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ICMS Workshop on Birational Geometry held in honour of Vyacheslav V. Shokurov's 60th birthday

Edinburgh, December 2010

In December 2010, more than 70 mathematicians (three Fields medallists among them) came to Edinburgh to participate in a conference dedicated to the 60th birthday of Slava Shokurov. The conference was a great success. So we decided to publish the proceedings of the conference as a volume of the *Proceedings of the Edinburgh Mathematical Society*. Twenty-nine mathematicians contributed to this volume: Valery Alexeev, Fedor Bogomolov, Frederic Campana, Paolo Cascini, Fabrizio Catanese, Yifei Chen, Benoit Claudon, Sir Simon Donaldson, Baohua Fu, Angela Gibney, Frederick Greenleaf, Chris Hacon, Jun-Muk Hwang, Atanas Iliev, Ludmil Katzarkov, Yujiro Kawamata, János Kollár, Yuri Manin, James McKernan, Shigefumi Mori, Mircea Mustaţă, Slava Nikulin, Yuri Prokhorov, Victor Przyjalkowski, Dmitrijs Sacovics, Maxim Smirnov, David Swinarski, Claire Voisin and Yuri Zarhin. We briefly describe their contributions.

In the paper 'Higher-level $\mathfrak{sl}_2$ conformal blocks divisors on $M(0;n)$', Alexeev *et al.* study a family of semi-ample divisors on the moduli space of n-pointed genus 0 curves $\bar{M}_{0,n}$ given by higher-level $\mathfrak{sl}_2$ conformal blocks. They derive formulas for the intersections of these divisors with a basis of 1-cycles, show that these divisors form a basis for the S_n-invariant Picard group, and generate a full-dimensional subcone of the S_n-invariant nef cone.

The paper 'A constructive proof of Brauer's theorem on induced characters in the group ring $\mathcal{R}[G]$' by Bogomolov and Greenleaf contains a new proof of the classical Brauer theorem for finite groups. The theorem states that any complex representation of a finite group is a sum with integer coefficients (both positive and negative) of representations induced from the characters of elementary subgroups. The main idea of the proof is that the result for any finite group follows from the result for the symmetric groups S_n, and in the latter case the proof reduces to the representation of the trivial one-dimensional representation as a sum in $\mathcal{R}[G]$ with integer coefficients of the induced representations above. The latter is constructed using elementary properties of elements and Sylow subgroups of symmetric groups.

In the paper 'Abelianity conjecture for special compact Kähler 3-folds', Campana and Claudon prove that the fundamental group of a special Kähler 3-fold always contains a free abelian subgroup of a finite index. It is conjectured that the same result (and evidence is provided) holds for all *special* Kähler manifolds introduced by Campana, which are those such that any exterior power of their cotangent bundles does not contain a rank 1 subsheaf with a space of sections growing in the tensor powered as the degree of the exterior power.

The paper 'The augmented base locus in positive characteristic' by Cascini *et al*. studies the augmented base locus of a nef Cartier divisor on a projective scheme in positive characteristic. The authors prove a theorem similar to a result of Nakamaye for smooth projective varieties in characteristic 0. More precisely, they show that if L is a nef Cartier divisor on a projective scheme X in positive characteristic, then the augmented base locus of L is equal to the union of the irreducible closed subsets V of X such that the restriction of L to V is not big. The methods used are quite different from those of Nakamaye. The main point is that, in positive characteristic instead of vanishing theorems, one can use the Frobenius morphism.

In the paper 'Cayley forms and self-dual varieties', Catanese studies the so-called Cayley forms (tangential Cayley forms, respectively); these are polynomials in the Plücker coordinates of $G(1;3)$, whose zero set is the set of lines intersecting a given space curve C (the lines tangent to a given surface S, respectively). Catanese gives another characterization of such forms, in terms of self-duality, and completes the classical result of Cayley, obtaining a full set of equations for the variety of Cayley forms.

In the paper 'Minimal rational curves on complete toric manifolds and applications', Chen *et al*. study families of rational curves on toric manifolds. Manifolds with sufficiently negative canonical classes, like uniruled or Fano manifolds, have a lot of rational curves lying on them. These curves cover the whole variety. Starting with a rational curve passing through a fixed point, one can consider a reducible curve that is a union of the initial curve and a rational curve transversally intersecting this curve. Deforming this reducible curve, one can obtain another rational curve of higher degree. To exclude such non-minimal cases, it seems natural to introduce the notion of a minimal rational curve, i.e. a curve passing through a fixed point such that any deformation of this curve keeping the fixed point is irreducible. Chen *et al*. study minimal components in the normalized space of rational curves, i.e. components of minimal rational curves (of fixed anticanonical degree) on manifolds covering these varieties.

The paper 'b-stability and blow-ups' by Donaldson is related to the following well-known questions: which Fano manifolds admit Kähler–Einstein metrics and, more generally, which polarized manifolds admit metrics of constant scalar curvature? Donaldson's paper makes progress towards a proof that these metric conditions imply that the manifold must be *b-stable*, an algebro–geometric stability condition introduced earlier by Donaldson.

