

AN ATTEMPT TO DETERMINE THE STRUCTURE OF THE SOLAR CORE FROM OBSERVED G-MODE FREQUENCIES

A.G. Kosovichev
Crimean Astrophysical Observatory
of the USSR Academy of Sciences
Crimea
USSR

1. INTRODUCTION

The purpose of this investigation is to estimate how well the structure of the solar core might be inferred by inverting g-mode frequencies.

Delache and Scherrer (1983), Isaak *et al.* (1984), Severny *et al.* (1984) reported detection and identification of solar g modes. Although the results seem not to be complete it is helpful to invert the available data in order to understand what new information on the structure of the solar core can be obtained from g-mode frequencies.

2. INVERSION PROCEDURE.

Let $\omega_{0,1}, \omega_{0,2}, \dots, \omega_{0,n}$ be observed solar-oscillation frequencies which were initially identified as corresponding to g-mode frequencies $\omega_{c,1}, \omega_{c,2}, \dots, \omega_{c,n}$ of a solar model with a spherical-symmetric density stratification $\rho_0(r)$. Our purpose is to find a correction $\delta\rho(r)$ to the initial density distribution $\rho_0(r)$ such that the frequencies of new model ($\rho(r) = \rho_0(r) + \delta\rho(r)$) are close to the observed ones.

A system of linear integral equations for $\delta\rho(r)$ was found by Backus and Gilbert (1967) from a variational formulation of the eigenvalue problem

$$\frac{\delta\omega_j^2}{\omega_{c,j}^2} = \int_0^R S_j(r) \frac{\delta\rho}{\rho} dr \quad (1)$$

A formula for the differential kernel $S_j(r)$ used in this investigation was given by Kosovichev (1986). It is necessary to add to system (1) the condition of conservation of total mass

$$\int_0^R \rho r^2 \frac{\delta\rho}{\rho} dr = 0.$$

Based on equation (1) one can develop an iterative procedure in form of a generalized Newton-Raphson method to determine the density stratification more precisely. The first step of the procedure is $\delta\omega_j^2 = (\omega_{0,j})^2 - (\omega_{c,j})^2$.

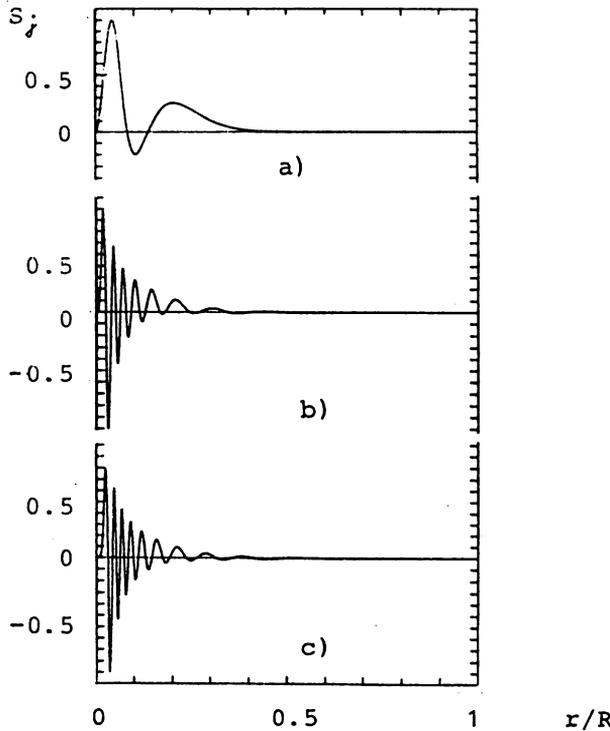


Figure 1. The differential kernels $S_j(r)$ for three g modes: a) $\ell=1, n=1$; b) $\ell=2, n=8$; c) $\ell=4, n=10$.

System (1) can be considered as a linear operator equation:

$$Az = u \tag{2}$$

where $z = \delta\rho/\rho, u = (u_1, \dots, u_n)$, and $u_j = \delta\omega_j^2/\omega_j^2$. As is well known the problem of finding z from (2) is ill-posed because its solution is not unique (Backus and Gilbert (1967), Gough (1984)). To solve equation (2) we have adopted Tikhonov's standard form of regularization theory (Tikhonov and Arsenin, 1977). The main idea is to restrict the class of admissible solutions using some a priori information. For the helioseismological inverse problem it is suitable to seek a solution which is continuous and provides closeness of the corrected solar model to the initial one. Under these assumptions there is a unique solution $z = z_\alpha$ of equation (2) minimizing Tikhonov's functional

$$M[z_\alpha, u] = \|Az_\alpha - u\|^2 + \alpha \|z_\alpha\|_{w_2}^2 \equiv \sum_{j=1}^n \left[\int_0^R S_j(r) z_\alpha dr \right]^2 + \alpha \int_0^R [z^2 + (z')^2] dr$$

where α is the so-called regularization parameter. Standard regularization

theory provides a technique for determining the best value of α . It can be found from equation

$$\|Az_\alpha - u\|^2 - [\delta_0 + \mu_0 \|z_\alpha\|_{w_2}]^2 = 0$$

where δ_0 is a measure of the relative errors in the data, and μ_0 is the error caused by the numerical computation of the integral operator A .

