5·026	4·763	4·530
26 26	7·022	6·506	6·063	5·678	5·342	5·046	4·784	4·551
27 27	7·038	6·524	6·081	5·697	5·361	5·066	4·805	4·573
28 28	7·056	6·542	6·100	5·718	5·383	5·089	4·829	4·598
29 29	7·075	6·562	6·121	5·740	5·406	5·113	4·854	4·625
30 30	7·095	6·584	6·144	5·763	5·431	5·140	4·882	4·654
31 31	7·118	6·608	6·170	5·790	5·459	5·169	4·913	4·686
32 32	7·143	6·634	6·197	5·819	5·489	5·200	4·946	4·721
33 33	7·170	6·662	6·227	5·851	5·523	5·235	4·982	4·759
34 34	7·200	6·694	6·260	5·885	5·559	5·273	5·022	4·801
35 35	7·233	6·728	6·296	5·923	5·599	5·316	5·066	4·847
36 36	7·268	6·766	6·336	5·965	5·643	5·361	5·115	4·897
37 37	7·307	6·807	6·379	6·011	5·691	5·411	5·167	4·953
38 38	7·351	6·853	6·427	6·061	5·744	5·467	5·225	5·013
39 39	7·399	6·903	6·480	6·116	5·801	5·528	5·289	5·080
40 40	7·450	6·957	6·537	6·176	5·865	5·594	5·358	5·152
41 41	7·507	7·017	6·601	6·243	5·934	5·667	5·435	5·232
42 42	7·570	7·084	6·670	6·316	6·011	5·747	5·519	5·320
43 43	7·638	7·156	6·745	6·395	6·094	5·834	5·610	5·416
44 44	7·714	7·236	6·829	6·483	6·186	5·931	5·711	5·521
45 45	7·797	7·322	6·921	6·579	6·287	6·036	5·821	5·636
46 46	7·888	7·418	7·021	6·685	6·398	6·152	5·942	5·762
47 47	7·988	7·523	7·132	6·801	6·519	6·279	6·075	5·901
48 48	8·097	7·638	7·253	6·928	6·652	6·419	6·220	6·052
49 49	8·218	7·765	7·387	7·068	6·798	6·572	6·380	6·219
50 50	8·350	7·905	7·533	7·221	6·959	6·739	6·554	6·400
51 51	8·496	8·058	7·694	7·390	7·136	6·923	6·746	...
52 52	8·656	8·226	7·870	7·575	7·329	7·125	...	...
53 53	8·832	8·412	8·064	7·778	7·541	...	...	...
54 54	9·026	8·615	8·278	8·001	...	...	...	...
55 55	9·238	8·838	8·512	...	...	...	...	...
56 56	9·472	9·084	...	..	...	...	...	...
57 57	9·729	...	...	...	...	...	...	...

Endowment Assurances for two Joint Lives (equal ages), Annual Premiums for 100, payable in n Years or at First Death.— H^M *3 per-cent* (Text-Book).

Duration	21	22	23	24	25	26	27	28
Ages								
20 20	4·206	4·016	3·844	3·688	3·547	3·418	3·301	3·194
21 21	4·232	4·042	3·871	3·716	3·575	3·447	3·330	3·224
22 22	4·256	4·066	3·895	3·741	3·601	3·473	3·357	3·252
23 23	4·278	4·089	3·918	3·765	3·625	3·499	3·384	3·279
24 24	4·300	4·111	3·942	3·789	3·650	3·525	3·411	3·307
25 25	4·321	4·134	3·965	3·813	3·675	3·551	3·438	3·335
26 26	4·343	4·156	3·989	3·838	3·702	3·578	3·467	3·365
27 27	4·366	4·181	4·015	3 865	3·730	3·608	3·497	3·397
28 28	4·393	4·208	4·043	3·895	3·761	3·640	3·531	3·433
29 29	4·420	4·238	4·074	3·926	3·794	3·675	3·568	3·471
30 30	4·450	4·269	4·107	3·961	3·831	3·713	3·607	3·512
31 31	4·484	4·304	4·144	4·000	3·871	3·755	3·651	3·558
32 32	4·521	4·343	4·184	4·042	3·915	3·801	3·699	3·607
33 33	4·561	4·385	4·228	4·087	3·962	3·851	3·751	3·662
34 34	4·604	4·431	4·276	4·137	4·015	3·905	3·808	3·721
35 35	4 653	4·481	4·328	4·193	4·072	3·965	3·870	3·785
36 36	4·705	4·536	4·386	4·253	4·135	4·030	3·938	3·857
37 37	4·764	4·597	4·449	4·319	4·204	4·102	4·012	3·934
38 38	4·827	4·663	4·518	4·391	4·279	4·180	4·094	4·018
39 39	4·896	4·736	4·594	4·470	4·361	4·266	4·183	4·110
40 40	4·972	4·815	4·677	4·556	4·451	4·359	4·279	4·210
41 41	5·056	4·902	4·768	4·651	4·549	4·461	4·385	4·319
42 42	5·148	4·998	4·867	4·754	4·656	4·572	4·500	4·439
43 43	5·247	5·102	4·976	4·867	4·773	4·693	4·625	...
44 44	5·357	5·216	5·094	4·990	4·901	4·825	...	...
45 45	5·477	5·341	5·224	5·125	5·040	...	...	...
46 46	5·609	5·478	5·366	5·272	...	..	..	...
47 47	5·753	5·627	5·521	...	. .	...	..	...
48 48	5·910	5·791	...	...	...	...	...	...
49 49	6·083	...	...	...	...	...	...	...

