BOOK REVIEWS

GRUENBERG, K. W. and WEIR, A. J., *Linear Geometry* (Van Nostrand, London, 1967), viii + 186 pp.

This is a carefully written algebraic exposition of the basic concepts of affine, projective and euclidean geometry. By omission of certain sections it can be used as a text on linear algebra.

A succinct survey of elementary vector space theory precedes the introduction of affine and projective geometries (Chapter 2). Isomorphisms of these geometries are discussed in Chapter 3. Linear mappings and bilinear forms are the topics of the next two chapters, including the introduction of quadrics and a brief mention of sesquilinear forms. The stage is then set for euclidean geometry. The final seventh chapter introduces modules, leading up to the classification of collineations.

The reviewer apologizes for the long delay in the appearance of this notice. During the time the book has been on his shelves he has found it a useful reference on a number of occasions. As a textbook for a course he feels it is rather stiff, at any rate for undergraduates. He confesses to being old-fashioned, and liking his geometry (linear or otherwise) with pictures. These the authors do not provide, arguing that diagrams can most profitably be drawn by the reader himself. Nevertheless the reviewer considers that an elementary traditional geometry course would be a necessary preliminary for many students. For those with such a background, or who already have substantial experience of the abstract approach, it should provide a rewarding course of study and introduction to more advanced work.

The compactness of presentation can be judged by taking the remark in the Preface that " the material can be adequately covered in about one hundred lectures " along with the observation that the main text occupies a mere 164 pages, of which the equivalent of perhaps 30 are taken up by the good supply of exercises! Fortunately the authors " do not expect it to be read in mathematical isolation ". Clearly the lecturer must supplement the text with further explanations and illustrations of the concepts. Following the main part of the book are 13 pages of solutions, a list of symbols and an excellent index. The book is available in both clothbound and paperback editions. It merits close study by those preparing systematic courses in the topics with which it deals.

D. MONK

FENYÖ, S. and FREY, T., *Modern Mathematical Methods in Technology*, Volume 1 (North-Holland 1969), xii + 403 pp., £8·20.

Contents: Extension of the classical concept of an integral, Lebesgue, Stieltjes integrals (67 pp.). The operational calculus (76 pp.). Fundamentals of distribution theory (134 pp.). Analysis of nonlinear differential equations. The theory of nonlinear vibrations (121 pp.). Numerous examples and exercises.

The basic problem in writing a book on mathematical methods is that of striking a proper balance between the mathematical development of the method and the explanation of the relationship between the method and its fields of application. In order to keep the size and the price of the book down it is not unknown for either proofs or applications to be excluded or relegated entirely to the limbo of an appendix. The present authors have taken a more laudable middle course and have included

both features within the text, possibly feeling that it is for the reader to decide for himself how he can best make use of the information.

The four main sections are essentially independent of each other, and each represents an excellent treatise on the topic in question. Perhaps the relationship between the largely alternative methods based on the operational calculus and the distribution theory approaches could have been discussed in more detail. Nearly all the proofs are given in their entirety, and are cast in an elementary, and sometimes rather lengthy form. However this does make less demands on the previous mathematical attainment of the reader, which is that to be expected of a fairly recent graduate in a scientific or engineering subject.

The book is a translation, with revisions, of the German edition of 1966. The standard of translation is very high, with only very infrequent lapses from an elegant style of English.

C. D. GREEN

COHEN, J. W., *The Single Server Queue* (North-Holland Publishing Company, Amsterdam, 1969), xiv + 653 pp.

This treatise deals thoroughly with an important area of applications of probability theory. The author, himself an active contributor to research in queueing over a number of years, has successfully drawn together a wide variety of complex problems in the field. In so doing, he has rendered us all a service: his book will provide a welcome guide in the proliferating literature of the subject.

The work consists of three parts. Part I is a 155-page review of topics in stochastic processes relevant to queueing theory. These include Markov chains in discrete and continuous time, birth and death processes, derived Markov chains (a topic originating with the author), renewal theory and regenerative processes, including elements of fluctuation theory.

In Part II, 205 pages are denoted to a detailed study of the single server queue. First the queueing model is developed, and Kendall's notation $-/-/-$ for the (Inter-arrival time distribution)/(Service time distribution)/(Number of servers), with independent inter-arrival and service times, is explained. The cases $M/M/1$, $G/M/1$, $M/G/1$, $G/G/1$, where M stands for a negative exponential and G for a general distribution, are considered in some detail. In every case, random variables such as the busy period, waiting time and queue length (among others) are discussed. Direct probabilistic and analytic methods are used for the first three queues; in the study of the $G/G/1$ queue, Pollaczek's integral equation is introduced and this is then used to investigate the $G/K_n/1$ and $K_m/G/1$ systems. Here, K_n denotes a distribution whose Laplace-Stieltjes transform is a rational function with denominator of degree n. Finally some special methods for the treatment of queueing problems are outlined: these include Lindley's integral equation method, the phase method for Erlang distributions of inter-arrival and service times, and Takács's combinatorial method.

Part III, the longest section consisting of 255 pages, deals with variants of the single server queue. A chapter is devoted to queues where the customers may arrive in groups and be served in batches, or where priority disciplines are in force. The queue for which actual waiting times are limited to a fixed maximum is considered. This is followed by a study of the queue with bounded virtual waiting time, or in storage terminology, the finite dam. The $M/G/1$ system with finite waiting room is then discussed; a final chapter on limit theorems for single server queues, including several for heavy traffic theory, concludes the book.

There follow a 7-page Appendix of useful mathematical theorems, some helpful explanatory notes on the literature, an impressive 9-page list of references on queueing, and finally author, notation and subject indices. Few misprints were noted.