To estimate the accuracy of the regularized solution, the following procedure of a 'quasi-real' numerical experiment has been used. At first a vector $u_\alpha = (u_{\alpha,1}, \dots, u_{\alpha,n})$ is calculated by substituting the regularized solution z_α into equation (2). Then the values of $u_{\alpha,j}$ are perturbed by adding a noise $\xi[\delta_0]$ with amplitude δ_0 . A new regularized solution is found for the perturbed data $\tilde{u}_\alpha = u_\alpha[1 + \xi(\delta_0)]$. This procedure is repeated many times, and changes in the values of \tilde{z}_α give a measure of the accuracy.

3. RESULTS.

a) Crimean Data

Periods of solar g modes that were detected by Severny *et al.* (1984) are given in Table 1. Errors in the data are about 0.5 min.

Table 1. Crimean g-modes data.

Mode ℓ, n	Observed periods P_0 (min)	Calculated periods for model C P_c (min)	Differences $P_0 - P_c$ (min)
$\ell=4, n=10$	116.4	117.1	-0.7
$\ell=4, n=12$	134.3	133.4	+0.9
$\ell=4, n=13$	142.0	141.7	+0.3
$\ell=4, n=14$	150.0	149.9	+0.1
$\ell=4, n=15$	159.3	158.2	+1.1
$\ell=4, n=16$	166.9	166.5	+0.4
$\ell=4, n=17$	175.6	174.8	+0.8
$\ell=4, n=18$	183.5	183.1	+0.4
$\ell=4, n=19$	192.0	191.6	+0.4
$\ell=4, n=20$	200.0	200.3	-0.3
$\ell=2, n= 8$	160.0	161.4	-1.4

Figure 1 shows the differential kernels $S_j(r)$ for three g modes: a) $\ell=1, n=1$; b) $\ell=2, n=8$; c) $\ell=4, n=10$.

The result of the inversion of the data is shown on Figure 2. Only one iteration has been required in this case. A small-scale structure in the density stratification appears due to the fact that the observed periods do not fit the asymptotic relation for g modes (Kosovichev, 1986). However, the uncertainty of the result is too large to draw a final conclusion on the structure of the solar core.

b) Artificial Data.

One of the artificial sets of data considered is given by the theoretical eigenfrequencies of model C of Christensen-Dalsgaard *et al.* (1979). The data

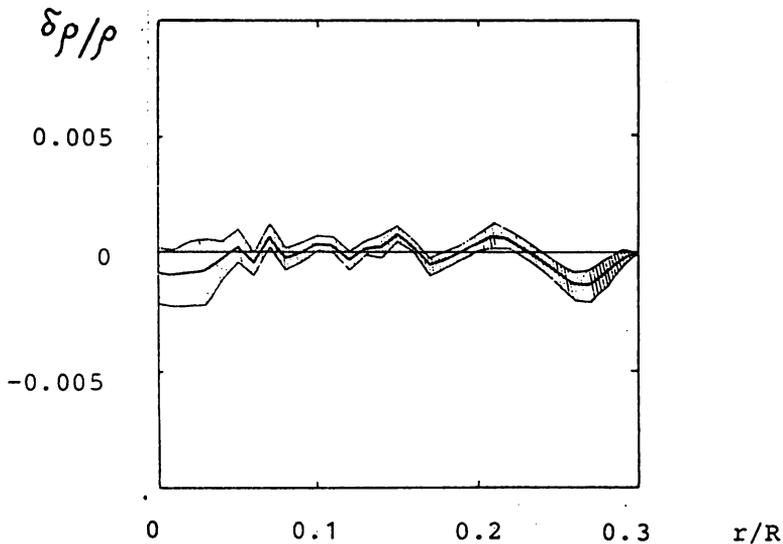


Figure 2. Results of the inversion of the Crimean g-mode data. The solid line shows the regularized solution of system (1), the shaded region is a result of the 'quasi-real' numerical experiment using errors with standard deviations of 0.5 min in the periods.

contain the 28 g modes mentioned by Severny *et al.* (1984).

The results of the inversion are shown in Figure 3. In this case a significant correction to the density stratification of the initial solar model has been obtained. In particular, the value of the density in the center of the corrected solar model coincides with the central density of model C of Christensen-Dalsgaard *et al.* (1979).

Figure 4 shows the density stratifications of our initial model and the one obtained as a result of the inversion. Our initial model has been obtained by recomputation of Christensen-Dalsgaard *et al.*'s (1979) model C. (Kosovichev and Severny, 1985).

CONCLUSION.

Useful information about the stratification of the solar core ($0 < r < 0.3 R$) can be obtained from inversions of g-modes frequencies (a range of periods 100-200 min) using Tikhonov's standard form of regularization theory. However, more reliable data of the g modes are required.