Endowment Assurances for two Joint Lives (equal ages), Annual Premiums for 100, *payable in n Years or at First Death.—* H^M 3 *per-cent* (Text-Book).

Duration	29	30	31	32	33	34	35	36
Ages								
20 20	3·097	3·007	2·926	2·851	2·782	2·719	2·662	2·609
21 21	3·127	3·038	2·957	2·883	2·815	2·754	2·697	2·646
22 22	3·156	3·068	2·988	2·915	2·848	2·788	2·732	2·682
23 23	3·184	3·098	3·018	2·946	2·881	2·821	2·767	2·717
24 24	3·213	3·127	3·049	2·978	2·914	2·856	2·802	2·754
25 25	3·242	3·158	3·081	3·012	2·948	2·891	2·840	2·793
26 26	3·274	3·191	3·115	3·047	2·985	2·929	2·879	2 834
27 27	3·307	3·225	3·152	3·085	3·025	2·970	2·922	2·878
28 28	3·344	3·264	3·191	3·126	3·068	3·015	2·968	2·926
29 29	3·384	3·305	3·234	3·171	3·114	3·063	3·018	2·978
30 30	3·427	3 350	3·281	3·220	3·165	3·116	3·073	3·034
31 31	3·474	3·399	3·333	3·273	3·220	3·173	3·132	3·096
32 32	3·526	3·453	3·389	3·331	3·280	3·236	3·195	3·162
33 33	3·583	3·512	3·450	3·394	3·346	3·303	3·266	3·234
34 34	3·644	3·576	3·516	3·463	3·417	3·377	3·341	3 312
35 35	3·712	3·646	3·589	3·538	3·495	3·457	3·424	…
36 36	3·785	3·722	3·667	3·620	3·578	3·543	…	…
37 37	3·865	3·805	3·753	3·708	3·670	…	…	…
38 38	3·952	3·895	3·846	3·804	…	…	…	…
39 39	4·048	3·994	3·948	…	…	…	…	…
40 40	4·151	4·100	…	…	…	…	…	…
41 41	4·264	…	…	…	…	…	…	…

Duration	37	38	39	40	41	42	43	44
Ages								
20 20	2·561	2·518	2·478	2·442	2·409	2·379	2·353	2·329
21 21	2·599	2·556	2·517	2·482	2·450	2·422	2·396	2·374
22 22	2·636	2·594	2·556	2·522	2·492	2·465	2·440	2·419
23 23	2·673	2·632	2·596	2·563	2·534	2·508	2·485	2·464
24 24	2·711	2·672	2·637	2·605	2·578	2·553	2·531	2·512
25 25	2·751	2·713	2·680	2 650	2·623	2·600	2·579	2·562
26 26	2·793	2·757	2·725	2·697	2·672	2·650	2·631	2·614
27 27	2·839	2·805	2·774	2·747	2·724	2·703	2·686	…
28 28	2·889	2·856	2·827	2·802	2·780	2·761	…	…
29 29	2·943	2·912	2·884	2·861	2·840	…	…	…
30 30	3·001	2·971	2·946	2·924	…	…	…	…
31 31	3·064	3·036	3·013	…	…	…	…	…
32 32	3·133	3·107	.	…	…	…	…	…
33 33	3·206	…	…	…	…	…	…	…

Duration	45	46	47	48	49	50
Ages						
20 20	2·307	2·289	2·272	2·257	2·244	2·233
21 21	2·353	2·336	2·320	2·306	2·295	…
22 22	2·400	2·383	2·368	2·356	…	…
23 23	2·447	2·431	2·418	…	…	…
24 24	2·496	2·481	…	…	…	…
25 25	2·546	…	…	…	…	…