In the paper 'On the log canonical inversion of adjunction', Hacon proves a version of inversion of adjunction that generalizes a result of Kawakita to higher codimension log canonical centres. If (X, B) is a log canonical pair and V is a log canonical centre, then Hacon defines a b-divisor $\boldsymbol{B}$ on the normalization W of V, and defines a notion of log canonicity of the pair $(W, \boldsymbol{B})$. He then proves that (X, B) is log canonical near V if and only if $(W, \boldsymbol{B})$ is log canonical. If V is a prime divisor on X, then one recovers the result of Kawakita as a special case.

In the paper 'Double solids, categories and non-rationality', Iliev *et al*. suggest a new approach to questions of rationality of 3-folds based on category theory. The main goal of this paper is to construct a series of interesting examples where the proposed approach might work.

The paper 'Hodge theory on generalized normal crossing varieties' by Kawamata extends some of the results of Hodge theory for smooth varieties to the context of generalized simple normal crossing varieties. Kawamata constructs a cohomological mixed Hodge $\boldsymbol{Q}$-complex on each generalized simple normal crossing pair and generalizes it to the relative setting. He then proves a vanishing theorem of Kollár type. This theory can be applied to certain aspects of the abundance conjecture that appeared in Kawamata's earlier work.

The paper 'Semi-normal log centers and deformations of pairs' by Kollár studies the log centres of a log canonical pair (X, Δ), that is, the irreducible subvarieties Z of X whose minimal log discrepancies are strictly less than 1, e.g. a log canonical centre is a log centre. He proves that if the minimal log discrepancy of Z is close enough to 0, then Z behaves in quite a similar way to a log canonical centre. For example, if $Z_1, \dots, Z_m$ are log centres whose minimal log discrepancies are all less than $\frac{1}{6}$, then the union $\bigcup_{i=1}^{m} Z_i$ is semi-normal. This is surprising, since the number $\frac{1}{6}$ is independent of the dimension of X.

In the paper 'Towards motivic quantum cohomology of $\bar{M}_{0,S}$', Manin and Smirnov explicitly calculate some Gromov–Witten correspondences determined by maps of labelled curves of genus 0 to the moduli spaces of labeled curves of genus 0, which is the first step towards studying the self-referential nature of motivic quantum cohomology.

The paper '3-fold extremal contractions of types (IC) and (IIB)' by Mori and Prokhorov studies a germ (X, C) that consists of a 3-fold X with terminal singularities and an irreducible reduced complete curve C such that there exists a contraction $f\colon (X, C) \to (Z, o)$ with $C = f^{-1}(o)_{\mathrm{red}}$ and with $-K_X$ f-ample. Mori and Prokhorov wrote many papers about this topic. The paper in this volume deals with the case when (X, C) contains a point of type (IC) or (IIB), and completes the classification of such germs in terms of a general member $H \in |\mathcal{O}_X|$ containing C.

In the paper 'Elliptic fibrations on K3 surfaces', Nikulin reviews, and gives applications of, his earlier results on elliptic fibrations on K3 surfaces. He discusses the case when a K3 surface S has an elliptic fibration, in particular, one with an infinite automorphism group. The existence of non-zero exceptional elements of the Picard lattice with respect to the automorphism group of a K3 surface is discussed. Next, Nikulin addresses the question of how many elliptic fibrations, and also elliptic fibrations with infinite automorphism groups, can a K3 surface have. In particular, when the Picard number of S is at least 3, he shows that a K3 surface S has an infinite number of elliptic fibrations, and also an infinite number of elliptic fibrations with infinite automorphism groups, if it has one such fibration, except in some special cases.

In the paper 'Five-dimensional weakly exceptional quotient singularities', Sacovics classifies weakly exceptional quotient singularities of dimension 5. Weakly exceptional singularities were introduced by Shokurov in his famous paper on 3-fold log flips. In fact, he proved that weakly exceptional two-dimensional singularities are just quotient singularities of type $\mathbb{D}$ and $\mathbb{E}$. In dimensions 3 and 4, weakly exceptional quotient singularities have been classified earlier by Sacovics.

In the paper 'Approximately rationally or elliptically connected varieties', Voisin investigates the relation between Kobayashi (non-) hyperbolicity of a smooth complex projective algebraic variety and the properties of algebraic curves covering a small topological neighborhood in a given projective space. She observes, in particular, that an arbitrarily small neighborhood of a Kobayashi hyperbolic variety does not contain rational or elliptic curves. On the contrary, she proves that chain rational connectedness of small neighborhoods is a property satisfied by uniruled varieties X, so this does not imply that the Kobayashi pseudo-distance of X is 0. On the other hand, she shows that though a neighborhood of an abelian variety cannot be covered by rational curves, it can be covered by elliptic curves. Similarly, while most elliptic surfaces contain only finitely many rational curves, arbitrarily small neighborhoods of them in projective space are covered by rational curves.

