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CORRIGENDUM

From electrodiffusion theory to the electrohydrodynamics of leaky dielectrics through the weak electrolyte limit – CORRIGENDUM

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This corrigendum provides a brief derivation that yields corrections to the migration speed of the drop ((5.115) and (5.116)) in Mori & Young 2018). We conclude that the velocity of the droplet migration is proportional to $\phi^{\Delta}E$ for small Galvani potentials ϕ^{Δ}. The original conclusion, that this is not necessarily the case, is revised.

1. Corrections in Appendix B and Chapter 5

The corrected equation (B.26) reads

$$\mu\left(\frac{\partial^2\tilde{v}_1^i}{\partial\xi'^2}+\kappa_0\frac{\partial\tilde{v}_0^i}{\partial\xi'}+2g^{ik}h_{kj}\frac{\partial\tilde{v}_0^j}{\partial\xi'}\right)-g^{ij}\frac{\partial\tilde{p}_{-1}}{\partial\eta^j}-2\xi' h^{ij}\frac{\partial\tilde{p}_{-2}}{\partial\eta^j}$$
$$=\tilde{q}_0g^{ij}\frac{\partial\tilde{\phi}_1}{\partial\eta^j}+\tilde{q}_1g^{ij}\frac{\partial\tilde{\phi}_0}{\partial\eta^j}+\tilde{q}_0 2\xi' h^{ij}\frac{\partial\tilde{\phi}_0}{\partial\eta^j}. \tag{1.1}$$

The corrected equation (5.87) reads

$$\mu\left(\frac{\partial^2\tilde{v}_1^i}{\partial\xi'^2}+2\kappa_0\frac{\partial\tilde{v}_0^i}{\partial\xi'}\right)-g^{ij}\frac{\partial\tilde{p}_{-1}}{\partial\eta^j}-2\xi' h^{ij}\frac{\partial\tilde{p}_{-2}}{\partial\eta^j}$$
$$=\tilde{q}_0g^{ij}\frac{\partial\tilde{\phi}_1}{\partial\eta^j}+\tilde{q}_1g^{ij}\frac{\partial\tilde{\phi}_0}{\partial\eta^j}+\tilde{q}_0 2\xi' h^{ij}\frac{\partial\tilde{\phi}_0}{\partial\eta^j}. \tag{1.2}$$

The h^{ij}-terms result from the expansion of the Coulomb-force term $\hat{g}^{ij}\tilde{q}(\partial\tilde{\phi}/\partial\eta^j)$ using the basic expansions of $\tilde{q}$ and $\tilde{\phi}$ and the expansion of the metric

$$\hat{g}^{ij}\sim g^{ij}+2\delta\xi' h^{ij}+\delta^2\xi'^2\tilde{h}^{ij}+O(\delta^3). \tag{1.3}$$

Note that h^{ij} and $\tilde{h}^{ij}$ are not equal to the inverses of h_{ij} and $\tilde{h}_{ij}$ as defined in (B2-3) respectively.

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The expansion of the force term thus reads

$$\hat{g}^{ij}\tilde{q}\frac{\partial\tilde{\phi}}{\partial\eta^j} \sim g^{ij}\tilde{q}_0\frac{\partial\tilde{\phi}_0}{\partial\eta^j} + \delta\left(\tilde{q}_0 g^{ij}\frac{\partial\tilde{\phi}_1}{\partial\eta^j} + \tilde{q}_1 g^{ij}\frac{\partial\tilde{\phi}_0}{\partial\eta^j} + \tilde{q}_0 2\xi' h^{ij}\frac{\partial\tilde{\phi}_0}{\partial\eta^j}\right) + O(\delta^2). \tag{1.4}$$

With these corrections we find that p_{-2} does not depend on η, the term $\tilde{p}_{-2}/\partial\eta^j$ in (1.1) and (1.2) vanishes. The corrected equation (5.27) now reads

$$\left[2\mu\frac{\partial\tilde{u}_{-1}}{\partial\xi'} - \tilde{p}_{-2} - \tilde{q}_0\frac{\partial\tilde{\phi}_0}{\partial\xi'}\right] = 0. \tag{1.5}$$