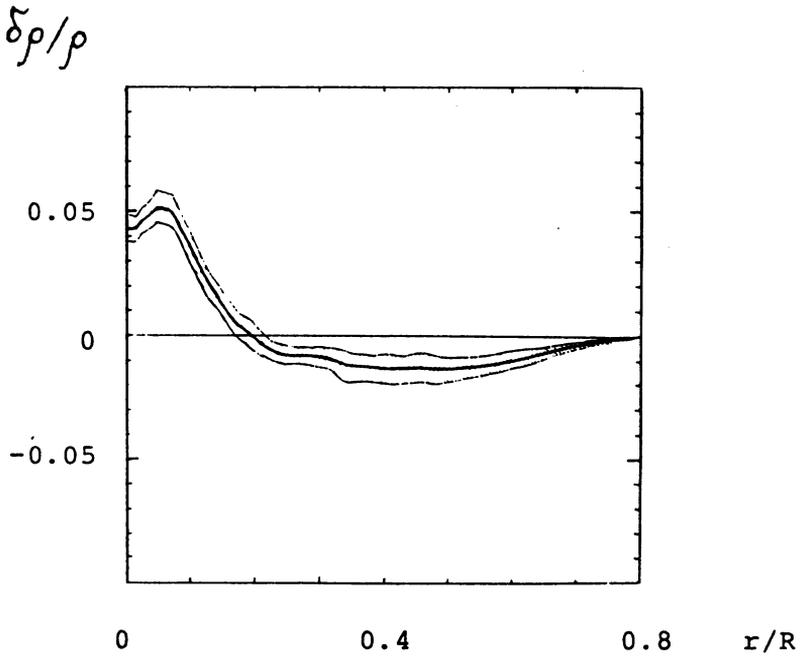


Figure 3. Results of the inversion of the artificial data.

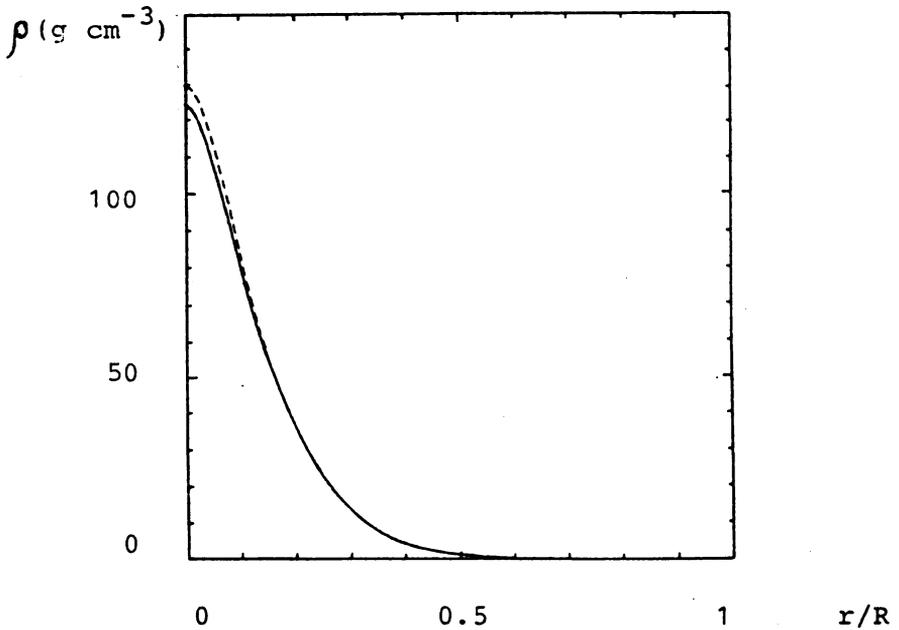


Figure 4. Density stratifications in the initial solar model (continuous line) and the model obtained as a result of the inversion of the artificial data (dashed line).

REFERENCES

- Backus, G. and Gilbert, F., 1967: *Geophys. J. R. astr. Soc.*, **13**, 247-276.
- Christensen-Dalsgaard, J., Gough, D.O. and Morgan, J.G., 1979: *Astr. Astroph.*, **73**, 121-128.
- Delache, P. and Scherrer, P.H., 1983: *Nature*, **306**, 651-653.
- Gough, D.O., 1984: *Solar Seismology from Space*, eds. R.K. Ulrich, J. Harvey, E.J. Rhodes, Jr., and J. Toomre (Pasadena: Jet Propulsion Laboratory), p. 49-78.
- Isaak, G.R., van der Raay, H.B., Palle, P.L., Roca Cortes, T., and Delache, P., 1984: *Mem. Soc. Astr. It.*, **55**, 91-97.
- Kosovichev, A.G. and Severny, A.B., 1985: *Izv. Krym. Astr. Obs.*, **72**, 188-198.
- Kosovichev, A.G., 1986: *Izv. Krym. Astr. Obs.*, **75**, 40-47.
- Severny, A.B., Kotov, V.A. and Tsap, T.T., 1984: *Nature*, **307**, 247-249.
- Tikhonov, A.N. and Arsenin, V.Y., 1977: *Solutions of Ill-Posed Problems*, (Winston, Washington, DC).