The Latus Rectum of the Parabola $(ax + by)^2 + 2gx + 2fy + c = 0$.

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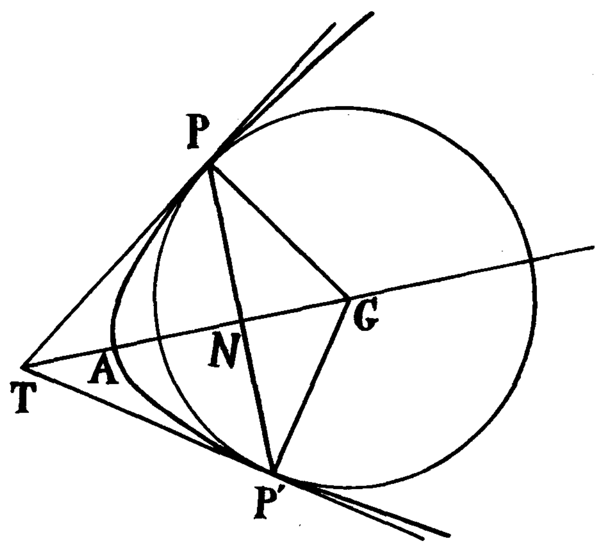
If a circle has double contact with a parabola, the points of contact being P and P', its centre, G, is the point of intersection of the axis with the normals at P and P', and the perpendicular from G to PP' is the subnormal GN, which is half the latus rectum.



Now the equation $(ax + by)^2 + 2gx + 2fy + c = 0$ can be written in the form

$$(a^2 + b^2)(x^2 + y^2) + 2gx + 2fy + c - (bx - ay)^2 = 0,$$

which shows that $(a^2 + b^2)(x^2 + y^2) + 2gx + 2fy + c = 0$ represents the circle which has double contact with the parabola so that the chord of contact $bx - ay = 0$ passes through the origin. The centre of the circle is the point $\left(\frac{-g}{a^2 + b^2}, \frac{-f}{a^2 + b^2}\right)$, and hence the length of the latus rectum of the parabola is $\pm 2 \frac{(bg - af)}{(a^2 + b^2)^{3/2}}$.

A similar method can be applied to the general equation in x, y, z, in the case where it represents a paraboloid of revolution, to obtain the latus rectum of the generating parabola.

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