As a result the corrected equation (5.32) becomes

$$p_+|_{\xi'=0+} = p_-|_{\xi'=0-}, \tag{1.6}$$

and the corrected equation (5.33) is

$$[p_{-2}] = 0. \tag{1.7}$$

Let us turn back to (1.2). We use (5.90) to find the corrected version of (5.91). Note, that in the original paper a sign mistake occurred when deriving equation (5.91) with (5.90)

$$\mu\left(\frac{\partial^2\tilde{v}_1^i}{\partial\xi'^2} + 2\kappa_0\frac{\partial\tilde{v}_0^i}{\partial\xi'}\right) = \tilde{q}_0 g^{ij}\frac{\partial\tilde{\phi}_1}{\partial\eta^j} + \tilde{q}_1 g^{ij}\frac{\partial\tilde{\phi}_0}{\partial\eta^j} + \tilde{q}_0 2\xi' h^{ij}\frac{\partial\tilde{\phi}_0}{\partial\eta^j} + g^{ij}\epsilon\frac{\partial^2\tilde{\phi}_1}{\partial\xi'\partial\eta^i}\frac{\partial\tilde{\phi}_0}{\partial\xi'}. \tag{1.8}$$

Using (5.10), (5.38) and (5.82), and exploiting $\partial^2\tilde{\phi}_0/(\partial\eta^j\partial\xi') = 0$, we obtain the correction of (5.92)

$$\mu\frac{\partial^2\tilde{v}_1^i}{\partial\xi'^2} = g^{ij}\epsilon\left(-\frac{\partial}{\partial\xi'}\left(\frac{\partial\tilde{\phi}_1}{\partial\xi'}\frac{\partial\tilde{\phi}_0}{\partial\eta^j}\right) - \frac{\partial}{\partial\xi'}\left(\frac{\partial\tilde{\phi}_0}{\partial\xi'}\frac{\partial\tilde{\phi}_1}{\partial\eta^j}\right) + \kappa_0\frac{\partial\tilde{\phi}_0}{\partial\xi'}\frac{\partial\tilde{\phi}_0}{\partial\eta^j} + 2\frac{\partial}{\partial\eta^j}\left(\frac{\partial\tilde{\phi}_1}{\partial\xi'}\frac{\partial\tilde{\phi}_0}{\partial\xi'}\right)\right) - \epsilon\frac{\partial\tilde{\phi}_0}{\partial\xi'^2}2\xi' h^{ij}\frac{\partial\tilde{\phi}_0}{\partial\eta^j}. \tag{1.9}$$

We integrate each of the six terms throughout the inner layer. The first four integrals were derived in the original paper. We here focus on the last two terms. The fifth integral is

$$\text{V:}\quad \int_{-\infty}^{\infty} g^{ij}\epsilon 2\frac{\partial}{\partial\eta^j}\left(\frac{\partial\tilde{\phi}_1}{\partial\xi'}\frac{\partial\tilde{\phi}_0}{\partial\xi'}\right)\mathrm{d}\xi' = 2g^{ij}\frac{\partial}{\partial\eta^j}\int_{-\infty}^{\infty}\epsilon\frac{\partial\tilde{\phi}_0}{\partial\xi'}\left(-\kappa_0\frac{\partial\psi_\kappa}{\partial\xi'} - J_0\frac{\partial\psi_J}{\partial\xi'} - \frac{J_0}{\Sigma}\right)\mathrm{d}\xi'$$
$$= 2g^{ij}\frac{\partial J_0}{\partial\eta^j}\left(\frac{\epsilon_{in}}{\sigma_{in}}\phi_{in}^{\Delta} + \frac{\epsilon_{ex}}{\sigma_{ex}}\phi_{ex}^{\Delta} - I_J\right). \tag{1.10}$$