The paper 'Theta groups and products of abelian and rational varieties' by Zarhin gives a negative answer to an old question by Vladimir Popov about groups of birational automorphisms of elliptic ruled surfaces. More precisely, a group G is said to be Jordan if there exists a constant J_G such that every finite subgroup of G contains a normal abelian subgroup whose index is at most J_G. The definition is inspired by the classical theorem of Jordan on general linear groups. Popov proved that the groups of birational automorphisms of almost all algebraic surfaces are Jordan. (The case of the projective space was done earlier by Serre.) Elliptic ruled surfaces are the only remaining case. Zarhin proves that, in the latter case, the group of birational automorphisms is not Jordan.

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Vyacheslav Vladimirovich Shokurov

Shokurov is an extremely innovative mathematician who has made fundamental contributions to algebraic geometry, in the tradition of the great Russian school of birational geometry. His work has more than once revolutionized birational geometry, introducing radical new insights, ideas, techniques and conjectures, many of which have pervaded the whole of algebraic geometry. On a personal level, Shokurov has a very generous nature and an engaging personality. He is always ready to discuss mathematics and to share his ideas. Not surprisingly, he is a very popular advisor and has had a number of graduate students and post docs.

Vyacheslav Vladimirovich Shokurov was born on 18 May 1950, in Moscow, Russia. He attended Moscow High School No 2, which has had many students who went on to distinguished careers, including a great many of the stars of Russian mathematics. Shokurov had the good fortune to attend a seminar at this school run by Vasily Iskovskikh. He began his university studies as an undergraduate at the Faculty of Mechanics and Mathematics of Moscow State University from 1967 to 1972. He stayed on at Moscow State University to work with Yuri Manin, from 1972 to 1975, where he obtained his PhD in 1976. He was awarded the degree of Doctor of Science in 1985 by the Steklov Mathematical Institute. His first job (1975–1982) was at the Institute of Technical and Scientific Information. He held a position at Yaroslavl State Pedagogical University from 1982 to 1990. He spent one year (1990–1991) at the Institute for Advanced Studies in Princeton, moving to Johns Hopkins University in 1991, where he is currently a full professor. He is also a non-tenured member of the Steklov Mathematical Institute in Moscow, where he spends most of the summer and part of the winter break. He was an ICM speaker in 1986, he is an editor of the American Journal of Mathematics, and he was one of the organizers of several very successful JAMI conferences in algebraic geometry held at Johns Hopkins University.

It was clear right from the beginning of his career that Shokurov was quite exceptional. While still an undergraduate, he gave a new proof of a stronger version of the celebrated Noether–Enriques–Petri theorem: he proved that the scheme cut out by those quadrics that contain the canonical curve is always reduced. His PhD thesis concerned the study of modular forms on Kuga varieties and he continued this line of research in a series of papers that have remained influential.

After receiving his PhD, Shokurov started to work more closely with Iskovskikh and he began the work on birational geometry for which he is best known. His first major papers in this area gave a proof that the anticanonical linear system of a smooth Fano threefold contains a smooth surface, as well as a complete classification of which smooth Fano threefolds contain lines. Together with the work of Fano, Iskovskikh, and later results of Mori and Mukai, this led to a complete classification of smooth Fano threefolds.

He also worked on rationality problems for conic bundles over rational surfaces. He gave a characterization of which Prym varieties are Jacobians of curves, following work of Beauville and Mumford, by showing that a Prym variety of dimension at least 7 is the Jacobian of a curve if and only if it is either hyperelliptic, trigonal or a curve with a single node whose normalization is hyperelliptic.

His work in birational geometry has focused on the minimal model program. Perhaps his most famous result is the non-vanishing theorem, which is a pivotal result in the subject: it is one of the key steps in proving the base point free and cone theorems which are now central to higher-dimensional geometry. He introduced the notion of the 'difficulty' and, using this, he showed that any sequence of threefold flips must terminate. In a groundbreaking paper he proved the existence of threefold log flips, and at the same time introduced some key conjectures and techniques. The work contained in this paper was the focus of a summer school organized at the University of Utah, in 1991, and resulted in the book *Flips and Abundance*. Over the last seventeen years, he has released a number of short papers on birational geometry, with the collective title 'Letters of a birationalist', which, together with his other papers, contain some fascinating conjectures and results that have motivated the subject ever since: connectedness for the non-kawamata log terminal locus, the ascending chain condition for the log canonical threshold and for the log discrepancy, etc.

Ten years after his paper on threefold log flips, Shokurov proved the existence of fourfold log flips and succeeded in giving a particularly simple and revealing proof of the existence of threefold log flips. This work was the subject of a month-long seminar at the Isaac Newton Institute in Cambridge, in 2002 (which resulted in the book *Flips for 3-folds and 4-folds*), and has had a profound influence on subsequent developments that continue to this day.

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