Here, we have used decomposition (5.46). The last term is integrated as

$$\text{VI:}\quad \int_{-\infty}^{\infty} -\epsilon\frac{\partial\tilde{\phi}_0}{\partial\xi'^2}2\xi' h^{ij}\frac{\partial\tilde{\phi}_0}{\partial\eta^j}\,\mathrm{d}\xi' = -2h^{ij}\frac{\partial\tilde{\phi}_0}{\partial\eta^j}\left(\left[\epsilon\frac{\partial\tilde{\phi}_0}{\partial\xi'}\xi'\right]_{-\infty}^{\infty} - \int_{-\infty}^{\infty}\epsilon\frac{\partial\tilde{\phi}_0}{\partial\xi'}\,\mathrm{d}\xi'\right)$$
$$= -2h^{ij}\frac{\partial\tilde{\phi}_0}{\partial\eta^j}(\epsilon_{in}\phi_{in}^{\Delta} + \epsilon_{ex}\phi_{ex}^{\Delta}). \tag{1.11}$$

For a spherical particle, the geometric relation

$$2h^{ij} = -\kappa_0 g^{ij} \tag{1.12}$$

holds. Term VI cancels with the fourth integral. The effective shear stress jump (equivalent to 5.95) now reads

$$\left[\mu\left(\frac{\partial v_0^i}{\partial \xi}+g^{ij}\frac{\partial u_0}{\partial \eta^j}\right)\right]=\left(\frac{\epsilon_{in}}{\sigma_{in}}-\frac{\epsilon_{ex}}{\sigma_{ex}}\right)J_0 g^{ij}\frac{\partial \tilde{\phi}_0}{\partial \eta^j}-2\left(\frac{\epsilon_{in}}{\sigma_{in}}\phi_{in}^{\Delta}+\frac{\epsilon_{ex}}{\sigma_{ex}}\phi_{ex}^{\Delta}-I_J\right)g^{ij}\frac{\partial J_0}{\partial \eta^j}. \tag{1.13}$$

The term on the left-hand side is the jump in the hydrodynamic stress, the first term on the right-hand side the jump in the electric stress. Thus, the last term is the jump of the total stress.

2. Spherical drop under uniform electric field

The corrected equation (5.101) reads

$$[\Sigma_{r\theta}]=\left(\frac{3\sigma_{ex}E}{2\sigma_{ex}+\sigma_{in}}\right)^2\left(\epsilon_{in}-\epsilon_{ex}\frac{\sigma_{in}}{\sigma_{ex}}\right)\sin\theta\cos\theta + \frac{6\sigma_{ex}E}{2\sigma_{ex}+\sigma_{in}}\left(\epsilon_{in}\phi_{in}^{\Delta}+\epsilon_{ex}\frac{\sigma_{in}}{\sigma_{ex}}\phi_{ex}^{\Delta}-I_J\right)\sin\theta. \tag{2.1}$$

The corrected equation (5.113) reads

$$-6\mu_{in}A_1+6\mu_{ex}D_1=\frac{6\sigma_{ex}E}{2\sigma_{ex}+\sigma_{in}}\left(\epsilon_{in}\phi_{in}^{\Delta}+\epsilon_{ex}\frac{\sigma_{in}}{\sigma_{ex}}\phi_{ex}^{\Delta}-I_J\right). \tag{2.2}$$

The resulting migration velocity reads (cf. (5.115))

$$V_{mgr}=\frac{2(5\epsilon_{in}\sigma_{ex}\mu_{ex}\phi_{in}^{\Delta}+\epsilon_{ex}(3\mu_{in}\sigma_{ex}+2\mu_{ex}\sigma_{in})\phi_{ex}^{\Delta})}{\mu_{ex}(3\mu_{in}+2\mu_{ex})(2\sigma_{ex}+\sigma_{in})}E - \frac{4\mu_{ex}\sigma_{in}\sigma_{ex}I_J}{\mu_{ex}(3\mu_{in}+2\mu_{ex})(2\sigma_{ex}+\sigma_{in})}E. \tag{2.3}$$

The corrected equation (5.116) reads

$$V_{mgr}=\frac{2(5\epsilon_{in}\sigma_{ex}\mu_{ex}\zeta+\epsilon_{ex}(3\mu_{in}\sigma_{ex}+2\mu_{ex}\sigma_{in}))}{\mu_{ex}(3\mu_{in}+2\mu_{ex})(2\sigma_{ex}+\sigma_{in})(1+\zeta)}\phi^{\Delta}E+O((\phi^{\Delta})^2). \tag{2.4}$$

The term multiplying $\phi^{\Delta}E$ is positive. The migration velocity V_{mgr} is proportional to $\phi^{\Delta}E$.

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